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THE

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THE PROBABLE VALUE OF THE CONSTANT OF ABERRATION,

BY S. C. CHANDLER.

The general history of our knowledge of the value of the constant of aberration is familiar to astronomers. It has had some singular vicissitudes. A half a century ago this element was supposed to have been fixed by STRUVE with an accuracy which almost discouraged further investigation of it. NYRÉN, in 1883, thoroughly unsettled this belief; and some ten years later, after the discovery of the inconstancy in the position of the pole — and especially after DOOLITTLE's striking results had begun to inspire still greater distrust of STRUVE's value — all illusion of security vanished, the question was thrown wide open, and there was a renewal of active general investigation. The premature attempt of the Almanacs in 1896 to legislate upon the matter fortunately did not stifle, or even perceptibly retard this inquiry; and it is now manifest that the conventional correction to STRUVE's value then hastily adopted is entirely insufficient, being not over a third of the correction really required. This is not a matter of mere individual opinion, but is, I think, the conclusion to which any astronomer must arrive upon examination of the very extensive material now available. For whatever divergencies of opinion there may be as to the relative weights of the various determinations, the number of these is now so considerable that even wide differences of judgement as to these weights can exercise little influence on the mean result, which comes out about the same however the weights are disposed; namely, somewhere between 20″.52 and 20″.53. Moreover, the body of existing evidence is now so large that the addition of new determinations, as they from time to time appear in print, influences the mean result of all in a scarcely perceptible degree. So that it would seem that the time has nearly or quite arrived when a conventional value generally acceptable to astronomers can be decided upon, if desirable.

During the past ten years the writer has made the scrutiny of all existing evidence bearing on the aberration-constant a matter of much care. To present the details of this revision, with a full statement of the reasons for the exclusion of many determinations and for the modification of the printed results for others, as well as the method of assigning weights, would unduly extend this paper. Nevertheless some particulars on these points should be given, if only as a guide for others who may desire to review the ground for themselves, and exercise an independent judgement of the correctness of the present procedure, and how far differences of opinion as to details may affect the conclusions here reached as to the real value of the constant.

First, a list is given of the determinations, made from zenith-distances measured with meridian instruments, which I have not thought it allowable to employ in finding the mean value of this constant.

BESSEL (*Fund.*, p. 123); 20″.797: and PETERS (*Recherches*); 20″.522: from BRADLEY's zenith-sector observations of γ *Draconis*, 1754–55, at Greenwich.

AUWERS (*Monatsb., Berlin Ak.*, 1869); 20″.385: from MOLYNEUX's zenith-tube observations of γ *Draconis*, 1725–27, at Kew.

AUWERS (*Ibid.*); 20″.460: from BRADLEY's zenith-sector observations of thirteen stars, 1727–29, at Wanstead.

BUSCH (Oxford Memoir, 1838); from the same series as above, Kew, 20″.250, Wanstead, 20″.205.

BRINKLEY (Phil. Trans., 1821); 20″.372: from his observations with 8-ft. vertical circle at Dublin.

RICHARDSON (Mem. R. A. S., 1828); 20″.503: Greenwich Mural Circle.

HENDERSON (Mem. R.A.S., 1839); 20″.41: from zen. dist. of *Sirius* at Cape G. H.

HENDERSON (*Ibid.*, 1842); 20″.523: from altitudes α *Centauri* at Cape G. H.

MACLEAR (*Ibid.*, 1851); 20″.531: from altitudes α *Centauri* at Cape G. H.

MACLEAR (*Ibid.*, 1852); 20″.594: from altitudes β *Centauri* at Cape G. H.

MAIN (*Ibid.*, 1859–60); 20″.335: from Greenwich Reflex Zenith-Tube, 1852–59.

DOWNING (*Mon. Not.* XLII); 20″.378: from Greenwich Reflex Zenith-Tube, 1857–75.

THACKERAY (*Mon. Not.*, LIX, p. 350); 20″.67 U.C., 20″.30 L.C., obsns. of *Polaris*, Greenwich, 1851–93.

The principal reason for the exclusion of most of the above series is that the aberration is inherently and necessarily indeterminate from them, irrespective of the quality of the observations themselves; as I have shown in various places; see *A.J.* 287, p. 178; *A.J.* 427, p. 151; *Mon. Not.*, LXII, p. 122.

Secondly, I give a list of determinations from right-ascension observations which I have not employed.

BESSEL (*Fund.*, p. 118–121); 20″.643 from *Sirius*, *Procyon*, *Vega* and *Altair*; 20″.755 from *Polaris*, BRADLEY's observations.

STRUVE (Dorpat, 1822); 20″.349; and PETERS (*Recherches*); 20″.361: differences R.A. of six circumpolars observed by STRUVE.

PETERS (*Num. Const.*, p. 56); 20″.425, from STRUVE's and PREUSS's Dorpat obsns. of *Polaris*, 1822–38.

LINDENAU (*Abhandl.*, *Berlin Ak.*, 1841); 20″.449: R.A. of *Polaris* at various observatories, 1750–1816.

NEWCOMB (*Astron. Const.*, p. 138); 20″.55: R.A. of *Polaris*, Washington, 1866–67.

NEWCOMB (*Ibid.*, p. 139); 20″.39: R.A. of circumpolars, Greenwich.

THACKERAY (*Mon. Not.*, LIX, p. 347); 20″.42: R.A., *Polaris*, Greenwich, 1851–93.

The want of control of systematic diurnal changes in the instrument is the source of want of confidence in results for aberration found from right-ascensions. Whether this should go to the extent of exclusion of such results is of course a matter of opinion; although I presume no one would be inclined to give them anything but a small weight. At any rate the only series I have retained are those with the Pulkowa Transit, as hereinafter given.

Thirdly, the determinations by means of prime-vertical transits which have not been used are the two following:

HALL (*A.J.* 169, p. 1); NEWCOMB (*A.J.* 263): Washington, 1862–67.

WANACH (*A.N.* 3092); Pulkowa, 1890–91.

The reason, in the first case, has been already referred to (*A.J.* 287, p. 178); and in the second, is found in WANACH's own remarks thereupon.

Finally, of the determinations obtained by TALCOTT's method, the only ones not included are those for Waikiki (see *A.J.* 517, 518, 519), and Bethlehem for the year 1890.

It will thus be seen that the number of the existing determinations of the aberration here excluded is twenty-four as against the forty-three employed, hereinafter given. Such a wholesale rejection of so large a proportion of observation, numerically considered, may on its face seem startling, arbitrary, and possibly indefensible; and to require for its justification a specific statement, in each instance, of the grounds therefor. It has not been done carelessly, however, but at the cost of labor of intimate examination quite as great as has been expended on those determinations which have led to positive results, and which have been actually employed in obtaining my final mean of this constant. I am sometimes inclined to suspect that some former collocations of the material for the same purpose have been perhaps too cursorily made. Whether the dimensions of the limbo which I have constructed are too ample is a question that can not be decided off-hand; and I feel justified in throwing the burden of proof upon those who are inclined to think that a higher weight than zero should have been accorded to some of the values that have here been left out of the account. I will only take space for the general remark that the constant with which we are dealing is one in which known sources of constant error exist that tend to operate in one direction, and that the means are not now at hand for applying corrections for them with any degree of confidence; so that the safer road lies in the direction of exclusion of results in which such vitiation is to be feared, rather than towards seeking to enlarge the number of available results, in the hope of balancing errors.

We now come to the determinations employed in this investigation. These are given below, classified by the methods of observation, with such details as may be useful as to the data on which they rest, so far as they can be supplied. In the last column are the weights that I have been led to assign, according to the best judgement which I could exercise of all the circumstances bearing thereupon, among which the quoted probable errors are of course only subordinate. This was naturally a most delicate and difficult part of the task, as it always is in investigations of this sort at the point where computation has to be abandoned and mere estimate resorted to of the relative preponderance which should be allowed to the various collateral circumstances to be brought to this point of view. It is here that criticism finds a wide gateway to enter, and any dispute that may arise can with difficulty be rationally settled. Consequently it is fortunate that in the present case such differences of opinion can have little effect; for it will be found from experiment that very considerable departures, within reason, may be made from the relative weights here used without perceptible influence on the final mean.

Talcott's Method.

Observer	Station	Date	Groups	Pairs	Aberr'n	p.e.	Reference	Wt.
Küstner	Berlin	84.5–85.5	–	244	20.611	–	A.J. 429, 430, 433	1
Marcuse	Berlin	89.1–90.7	IX	1762	.490	±0.012	A.N. 3015	3
Marcuse	Berlin	91.1–93.0	IX	2831	.507	± .011	B.d. *Int. Erdm.*, 1899	6
Ristenpart	Karlsruhe	92.8–94.6	IV	1978	.499	–	VJS. 29	5
Weinek and Gruss	Prague	89.2–92.4	IX	3062	.504	± .027	A.J. 520	6
Becker	Strassburg	91.4–94.3	VII	386	.475	± .012	VJS. 30	2
Davidson	San Francisco	91.4–92.6	VIII	6768	.555	± .021	A.J. 520	10
Gill and Fearnley	Cape G. H.	92.1–94.2	VI	621	.575	± .008	Mon. Not., LVIII	4
Fergola	Naples	93.4–94.5	IV	2271	.533	–	Naples Acad., 1897	8
Schnauder and Hecker	Potsdam	93.9–98.0	X	4400	.498	± .016	A.J. 520	10
Doberck	Hongkong	–	–	–	.477	± .040	A.N. 3504	2
Stein	Leyden	99.4–00.5	X	1590	.541	± .011	Comm. géod. Néerl.	3
Kowalski	Kasan	92.4–93.5	IX	1223	.539	± .023	A.J. 520	3
Gratchew and Trozki	Kasan	93.6–95.0	IX	1684	.522	± .022	A.J. 520	3
Gratchew	Kasan	95.1–97.5	IX	2819	.594	± .018	A.J. 520	6
Rees, Jacoby and Davis,	New York	93.3–94.5	IV	1774	.457	± .013	A.J. 401	6
Rees and Davis	New York	94.5–96.0	IV	1081	.453	± .013	A.J. 451	5
Rees and Davis	New York	96.0–98.0	IV	1839	.469	± .011	A.J. 451	7
Rees and Davis	New York	98.0–00.0	IV	1824	.470	± .011	A.J. 474	7
Doolittle	Bethlehem	92.8–94.0	XI	2796	.551	± .009	Pub. Sayre Obs. 1902	4
Doolittle	Bethlehem	94.1–95.6	IV	2690	.537	–	Pub. Sayre Obs. 1901	7
Doolittle	Philadelphia	96.8–98.6	IV	3213	.580	± .009	A.J. 453, 490	10
Doolittle	Philadelphia	98.7–99.9	IV	1919	.540	± .010	A.J. 490, U.P. Pub. II	8
Doolittle	Philadelphia	00.4–01.7	IV	2657	.561	±0.008	A.J. 509, U.P. Pub. II	9
Intern'l Service	Six stations	99.8–02.0	XII	20302	20.512	–	A.N. 3808	16
			Mean		20.523	±0.005		151

Prime Vertical Transits.

Observer	Place	Date	Stars	Obsns.	Aberr'n	p.e.	Reference	Wt.
Struve	Pulkowa	40.3–42.8	7	285	20.50 :	–	A.J. 296, improved	3
Nyrén	Pulkowa	75.5–79.0	4	246	.547	–	A.J. 297, improved	6
Nyrén	Pulkowa	79.9–82.1	24	566	.521	–	A.J. 297, improved	15
			Mean		20.525			24

Meridian Zenith-Distances.

Pond	Greenwich	1812–19	*Pol.*	–	20.578	±0.043	A.J. 520	1
Pond	Greenwich	1825–36	8	2629	.512	± .019	A.J. 515	3
Struve and Preuss	Dorpat	1822–38	*Pol.*	1144	.551	± .043	Lundahl, 1842	2
Peters	Pulkowa	1842–44	*Pol.*	346	.510	± .021	A.J. 287	4
Peters	Pulkowa	1842–43	7	416	.467	–	A.J. 293	1
Gyldén	Pulkowa	1863–70	*Pol.*	195	.411 .469	–	A.J. 293, 298	3
Gyldén and Nyrén	Pulkowa	1863–73	10	182	.520	–	A.J. 298	1
Nyrén	Pulkowa	1871–75	*Pol.*	163	.505	–	A.J. 293	4
Becker	Strassburg	1885–90	*Pol.*	558	.577	± .033	Ann. Strassb. Obs. I	2
Hall	Ann Arbor	1898–00	*Pol.*	290	.68	± .03	A.J. 518	1
			Mean		20.514			22

Right-Ascensions.

Schweizer	Pulkowa	1842–44	*Pol.*	396	20.56?	–	A.J. 444, 462, A.N. 3562	2
Wagner (E & E)	Pulkowa	1861–72	*Pol.*	439	.50?	–	" " " "	2
Wagner (Chron.)	Pulkowa	1861–72	*Pol.*	429	.52?	–	" " " "	2
			Mean		20.53 :			6

PRISMATIC APPARATUS.

Observer	Place	Date	Pairs	Obsns.	Aberr'n	p.e.	Reference	
Loewy and Puiseux	Paris	1890–91	2	109	20.447	±0.024	Compt. Rend. CXII	1
Comstock and Flint	Madison	1890–92	20	752 :	.443 / .499	±0.010	Publ. Washb. Obs. IX	4
				Mean	20.48 :			5

The double result quoted for the last (COMSTOCK's) determination depends on the way the peculiar systematic personal difference is treated. An independent discussion of these observations has led me to the value 20″.49. The double result quoted for the *Polaris* vertical-circle observations of *Polaris*, 1863–70, arises from the curious contradiction between NYRÉN's and my solutions, one of which I think must be affected with some purely numerical error.

Collecting and combining the results of the foregoing tables we have the following summary:

	No. Det.	Aberration	Wt.
Talcott's method,	25	20.523	151
Prime-vertical transits,	3	.525	24
Meridian zenith-distances,	10	.514	22
Right-ascensions,	3	.53 :	6
Prismatic apparatus,	2	.48 :	5
General mean,	43	20.521	208
Probable error,		± .005	

The brute mean of the forty-three individual results is 20″.523, differing only 0″.002 from the above weighted mean; the brute mean of the twenty-five results by TALCOTT's Method is 20″.522, differing but 0.001 from the above weighted mean; and the brute mean of the ten results by Meridian Zenith-Distances is 20″.534, which is 0″.020 greater than the weighted mean.

Various experiments with the weights varied within reasonable limits all give means between 20″.522 and 20″.526. In short, after considering the data in various ways, the impression on my own mind is strong that the real value of this much disputed constant is likely to be found near or slightly above 20″.52.

The above weighted mean, 20″.521, which may be regarded as the result of this investigation, corresponds to the solar parallax 8″.781.

It is interesting and apposite to remark that the discordance which formerly appeared to exist between the values of the aberration afforded by the three principal methods on which we must rely for its determination — namely, TALCOTT's Method, Prime Vertical Transits and Meridian Zenith-Distances — is now dissipated; and that the reasonably distrusted method of Right-Ascensions may, in at least a plausible way, be also brought into harmony with the others.

Nor can I neglect to emphasize the importance of the stride forward in our knowledge of this constant due to the suggestion and development by KÜSTNER of his method, which has enriched our astronomy with a means of investigating the aberration less susceptible than any other to the disturbing influence of systematic error arising from known or imaginable causes, and which has been so prolific of result in its application.

From the point of view of the principles above adopted in the use of the data this investigation should stop here, and the task of demonstrating that improvement can be expected by utilizing some or all of the material here excluded should be left to those who would advocate that view. Nevertheless it seems appropriate and desirable to show how much the final result would have been affected by incorporating the determinations here excluded. I therefore give in the following tabulation, which is arranged in the same manner as the one already presented, the results of the twenty-four excluded series. They are used as given by the authors (rounded only to the nearest hundredth) without any attempt to apply corrections for variations of latitude, with which many of them are necessarily affected. An attempt has been made to assign relative weights such as I suppose would not be materially gainsaid by those who might be inclined to favor the employment of these determinations in their definitive mean. We thus have:

TALCOTT'S METHOD.

	Aberr'n	Wt.
Waikiki,	20.43	½
Bethlehem, 1890,	.45	½
Mean,	20.44	1

PRIME VERTICAL TRANSITS.

	Aberr'n	Wt.
Washington, 1862–67,	20.46	1
Wanach,	.40	1
Mean,	20.43	2

MERIDIAN ZENITH-DISTANCES.

	Aberr'n	Wt.
Peters (Bradley, 1754–55),	20.52	¼
Auwers (Kew),	.38	¼
Auwers (Wanstead),	.46	¼
Busch (same),	–	0
Brinkley,	.37	¼
Richardson,	.50	1
Henderson (*Sirius*),	.41	0
Henderson (α *Cent.*),	.52	¼
Maclear (α *Cent.*),	.53	¼
Maclear (β *Cent.*),	.59	¼
Main,	.34	¼
Downing,	.38	1
Thackeray,	.49	1
Mean,	20.46	5

RIGHT-ASCENSIONS.

	Aberr'n	Wt.
Bessel (Bradley),	20″.70	¼
Peters (Dorpat, 6 st.),	.36	½
Peters (Dorpat, *Pol.*),	.42	1
Lindenau,	.45	¼
Newcomb (Wash.),	.55	1
Newcomb (Greenw.),	.39	1
Thackeray,	.42	1
Mean,	20.45	5

Whence we get the following summary of these excluded determinations:

	No. Det.	Aberr'n	Wt.
Talcott's Method,	2	20″.44	1
Prime-vertical transits,	2	.43	2
Meridian Zenith-Distances,	13	.46	5
Right-ascensions,	7	.45	5
General mean,	24	20.450	13
The brute mean of the 24 values is		20.457	

Combining this result with the one previously found from forty-three accepted determinations we find:

	No. Det.	Aberr'n	Wt.
Mean from accepted values,	43	20″.521	208
Mean from rejected values,	24	.450	13
General mean,	67	20.517	221

It thus appears that without excluding any of the data we should have a value (20″.517) only ~~0″.04~~ smaller than 0″.004 the one (20″.521) which has been derived as the definitive result of this investigation, by the exercise of a critical choice which it seems to me is demanded by the circumstances for the purpose of arriving at the most acceptable result. And here again it will be manifest upon trial that wide differences of opinion as to weights can be allowed full play without significant influence on the conclusion.

From the best point of view I can compass, it seems to me that we are driven by the facts either to 20″.52 if a rounded decimal is preferred for a conventional value of this constant, or to something slightly above it, if for any reason it is deemed that any slight practical advantage, or a possible approach to accuracy, is gained by splitting the hundredth.

The reduction to such a value of the aberration-constant can be effected by the addition of 0.0016 to the values of $\log C$ and $\log D$, or of $\log h$ and $\log i$, in the ephemerides where STRUVE's constant (20″.445) is employed in the tables for reduction to apparent place; or of 0.0011 in those where the value of the Paris Conference of 1896 (20″.47) is the basis.

NOTES ON SOME RECENTLY DISCOVERED VARIABLE STARS,

By A. STANLEY WILLIAMS.

The observations upon which the following notes are based were chiefly made with a 6½-inch reflector, though occasionally a 2¾-inch refractor was used. The results are necessarily at present of a somewhat provisional character, though as the observations have been carefully reduced it is not probable that any material alteration will have to be made in the stated times of maxima and minima. Light-scales were formed for most of the variables, either by means of the comparisons between the variable and the comparison-stars, or by means of sequences of steps specially made for that purpose. It should be added that cloud and fog have been abnormally prevalent during the past year, rendering it difficult to secure satisfactory and uninterrupted series of observations of *Algol*-type or very rapid variables.

562. *Y Andromedae.*

The following are approximate photographic magnitudes of this star. 1899 Nov. 10, not visible, $<11^{\text{M}}.5$; 1900 Dec. 19, not visible, $<12^{\text{M}}$; Dec. 21, not visible, $<12^{\text{M}}$; 1901 Jan. 15, not visible, $<12^{\text{M}}$; Dec. 10, 10^{M}; Dec. 18, 10^{M}; 1902 Jan. 30, $11\frac{1}{2}^{\text{M}}$.

1205. *Y Persei.*

Observations on 45 nights between 1901 Mar. 12 and 1902 Dec. 2 yield the following maxima and minima:

	Date	J.D.	Mag.
Maximum	1901 April 15	241 5490	8.8
Minimum	Aug. 28	5625	9.6
Maximum	Dec. 21	5740	8.7
Minimum	1902 April 23	5863	9.8
Maximum	Sept. 12	6005	8.3

Dr. HARTWIG in 1901 found the period to be 236 days, but one slightly longer than this (251 days) better satisfies the above observations. The light variations are, however, by no means quite regular, and there are considerable differences in the form of the light-curve at different maxima. Thus, the curves of the first two maxima are rather flat, particularly that of the second one, whilst that of the third is comparatively sharply accentuated. The magnitudes given above are practically according to the scale of HAGEN's Second Chart and Catalogue for observing *Nova Persei*, but it should be mentioned that red stars usually appear decidedly fainter to me than they do to most observers, particularly as regards observations made

with the 2¾-inch refractor. The first two maxima and the two minima were observed with this instrument, and the third maximum with the 6½-inch reflector. Several observers have published observations made about the time of the first maximum, according to which the star must then have been about 8ᴹ.3. That is half a magnitude brighter than my observations make it. On four nights in 1901 both photographic and visual observations were made, the differences, visual — photographic, being —1.8, —1.8, —1.5 and —1.4. The mean difference is —1ᴹ.6, and the photographic observations published in the *A.N.* 3698 agree very well with the later visual ones made with the 2¾-inch refractor when corrected by this amount; though in order to make them comparable with the reflector observations, and those of most other observers, the correction should be a full two magnitudes.

6685. *Y Lyrae.*

Four good series of observations of this rapid variable were secured in 1902. Below will be found the observed heliocentric Greenwich mean times of maximum, and the like observed times of T_0. The times T_0 are the times when the variable in its rapid rise from minimum attains to equality with the comparison-star 1 (12ᴹ.1)*. The computed times according to the elements in the *Monthly Notices*, Vol. 62, p. 208, have been added for the purpose of comparison, together with the resulting residuals. The latter are not large, and it hardly seems necessary at present to attempt any revision of the elements.

Date	Computed T_0	Observed T_0	C — O
	h m	h m	m
1902 June 2	11 5.9	11 15.7	— 9.8
Aug. 22	9 29.0	9 32.2	— 3.2†
Sept. 7	11 32.8	11 50.6	—17.8
8	11 40.6	11 53.5	—12.9

Date	Computed maximum	Observed maximum	C — O
	h m	h m	m
1902 June 2	12 5.9	12 9.7	— 3.8
Aug. 22	10 29.0	10 44.2	—15.2
Sept. 7	12 32.8	12 44.6	—11.8
8	12 40.6	12 52.5	—11.9

6816. *Z Lyrae.*

In 1901 observations were made on 29 nights between May 21 and Oct. 6, and these indicate a pretty well defined maximum for July 1, mag. = 9.4 (equal to DM. +34°3385). Hartwig, from his own observations, makes it June 20 (see *A.N.* 3744, col. 370). In 1902 the star's brightness decreased from 11ᴹ.2 on May 27 to invisibility in a 6½-inch reflector (or below about 13ᴹ) after July 25. The star was re-observed on Oct. 26, mag. = 12.3. By means of the light-curve of 1901 the date of maximum may be fixed for 1902 April 7±. Some photographic observations of 1900 and 1899 were published in the *A.N.* 3671. Those for 1900, with the help of the visual light-curve of 1901, fix the time of maximum pretty exactly for 1900 Sept. 2. The last two observations of 1899, with the help of the visual light-curve of 1901 and the photographic light-curve of 1900, indicate a maximum for 1899 Nov. 21±, on the assumption that this occurred between two observations*. Taking the mean of Hartwig's and my own determinations for 1901, we have the following four observed maxima:

Date	J.D.	C — O
		d
1899 Nov. 21±	241 4980±	+2
1900 Sept. 2	5265	+7
1901 June 26	5562	0
1902 Apr. 17±	5847±	+5

These yield the following approximate elements of variation:

$$\text{Maximum} = \text{J.D. } 241\,5562 + 290^d\ E.$$

The last column above contains the residuals according to these elements.

6827. *RT Lyrae.*

The star rose from 12ᴹ.5 on 1902 May 24 to a sharply defined maximum (9ᴹ.8) on July 22, and then almost immediately declined rapidly and steadily to 12ᴹ.7 on Sept. 25. Observations were made on 16 nights between the above limiting dates. Some earlier photographic observations were published in the *A.N.* 3783. Those made in 1901 indicate a maximum about Nov. 11, but there is a good deal of uncertainty as to the exact date. The interval between the two maxima is 253 days; and assuming this to be period, maxima should have occurred on 1901 Mar. 3, 1900 June 23 and 1899 Oct. 13. The invisibility of the star on the photographs taken between 1900 Sept. 2 and Nov. 22 is in accordance with the first two maxima, but its faintness (12ᴹ.08) on the plate of 1899 Sept. 28 shows that this period cannot be quite correct, since the computed maximum occurs only 15 days later. It is uncertain at present whether the period of 253 days is a little too short,

*See the diagram in the *Monthly Notices*, Vol. 62, p. 201, and in *Popular Astronomy*, 1902, p. 216. Dr. Hartwig has published the times of several maxima of this star observed by him in 1901 in the *Vierteljahrsschrift der Astr. Gesell.*, *Jahrgang* 36, p. 268. They were received too late for inclusion in the writer's discussion of the observations of this star.

† On August 22 the comparison-star 1 was very faint, and sometimes even invisible owing to moonlight, so that the time of T_0 could not be satisfactorily observed.

*The first two observations of 1899 (Sept. 2 and 22) should be struck out. The plate of Sept. 2 is a trial one before the instrument was in adjustment, and the star images being drawn out into long trails it is difficult to identify the fainter stars, and it is doubtful if a faint trace is really due to this star. The plate of Sept. 22 is a poor one, and here also it is doubtful whether a faint mark is really due to the star.

or a little too long, though the probability is that it is too long. The next maximum should occur about 1903 Apr. 1.

6895. *RU Lyrae.*

This star rose from $12^{M}.7$ on 1902 May 27 to a sharply defined maximum (mag. = 9.9) on Aug. 21, and by Sept. 25 it had declined to $11^{M}.3$. Observations were made on 14 nights between the above limiting dates. Some earlier photographic observations were published in the *A.N.* 3796, and from these it was inferred that the period is almost exactly equal to a year, and that the next maximum would probably occur in August of the present year, as has actually happened. The period cannot be stated with greater exactness at present.

6915. *RV Lyrae.*

The following minimum of this *Algol*-variable has been observed and is in addition to those already published in the *A.N.* 3811.

1902 Sept. 3 $11^{h}\,42^{m}$

Twelve observations were obtained between $8^{h}\,44^{m}$ and $14^{h}\,18^{m}$, but cloud interfered a good deal after 11^{h}, though the above result is on the whole a fairly satisfactory one. The observation is not reduced to the sun. The computed time from the elements in the *A.N.* 3811 is $11^{h}\,36^{m}$.

6927. *U Sagittae.*

The undermentioned minima of this *Algol*-star have been observed, but the times are somewhat provisional, as owing to the unfavorable weather the observations are not sufficient to give a very satisfactory light curve. The heliocentric times of minimum according to EBELL's ephemeris in *A.N.* 3771 have been added for comparison with the differences O — C. The observed times are not, however, reduced to the sun.

Epoch	Date	Observed h m	Computed h m	O — C m
87	1902 Aug. 22	9 3	9 28	—25
90	Sept. 1	12 33	12 53	—20
92	8	6 56	7 9	—13
95	18	10 20	10 34	—14

Notes. E. 87, 12 observations between $8^{h}\,29^{m}$ and $11^{h}\,14^{m}$. E. 90, 9 observations between $8^{h}\,27^{m}$ and $11^{h}\,3^{m}$. E. 92, 7 observations between $8^{h}\,11^{m}$ and $10^{h}\,23^{m}$. E. 9.5, 11 observations between $8^{h}\,43^{m}$ and $11^{h}\,52^{m}$.

The star remains stationary at minimum for about $2^{h}\,10^{m}$. Normally the variable is a white star, but during the stationary period at minimum, and for a few minutes before and after, it is of a decided reddish color.

7019. *TY Cygni.*

In 1901 the star rose from 12^{M} on Aug. 6 to a sharply defined maximum on Nov. 11 (mag. = $9\frac{1}{2}$). In 1902 the star rose from $12\frac{1}{2}^{M}$ on July 7 to a well defined maximum on Oct. 27 (mag. = 9). Three earlier photographic observations were published in the *A.N.* 3687. With the help of the visual light-curve these observations indicate a maximum for 1900 Nov. 22. From these three maxima the following elements have been derived:

$$\text{Maximum} = \text{J.D. } 2416051 + 352^{d}\ E$$

The star is not visible on a photograph taken by Prof. MAX WOLF on the nights of Sept. 25 and 30, 1891 (exp. = 12^{h}), and it cannot therefore have been photographically so bright as 14^{M}. According to the elements the star should have been almost exactly at minimum then, so that the invisibility of the star on this photograph is just what might be expected.

7318. *UW Cygni.*

The observations of the undermentioned minima of this *Algol*-variable were all more or less hindered by cloud or other causes. The last minimum is the only one at all properly observed during both the decreasing and increasing phases. The times, which are geocentric Greenwich mean times, have been derived by means of a provisional light-curve based on three minima observed last year.

	Date	Minimum h m	Quality
(a)	1902 July 18	10 12	Fair.
(b)	Aug. 25	9 22.7	Unsatisfactory.
(c)	Sept. 1	6 50	Somewhat approximate.
(d)	18	12 56	Approximate.
(e)	25	10 32	Pretty good.

Notes. (a) 15 observations between $10^{h}\,48^{m}$ and $14^{h}\,35^{m}$. (b) 9 observations between $8^{h}\,18^{m}$ and $13^{h}\,15^{m}$, much hindered by cloud. (c) 10 observations between $8^{h}\,18^{m}$ and $13^{h}\,15^{m}$. (d) 9 observations between $8^{h}\,50^{m}$ and $11^{h}\,21^{m}$. The decrease could not be followed further owing to the brightness of the moon. (e) 17 observations between $8^{h}\,31^{m}$ and $12^{h}\,5^{m}$; hazy between $10^{h}\,27^{m}$ and $11^{h}\,50^{m}$.

7505. *UX Cygni.*

This star has been invisible in the $6\frac{1}{2}$-inch reflector, and consequently fainter than about $12\frac{1}{2}^{M}$, between 1902 July 11 and Nov. 17. Some photographic observations were published in the *A.N.* 3752, and to these may be added the following: — 1901 Nov. 15, mag. = 10.98. The observations of 1901 indicate a fairly definite maximum for 1901 Oct. 23; photographic mag. = 9.7. It is difficult at present to come to a satisfactory conclusion respecting the period of variation, though this is evidently long and the range of variation considerable.

7571*a*. *TW Cygni.*

Observations on 22 nights in 1901 indicate a well defined though somewhat flat maximum on Sept. 1. The star rose from $12^{M}.0$ on May 24 to $8^{M}.8$ at maximum, and by Nov. 3 had declined to $11^{M}.0$. In 1902 observations on 9 nights show a very definite maximum for Aug. 15, mag. = 9.6. Dr. HARTWIG, in 1901, suggested a period of $8\frac{1}{2}$

months (see *A.N.* 3744), but this is evidently, however, somewhat longer. The photographic magnitude is 9.85 on two photographs taken on Oct. 6 and 9, 1899. Comparison of the brightness of the star upon three plates taken on three separate nights with corresponding visual observations, gives −1.5, —1.6 and −1.1 as the correction to the photographic magnitudes in order to reduce them to the visual ones. Means = $-1^{M}.4$. Applying this correction to the above mentioned observations of 1899, we get $8^{M}.45$ as the photographic reduced to the visual brightness, so that the star was probably at or very near a maximum on 1899 Oct. 7. The following elements satisfactorily satisfy the observations.

Maximum = J.D. 241 5977 + 347^{d} E.

It is noteworthy that there seems to have been a progressive diminution in the brightness of the star at maximum.

8610. *Z Pegasi.*

The following are the approximate photographic magnitudes of this variable. 1899 Nov. 6 and 10 not visible, $<12^{M}$; Nov. 29, $<11^{M}$; 1900 Nov. 18, $<12^{M}$; Nov. 22, $<11\frac{1}{2}^{M}$; Dec. 13, $<10\frac{1}{2}^{M}$; Dec. 15, $11\frac{1}{2}^{M}$; 1901 Jan. 14, 10^{M}.

20 Hove Park Villas, Hove, 1902 *Dec.* 20.

ON THE VARIABILITY OF DM. —1°1182,

By ZACCHEUS DANIEL.

The star DM. —1°1182, $9^{M}.3$, occurs in the list of suspected variables at the end of CHANDLER's *Third Catalogue of Variable Stars* (*A.J.* 379) where it has the number (2235), and the observed range is given as 8^{M} to $9^{M}.3$.

This star was observed nine times at Harvard College Observatory with the Meridian Photometer during the years 1886 to 1888. Of these observations, seven by Professor E. C. PICKERING, from 1886 Feb. 16 to 1888 Feb. 28, yield a mean magnitude of 8.96. The mean deviation of each observation from this result is $0^{M}.19$, and the observed range is $0^{M}.8$. One observation by PICKERING on 1886 Jan. 10, and one by WENDELL on 1886 Jan. 26, which are not included in the mean, both give a residual of $-0^{M}.8$ from $8^{M}.96$. This makes a total observed range of $1^{M}.2$, or from $8^{M}.16$ to $9^{M}.36$.

These results are extracted from *H.C.O. Annals*, Volumes 23 and 24, where the star is designated M.P. 821.

I observed this star with the 10-inch Clark equatorial on four dates in 1901, from Jan. 19 to April 16, and on fourteen dates in 1902, from Jan. 3 to May 8. The mean result of the eighteen observations is $9^{M}.16$. The mean deviation of each observation from this value is $0^{M}.05$. The brightest observation is $9^{M}.07$; the faintest is $9^{M}.31$, which make an observed range of $0^{M}.24$.

All my observations agree in affording no evidence of variability. They are very accordant, and show no marked fluctuation.

The following are my comparison-stars. The magnitudes are the result of thirty-eight separate comparisons, and are adjusted closely to the DM. scale.

Star	Desig.	DM.	D	Grades
		M	M	
a =	DM. −1°1160	8.3	8.33	0.0
b =	−1°1170	9.3	8.93	6.0
c =	−1°1185	9.0	9.21	8.8
d =	−1°1177	9.5	9.61	12.8

Bucknell University, Lewisburg, Penna., 1902 *Dec.* 27.

OBSERVATION OF COMET *d* 1902 (GIACOBINI),

By E. E. BARNARD.

1902	90° time	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ	
	h m s			m s	′ ″	h m s	° ′	s	s
Dec. 30	11 11 41	1	3	. . . .	−4 22.5	. . . .	+3 36.0	. .	0.747
	11 26 28	1	18	+0 23.14	. . . .	7 3 33.1	. . . .	n9.049	. .
	11 36 51	1	3	. . . .	−4 6.0	. . . .	+3 36.3	. .	0.745

*	α 1902	δ 1902	Red. to app. place α	Red. to app. place δ	Authority
	h m s	° ′	s	″	
1	7 3 5.1	+3 40.7	+5.07	−16.5	DM. +3°1563*

The position was measured with the large telescope. The comet was about 12^{M}, with elongated brightening at middle. There was a slight brushing out of the nebulosity following.

This object was observed on Dec. 3, with the 12-inch, but as there is no micrometer to that instrument, an accurate position could not be obtained. No other opportunity has offered to observe it with the large telescope. The catalogues here do not contain an accurate place of the comparison-star.*

* This star is Albany A.G. 2635 ; $\alpha = 7^{h}\ 3^{m}\ 13^{s}.20$, $\delta = +3°\ 40'\ 39''.1$ (1902.0). — ED.

Yerkes Observatory, Williams Bay, Wis., 1903 *June* 3.

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OBSERVATIONS OF VARIABLE STARS OF LONG PERIOD,

By W. C. BRENKE.

The following observations were made by the method of sequences with the aid of the twelve-inch refractor of the University of Illinois Observatory. The comparison-stars have been those of HAGEN's *"Atlas Stellarum Variabilum,"* except a few faint ones which were used during times of minima, and which have been connected by sequence of steps with HAGEN's stars, making the scale of magnitudes uniformly that of the *"Atlas."* The magnitudes are given to two decimal places whenever two or more accordant observations were secured, while the first decimal only is given in cases of single observations, or of the mean of several estimates sufficiently discordant to make this decimal somewhat uncertain. The dates of maxima and minima were determined from curves, and the theoretical times have been added for comparison. Brackets are used to indicate those dates which are not well determined by the observations. The uncertainty in these cases is about $\pm 10^d$.

Fractions of a day in the observed dates indicate the approximate G.M.T. of the observations.

S Coronae.

Date	Mag.	Date	Mag.
1901 Apr. 8.7	8.0	1901 July 1.6	9.30
15.7	8.0	8.6	9.40
18.6	7.8	12.6	9.7
19.7	8.15	18.7	9.6
25.6	8.17	19.7	9.6
26.7	8.10	25.6	10.0
May 1.6	8.00	Aug. 5.6	10.1
4.6	8.33	14.7	11.2
13.7	8.32	Sept. 13.7	11.2
14.6	8.27	21.6	11.40
20.7	8.57	Oct. 2.6	12.03
28.6	8.37	4.6	12.27
30.6	8.50	14.6	12.02
June 6.6	8.68	Nov. 1.6	12.5
13.7	8.80	1902 Mar. 5.9	8.5
17.7	8.93	13.7	7.2
26.7	9.05	Apr. 2.7	8.4

R Herculis.

Date	Mag.	Date	Mag.
1901 May 30.7	9.86	1901 Sept. 13.7	invisible
June 6.6	10.05	21.6	"
13.7	10.67	Oct. 2.6	"
17.7	10.98	4.6	"
26.7	11.20	14.6	"
July 1.6	11.43	Nov. 1.6	"
8.6	11.60	1902 Feb. 17.9	8.65
12.6	11.57	Mar. 5.9	8.55
19.7	12.03	13.8	8.77
25.6	*invisible	Apr. 2.7	9.5
Aug. 5.6	12.20	13.7	9.75
15.6	12.6	June 5.7	11.5

* Bright moon.

U Herculis.

Date	Mag.	Date	Mag.
1901 May 28.7	10.5	1901 Sept. 13.7	10.07
30.7	10.5	21.6	10.05
June 6.7	10.45	Oct. 2.6	10.03
13.7	10.50	5.6	9.70
17.7	10.70	14.6	9.76
26.7	10.60	15.6	9.80
July 1.6	10.70	Nov. 1.6	9.40
8.6	11.03	1902 Feb. 17.9	7.55
12.6	10.62	Mar. 5.9	7.6
19.7	11.03	13.8	8.3
25.6	10.67	Apr. 2.9	8.4
Aug. 5.6	11.33	13.7	9.2
15.6	13.6		

W Herculis.

Date	Mag.	Date	Mag.
1901 May 28.7	12.02	1901 Sept. 13.7	8.05
30.7	11.9	21.6	8.0
June 6.7	12.06	Oct. 2.6	7.77
13.7	12.10	5.6	7.55
17.7	11.78	14.6	8.1
26.7	12.0	15.6	8.03
July 1.6	11.65	Nov. 1.6	8.37
8.6	11.25	1902 Feb. 17.9	12.2
12.6	10.95	Mar. 5.9	12.17
19.7	11.06	13.8	12.15
25.6	11.40	Apr. 2.9	11.9
Aug. 5.6	10.08	13.7	11.5
15.6	9.04	June 3.7	8.6

V Coronae.

Date	Mag.	Date	Mag.
1901 Apr. 8.7	8.67	1901 July 1.6	8.80
15.7	8.70	8.6	8.93
18.6	8.87	12.6	9.12
19.7	8.80	18.7	9.23
24.7	8.55	19.7	9.32
25.6	8.63	25.6	9.40
26.7	8.50	Aug. 5.6	9.70
May 1.6	8.70	14.7	10.30
4.6	8.56	Sept. 13.7	10.47
13.7	8.78	21.6	10.47
14.6	8.93	Oct. 2.6	11.00
20.7	8.63	4.6	11.00
28.6	8.63	14.6	11.34
30.6	8.68	Nov. 1.6	12.30
June 6.6	8.80	1902 Feb. 18.9	8.43
13.7	8.83	Mar. 5.9	8.35
17.7	8.86	13.7	8.35
26.7	8.97	Apr. 2.7	8.56

S Herculis.

Date	Mag.	Date	Mag.
1901 May 28.7	7.80	1901 Aug. 15.6	9.00
30.7	8.15	Sept. 13.7	10.05
June 6.7	8.37	21.6	10.08
13.7	8.20	Oct. 2.6	10.4
17.7	8.37	5.6	10.70
26.7	8.40	14.6	11.23
July 1.6	8.70	15.6	11.53
8.6	8.57	Nov. 1.6	12.20
12.6	8.63	1902 Feb. 17.9	8.60
19.7	8.65	Mar. 5.9	8.37
25.6	8.55	13.8	8.50
Aug. 5.6	8.70	Apr. 2.9	8.4

T Herculis.

Date	Mag.	Date	Mag.
1901 May 30.7	12.0	1901 Sept. 21.6	8.76
June 6.7	12.0	Oct. 2.6	9.18
13.7	11.9	5.6	9.25
17.7	11.13	14.6	9.92
26.7	10.50	15.6	9.92
July 1.6	10.13	Nov. 1.6	11.16
8.7	9.57	18.7	11.95
12.6	9.33	29.7	11.25
19.6	9.17	1902 Feb. 17.9	8.17
25.6	8.62	Mar. 5.9	8.45
Aug. 5.6	8.23	13.8	9.42
15.6	8.05	Apr. 3.9	10.3
Sept. 13.7	8.17	May 29.7	9.8

Maxima and Minima.

Ch. No.	Star	Ph.	Mag.	Date Cal. [Ob.]	Date Julian [Ob.]	Date Comp'd
5504	*S Coronae*	Min.	<12.5	1901 Dec. 1]	5720]	5685
5675	*V Coronae*	Min.	<12	1901 Nov. 21]	5710]	5664
5770	*R Herculis*	Max.	8.5	1902 Feb. 27	5808	5798
5889	*U Herculis*	Min.	<13.5	1901 Aug. 18	5615	5689
5889	*U Herculis*	Max.	7.5	1902 Feb. 19	5800	5856
5950	*W Herculis*	Max.	7.5	1901 Oct. 5	5663	5669
5950	*W Herculis*	Min.	12.2	1902 Mar. 1]	5810]	5840
6044	*S Herculis*	Min.	<12	1901 Dec. 11]	5730]	5734
6512	*T Herculis*	Min.	12.0	1901 June 4	5540	5517
6512	*T Herculis*	Max.	8.0	1901 Aug. 28	5622	5595
6512	*T Herculis*	Min.	12.0	1901 Nov. 16	5705	5681
6512	*T Herculis*	Max.	8.0	1902 Feb. 9	5785	5759

University of Illinois Observatory, Urbana, Ill., 1902 *December.*

NOTE ON THE SECULAR PERTURBATIONS OF THE PLANETS,

By A. Hall.

About thirty years ago, at the request of Professors Coffin and Newcomb, the Smithsonian examiners, I made some computations to test the results of Stockwell's memoir on the secular perturbations of the principal planets of our solar system. The numerical results proved to be very accurately deduced from the assumed data. Stockwell made calculations to show the effect of changes in the masses of the planets, and his memoir contains formulas for this purpose. It appeared to me that this might be done more directly, and Leverrier has pointed out a method. The arrangement of the planets is such that the equation of the eighth degree, whose roots furnish the coefficients, may, for the first approximation, be divided into two equations of the fourth degree; one of these equations belonging to the four interior planets, and the other to the four exterior planets. The general biquadratic can be written,

$$g^4 + p_1\, g^3 + p_2\, g^2 + p_3\, g + p_4 = 0.$$

The values of the coefficients in terms of the roots are,

$$\begin{aligned} p_1 &= -g - g_1 - g_2 - g_3 \\ p_2 &= +gg_1 + gg_2 + gg_3 + g_1g_2 + g_1g_3 + g_2g_3 \\ p_3 &= -gg_1g_2 - gg_1g_3 - gg_2g_3 - g_1g_2g_3 \\ p_4 &= +gg_1g_2g_3 \end{aligned}$$

Differentiating these equations, and eliminating all the variations of g but one, be have,

$$\delta g = -\frac{\delta p_1 \,.\, g^3 + \delta p_2 \,.\, g^2 + \delta p_1 \,.\, g + \delta p_4}{(g - g_1)\,(g - g_2)\,(g - g_3)}$$

This is Leverrier's result, but it does not appear to be a good form for computing, since small divisors may enter the denominator. Stockwell's equations seem to be safer, and to test them I have computed the roots from the masses adopted by G. W. Hill in his work on secular perturbations, and have compared these values with the roots found by Hill for the eccentricities and perihelia.

	Hill's $\frac{1}{m}$	log μ	Corr. to S.	Root g	Hill's value
			$''$	$''$	$''$
Mercury	10500000	9.729647n	+0.034496	+ 5.498299	+ 5.497704
Venus	408000	8.644612n	+0.040516	7.288943	7.283189
Earth	328000	9.093603	+0.302467	17.316840	17.322014
Mars	3093500	9.125356n	+0.219289	18.003745	18.003942
Jupiter	1047.355	6.699237	+0.020918	0.637602	0.634609
Saturn	3501.6	$-\infty$	−0.013435	2.714224	2.707414
Uranus	22869	8.949531	+0.014177	3.730784	3.722375
Neptune	19314	8.441669n	+0.028722	+22.489569	+22.418997

The form of STOCKWELL's mass was assumed to be $\frac{1+\mu}{m}$, and by comparing with HILL's values log μ was found.

The agreement is good except in the last case, where some error of calculation may exist. The solution may be completed by a method like that of NEWTON. We may, I think, consider the equation of the eighth degree as disposed of.

Goshen, Conn., 1902 *Nov.* 26.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A 4½-INCH REFRACTOR,

BY A. W. QUIMBY.

1902	Time	New Grs.	Total Grs.	Total Spots.	Fac. Grs.	Def.	1902	Time	New Grs.	Total Grs.	Total Spots.	Fac. Grs.	Def.	1902	Time	New Grs.	Total Grs.	Total Spots.	Fac. Grs.	Def.
July 12	4	1	1	1	1	good	Oct. 7	8	1	2	28	1	fair	Oct. 30	8	–	2	2	2	poor
13	5	–	–	–	1	fair	8	8	–	2	24	–	poor	31	9	–	–	–	2	poor
Aug. 16	4	1	1	2	1	fair	9	8	–	2	16	–	poor	Nov. 3	8	–	–	–	1	fair
17	8	–	1	1	–	fair	10	9	–	1	11	–	fair	4	9	–	–	–	1	fair
18	9	–	1	1	–	fair	12	9	–	1	1	–	poor	9	8	–	–	–	1	fair
19	8	–	1	1	–	good	13	7	–	1	1	–	poor	10	8	–	–	–	1	fair
Sept. 13	8	–	–	–	1	fair	14	8	–	1	1	–	poor	14	9	1	1	2	–	poor
14	8	–	–	–	1	fair	15	8	–	1	2	1	fair	15	10	2	3	11	3	fair
17	8	–	–	–	1	fair	16	8	–	1	1	1	fair	16	8	–	3	4	3	poor
19	10	1	1	8	–	good	17	10	–	–	–	–	poor	17	8	–	1	2	–	poor
20	11	–	1	4	–	poor	18	9	–	–	–	–	fair	19	11	–	1	20	–	poor
22	9	2	3	10	2	good	19	2	–	–	–	1	poor	20	8	–	1	26	–	fair
23	8	–	3	10	3	good	20	9	–	–	–	1	poor	21	8	–	1	33	–	fair
24	7	–	2	14	2	poor	21	8	1	1	1	5	fair	22	9	–	1	24	–	fair
27	3	–	1	6	–	fair	22	8	–	–	–	1	fair	23	8	–	1	22	–	fair
28	5	–	1	4	–	poor	23	9	1	1	9	–	fair	24	10	–	1	16	–	poor
29	7	–	1	4	–	poor	24	10	1	2	41	1	fair	Dec. 9	3	–	–	–	1	good
30	9	–	1	3	–	poor	25	8	–	2	30	1	poor	17	8	1	1	2	1	fair
Oct. 1	11	–	1	2	1	poor	26	8	–	2	17	–	poor	18	9	–	1	1	1	fair
2	8	–	1	2	1	fair	27	3	–	2	18	1	fair	20	12	–	–	–	1	good
3	8	–	1	1	1	fair	28	2	–	2	12	1	poor	22	8	–	–	–	1	good
6	8	1	1	32	1	fair	29	8	–	2	4	1	poor							

Observations were made on 105 other days of the semester, beginning July 1, when neither spots nor faculae were seen. The sun was invisible on July 30; Sept. 21, 25, 26; Oct. 4, 5, 11; Nov. 11, 18, 25, 26; Dec. 11, 21, 29.

THE MISSING DURCHMUSTERUNG STAR +30°583,

BY ZACCHEUS DANIEL.

In *A.J.* 430, page 179, Professor MARY W. WHITNEY states that no star was seen in the position for DM. +30°583 on either 1897 Nov. 27, or 1898 Feb. 28.

The *Bonner Sternverzeichniss* gives the position,

$$\alpha = 3^h\ 43^m\ 12^s.7 \quad ; \quad \delta = +30^\circ\ 4'.6\ (1855)\ ;$$

and the magnitude, 9.5.

With a 4-inch refractor, I looked for this star on nine dates, from 1898 April 21, to 1899 April 5, inclusive, but I never could see any star brighter than the twelfth magnitude near the given position, although all other DM. stars near were always seen and identified. However, on 1898 Sept. 12, I saw a star of about the twelfth magnitude near the place. I also examined the region with the 10-inch refractor on nine dates, from 1900 Oct. 24, to 1902 Nov. 19, inclusive, with the same result. In good seeing, the 10-inch telescope always showed the twelfth-magnitude star and several fainter stars near it.

Bucknell University, Lewisburg, Penna., 1902 *Dec.* 22.

QUESTIONS RELATING TO STELLAR PARALLAX, ABERRATION AND KIMURA'S PHENOMENON,

By S. C. CHANDLER.

1. In view of the narrow range within which it would appear, from *A.J.* 529, that we can now define the constant of aberration, the effect of stellar parallax on its determination by the KÜSTNER–TALCOTT method ought not to go unexamined. It has hitherto been neglected both by myself and others on the presumption that it is unimportant.

To obtain a convenient and sufficiently accurate formula for this correction let us take the expression for parallax in declination,

$$-\pi\sin\odot(\cos\epsilon\sin\delta\sin\alpha-\sin\epsilon\cos\delta)-\pi\cos\odot\sin\delta\cos\alpha$$

in which the earth-sun radius is treated as constant, and pursue an analogous transformation to that adopted for the aberration in *A.J.* 517 and 520. For a pair of stars of equal zenith-distance, north and south, in the same right-ascension, this becomes

$$(1)\qquad \pi\cos\zeta(m\sin\odot-n\sin\odot\sin\alpha-\sin\varphi\cos\odot\cos\alpha)$$

where $m=\sin\epsilon\cos\varphi\quad,\quad n=\cos\epsilon\sin\varphi$

Introducing the apparent solar time of observation, $T=\alpha-\odot$, we get

$$(2)\qquad \pi\cos\zeta\,[m\sin\odot-n\cos T -\sin\varphi\,(\cos\epsilon-1)\,(\tfrac{1}{2}\sin2\odot\sin T-\cos^2\odot\cos T)]$$

which is of the same form as eq. (2) for aberration, *A.J.* 520.

For our present purpose the term in $\sin\varphi(\cos\epsilon-1)$ is negligible. Using the subscripts 1 and 2 to designate evening and morning groups of observations we have, as the corrections of the latitude for error of assumed aberration and for parallax,

(3)

$$dk\,.\cos\zeta_1(n\sin T_1-m\cos\odot)+\pi\cos\zeta_1(m\sin\odot-n\cos T_1)$$
$$dk\,.\cos\zeta_2(n\sin T_2-m\cos\odot)+\pi\cos\zeta_2(m\sin\odot-n\cos T_2)$$

We can put without appreciable error in practice,

$$\cos\zeta_1=\cos\zeta_2=\cos\zeta_0$$

and the difference of these expressions for observations on the same night is therefore

$$(4)\qquad dk\,.\,n\cos\zeta_0(\sin T_1-\sin T_2)-\pi n\cos\zeta_0(\cos T_1-\cos T_2)$$

Then, in the determination of the aberration by KÜSTNER's method from the cyclical sum of all the group-combinations, denoting the coefficients of dk and π by A and B, respectively, and the absolute terms by ν, we have

$$\Sigma A\,.\,dk+\Sigma B\,.\,\pi+\Sigma\nu=0$$

whence the aberration-correction

$$dk=-\frac{\Sigma\nu}{\Sigma A}-\pi\frac{\Sigma B}{\Sigma A}$$

But, since the average time of the observations will be about the same for all the group-combinations, we have

$$\frac{\Sigma B}{\Sigma A}=\frac{B}{A}=-\frac{\cos T_1-\cos T_2}{\sin T_1-\sin T_2}=+\tan\tfrac{1}{2}(T_1+T_2)$$

consequently, denoting by dk' the correction of the aberration-constant as ordinarily found by neglecting the effect of stellar parallax, the corrected value will be

$$dk=dk'-\pi\tan\tfrac{1}{2}(T_1+T_2)\qquad(5)$$

From this it appears that the correction for stellar parallax is zero when the average apparent time of observation is 12^h, *i.e.*, when the evening and morning groups are symmetrically disposed as to apparent midnight. This is rarely practically the case. In most of the series for which aberration-determinations by this method are given on p. 3 of *A.J.* 529, this average falls before midnight, and the correction for parallax for most of them is therefore positive. Fortunately the printed data enable us to find these times approximately enough except for Strassburg and Hongkong. They are given in the following table, where the first column sufficiently designates the respective series, which are in the same order as in *A.J.* 529. Then follow the corrections, by eq. (5), of the aberration-determinations, expressed in terms of the unknown π; and in the last column their values in arc on the assumption $\pi=0''.02$.

	T_1	T_2	Correction	
	h	h		"
Berlin,	–	–	–	+0.005
Berlin,	10.1	12.8	+0.14π	+ .003
Berlin,	10.1	12.8	+ .14	+ .003
Karlsruhe,	8.8	14.8	+ .05	+ .001
Prague,	10.1	12.8	+ .14	+ .003
Strassburg,	–	–	–	–
San Francisco,	11.0	13.9	– .12	– .002
Cape Good Hope,	8.3	16.4	– .09	– .002
Naples,	8.5	14.8	+ .09	+ .002
Potsdam,	9.3	11.7	+ .41	+ .008
Hongkong,	–	–	–	–
Leyden,	9.8	12.1	+ .28	+ .006
Kasan,	10.0	12.5	+ .19	+ .004
Kasan,	9.6	12.3	+ .28	+ .006
Kasan,	8.9	11.6	+ .49	+ .010
New York,	8.5	14.8	+ .09	+ .002
Bethlehem,	8.8	14.8	+ .05	+ .001
Philadelphia,	8.8	14.8	+ .05	+ .001
Int'l 6 stations,	10.0	12.0	+0.27	+0.005
	Weighted mean,		+0.14π	+0.003

The assumption $\pi=0''.02$ is taken as a sort of measure of the superior limit which could with much probability, according to accepted notions, be assigned to this element for the class of stars employed. Some astronomers might

be inclined to reduce the estimate to one-half this quantity. From more than six hundred stars, actually used in five of the series, I find the average proper motion in declination to be about 0″.05, or in arc of a great circle, total motion nearly 0″.08; reduced to Boss's system about 0″.09. The average magnitude is about the sixth. KAPTEYN's formulas would give for this case $\pi = 0''.017$. Prof. Boss, whom I consulted on the matter, is in favor of a decidedly lower value. The value 0″.020 seems a fair estimate considered as a superior limit. But the main point is that the correction in question for the value of the aberration derived in *A.J.* 529 for these twenty-five series is essentially positive, so that their corrected mean would be 20″.525 or 20″.526 instead of 20″.523 as there given. The general mean from the forty-three accepted series would therefore become 20″.523, and I beg that this be regarded as the definitive mean of my investigation in that paper, corresponding to the value of the solar parallax 8″.780, instead of the quantities there given. The change is of course trivial, but, being admittedly real, is necessary. So far as it goes it reinforces the likelihood that any rounded conventional value for this constant should be taken at least as high as 20″.52.

It may be noted that the prime-vertical and meridian zenith-distance determinations given in the paper require no correction on this account, since the parallax was eliminated or simultaneously determined in the solutions.

2. The development of the foregoing formulas leads naturally to the suggestion that, in the KÜSTNER–TALCOTT method, we should have a means of finding the average absolute parallax of a set of stars observed in common at a belt of stations in widely different longitudes, such as has been contrived and is now in successful operation for the determination of variations of latitude. The parallax so determined would be independent of errors in the star-declinations and of the latitude-variation. The high precision to which such observations have been brought, and the enormous mass of them, ought to make the method adequate for this purpose. I have therefore had the curiosity to develop and apply it, in the manner now to be shown.

Taking the mean of equations (3) we have

$$A.dk+B\pi=n\tfrac{1}{2}\cos\zeta\,[dk(\sin T_1+\sin T_2)-\pi(\cos T_1+\cos T_2)] \\ -m\cos\zeta\,[dk.\cos\odot-\pi\sin\odot] \tag{6}$$

Now, with observations symmetrically disposed as to apparent midnight, the term in sin T will disappear from the mean observed on a given night; also the mean latitude for each station, deduced from a year's observations, will be affected by the constant value of the term in cos T; so that the variations of latitude at a station in longitude λ (reckoned positive west from Greenwich), as found in the ordinary manner by subtracting this mean latitude from the observed values, may be expressed by

$$\varphi-\varphi_0 = x\sin\lambda - y\cos\lambda + z \tag{7}$$

where I have put

$$z = m\cos\zeta\,(dk.\cos\odot-\pi\sin\odot) \tag{8}$$

and the rectangular coordinates are reckoned, $+y$ towards Greenwich, $+x$ towards 90° east.

The values of x, y, z, can be determinately found from a belt of stations such as that of the International latitude-series for each date or group of dates. Then we may find dk and π from the equations of condition

$$m\cos\zeta\cos\odot.dk - m\cos\zeta\sin\odot.\pi = z \tag{9}$$

We therefore arrive at the curious result that the quantity z is nothing more than the empirical term, independent of the longitude of the station and varying with the time of year, which KIMURA has discovered and announced in *A.J.* 517, and which has been confirmed by ALBRECHT by means of the International latitude-series. It indubitably appears in the results for both 1900 and 1901.

Consequently, if the effect of stellar parallax furnishes the true explanation of this empirical term, we can find the value of π, or the average parallax of the observed stars, by introducing the values of z given by KIMURA and ALBRECHT as the absolute terms of equation (9). For the series 1900–1901 we take from *A.N.* 3808,

VALUES OF z.

y	1899 ″	1900 ″	1901 ″	1902 ″	Mean ″	Computed ″
0.0	–	+.028	+.062	+.044	+.045	+.043
.1	–	+.021	+.055	–	+.038	+.036
.2	–	+.008	+.022	–	+.015	+.017
.3	–	−.014	−.005	–	−.009	−.008
.4	–	−.029	−.026	–	−.028	−.028
.5	–	−.033	−.036	–	−.034	−.037
.6	–	−.025	−.032	–	−.029	−.030
.7	–	−.008	−.016	–	−.012	−.011
.8	–	+.019	+.007	–	+.013	+.014
0.9	+.031	+.047	+.025	–	+.034	+.034

For the coefficients in eq. (9) we take $\cos\zeta = 0.98$, and $m = 0.309$ ($\varphi = 39°\ 8'$); whence the observation-equations following, where w is an arbitrary constant to reduce the residual sum to zero, and the absolute terms are the mean observed values in the above table of z.

$$\begin{array}{lrlrlr}
w & +.053 & dk & +.298 & \pi = & +''.045 \\
 & +.218 & & +.213 & = & +.038 \\
 & +.300 & & +.042 & = & +.015 \\
 & +.268 & & -.142 & = & -.009 \\
 & +.133 & & -.273 & = & -.028 \\
 & -.058 & & -.298 & = & -.034 \\
 & -.218 & & -.213 & = & -.029 \\
 & -.300 & & -.042 & = & -.012 \\
 & -.268 & & +.142 & = & +.013 \\
 & -.133 & & +.273 & = & +.034
\end{array}$$

The solution by equal weights gives

$$w = +0''.003 \quad , \quad dk = +0''.028 \quad , \quad \pi = +0''.128$$

from which we have the computed values in the last column of the table of z. The extraordinary accordance with the observed mean values must be largely fortuitous. If the result for parallax were reasonable this close agreement might be taken as an index of the efficiency of this method of finding parallax.

A similar computation for the data given by KIMURA for the other series in *A.J.* 517 gives us four other values of π, so that we have

1891–92	$\pi =$	$+0''.06$
95–96		$+\ .02$
96–97		$+\ .08$
98–99		$+\ .14$
1900–01		$+\ .13$
Mean	$\pi =$	$+0.086$

Now, it must be at once admitted that such parallaxes as these, for stars of the sixth magnitude and average proper motion of about $0''.08$ or $0''.09$, are inadmissible, according to orthodox notions. They are in flat contradiction of the inferences from the relations, apparently demonstrated by STUMPE and BOSS, between stellar proper motion and solar parallactic motion, taken in connection with the spectroscopic measurements of the sun's linear velocity. KAPTEYN's formulas would give for these stars an average parallax of not over $0''.017$.

If we abandon the idea of ascribing more than a moderate portion of KIMURA's phenomenon to the effect of stellar parallax we must seek the cause elsewhere for the principal part.

Let us see whether the observed phenomenon will correspond with a parallactic effect of another kind, namely, in a change of direction in our line of reference. Take the hypothesis that I have suggested in *A.J.* 524, that the earth's center of gravity may possibly be subject to an annual vibration along the line of the terrestrial axis. This does not seem to me intrinsically absurd. Let h be the linear semi-amplitude, expressed in feet, of such a vibration, and H the sun's longitude on the date corresponding to its southernmost point. The effect on measured latitudes will be the same for all longitudes, and will be

$$\varphi - \varphi_0 = \frac{h}{\rho \sin 1''} \cos\varphi \cos(\odot - H) \tag{10}$$

where $\dfrac{1}{\rho \sin 1''} = 0''.01$

Solving for the constant w, and for h and H, using the observed values of z in the table we find

$$w = +0''.003 \quad , \quad h = 5.1 \text{ ft.} \quad , \quad H = 282°.6 \text{ (January 2)}$$

the substitution of which gives us the same computed values in the last column of the z-table as before. By this hypothesis then there would be an oscillation of five feet from a mean position, the southernmost and northernmost points being reached on January 2 and July 2, respectively.

It is to be remarked that, as I have shown, stellar parallax is legitimately responsible for a part of the observed effect, so that the above numerical value of the linear semi-amplitude of the hypothetical oscillation would be correspondingly reduced, say to three or four feet.

Numerically, therefore, this hypothesis fits the facts, and on that account merely is perhaps worth suggesting for examination; but I presume that the required amount of shift is so great as to make the supposition unacceptable, as an explanation of the phenomenon. Remains, the possibility of anomalous refraction, already suggested by ALBRECHT; but this, by its nature, cannot be intelligibly formulated and tested at present. So as to the remote possibility that there is still lurking a weakness in the joints of the star-group combinations, not protected against by the existing scheme of observation. What seems to me certain is the desirability of enlarging and varying this program to meet and solve if possible this unforeseen dilemma. To dismiss it on the assumption that it is merely some form of purely subjective error for which no imaginable cause can be assigned would be to repeat a mistake that has confused some other astronomical questions at issue within recent memory.

There are three things that can be done, all of them, unfortunately, expensive and laborious. First, establishment of new equatorial and high northerly and southerly stations, either singly or in belts. Secondly and more feasibly accomplished, provision in the existing belt and at the same stations, or at a part of them, of a second observer; in order that the observations, instead of being as now confined to four hours of the night, can embrace the whole diurnal arc between sunset and sunrise, as nearly as possible. This could be accomplished by four-hour shifts for each observer on each night as at present, properly alternated on successive nights or pairs of nights; so as to eliminate personal differences as well as to cover practically the whole visible arcs. Thirdly, the easier but necessary undertaking of the reduction, of all the numerous series of observations made during the past twelve years by TALCOTT's method, to the correct value of the aberration-constant; so that by comparison of homogeneous results we may arrive at the best conclusions about this new and most interesting residual phenomenon, which, by the distinctness of the evidence that supports it, is most emphatic witness to the high precision to which astronomical measurement has been brought.

OBSERVATIONS OF COMET *b* 1902 (PERRINE),

MADE WITH THE 11-INCH EQUATORIAL AT THE SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS.,

By MARY E. BYRD.

1902 Greenwich M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Sept. 2 19 27 37	1	12 , 6	+0 20.36	−3 11.8	3 16 13.55	+35 21 32.3	*n*9.402	0.172	+3.97	+ 1.2
5 16 7 14	2	14 , 8	−0 20.54	+5 58.1	3 12 55.90	+36 37 10.5	*n*9.718	0.569	+4.10	+ 1.4
7 16 2 44	3	12 , 8	+0 9.68	−0 17.1	3 9 55.28	+37 36 16.3	*n*9.720	0.546	+4.23	+ 1.6
8 17 21 7	4	13 , 8	+2 18.70	−0 11.4	3 8 3.45	+38 9 54.3	*n*9.639	0.334	+4.31	+ 2.0
10 18 17 14	5	12 , 8	−1 15.63	+5 56.7	3 3 46.47	+39 20 21.7	*n*9.508	0.069	+4.43	+ 2.1
11 16 53 36	6	12 , 8	+0 24.22	+5 59.6	3 1 27.96	+39 55 28.4	*n*9.663	0.318	+4.51	+ 2.5
14 17 17 48	7	12 , 7	+1 34.22	+4 46.9	2 52 13.47	+42 0 51.2	*n*9.601	0.075	+4.76	+ 3.7
Oct. 8 13 57 12	8	12 , 8	−0 17.22	−4 5.3	19 49 6.49	+41 25 13.1	9.534	9.972	+2.38	+33.6
10 15 27 38	9	11 , 8	−1 6.88	+1 38.8	19 14 20.86	+34 3 46.2	9.691	0.557	+2.15	+30.5
14 14 3 43	11	12 , 8	−0 15.66	−3 27.5	18 30 22.95	+21 21 26.9	9.620	0.646	+2.02	+24.0
16 13 52 1	12	12 , 7	−2 8.04	+1 0.1	18 15 19.47	+16 6 55.8	9.619	0.695	+2.02	+21.5
20 12 51 40	13	12 , 8	+1 58.69	−2 49.4	17 53 31.37	+ 7 55 24.5	9.495	0.717	+2.00	+17.0
25 11 44 55	14	15 , 8	−0 4.15	−3 26.1	17 34 55.55	+ 0 45 28.4	9.542	0.767	+2.05	+13.2
31 11 8 16	15	12 , 9	−0 37.11	+6 36.5	17 18 27.47	− 5 7 15.2	9.549	0.793	+2.03	+10.1
Nov. 1 11 11 59	16	12 , 7	+1 15.71	−6 41.8	17 16 2.46	− 5 55 7.6	9.562	0.795	+2.02	+ 9.6
2 11 7 6	17	11 , 8	+1 40.55	+0 19.2	17 13 37.47	− 6 40 10.1	9.565	0.797	+2.02	+ 9.2

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	3 15 49.22	+35 24 42.9	Lund, A.G. 1731	10*	19 14 34.40	+33 57 28.7	Leiden, A.G. 7247
2	3 13 12.34	+36 31 11.0	Lund, A.G. 1712	11	18 30 36.59	+21 24 30.4	Berlin, B.A.G. 6545
3	3 9 41.37	+37 36 31.7	Lund, A.G. 1680	12	18 17 25.49	+16 5 34.2	Berlin, A.A.G. 6745
4	3 5 40.44	+38 10 3.7	Lund, A.G. 1643	13	17 51 30.68	+ 7 57 56.9	Leipzig II, A.G. 8176
5	3 4 57.67	+39 14 22.9	Lund, A.G. 1634	14	17 34 57.65	+ 0 48 41.3	Nicolajew, A.G. 4379
6	3 0 59.23	+39 49 26.3	Lund, A.G. 1595	15	17 19 2.55	− 5 14 1.8	Paris III, 22043
7	2 50 34.49	+41 56 0.6	Bonn, A.G. 2502	16	17 14 44.73	− 5 48 35.4	Paris III, 21919
8	19 49 21.33	+41 28 44.8	Bonn, A.G. 13492	17	17 11 54.90	− 6 40 38.5	Ottakring, A.G. Zones
9	19 15 25.59	+34 1 36.9	Leiden, A.G. 7258				

* Owing to fog, second measures for Δδ were made from *10 whose difference in declination from *9 was measured by micrometer.

EPHEMERIS OF COMET *d* 1902.

1903 Gr. M.T.	App. α	App. δ	log Δ	1903 Gr. M.T.	App. α	App. δ	log Δ
	h m s	° ′			h m s	° ′	
Feb. 1.5	6 41 31	+13 59.0	0.2852	Feb. 21.5	6 36 21	+20 19.6	0.3166
3.5	40 34	14 39.0		23.5	36 25	20 54.5	
5.5	39 43	15 18.6	0.2899	25.5	36 34	21 28.6	0.3248
7.5	38 56	15 57.9		27.5	36 51	22 2.0	
9.5	38 15	16 36.8	0.2954	Mar. 1.5	37 14	22 34.7	0.3335
11.5	37 41	17 15.4		3.5	37 44	23 6.7	
13.5	37 13	17 53.5	0.3018	5.5	38 21	23 37.8	0.3424
15.5	36 51	18 30.9		7.5	39 4	24 8.2	
17.5	36 34	19 7.8	0.3089	9.5	6 39 54	+24 37.9	0.3516
19.5	6 36 25	+19 44.0					

Computed from RISTENPART's elements, A.N. 3838. — ED.

COMET a 1903.

[From Ritchie's Circular, No. 183, of January 27.]

A message received January 20, from Dr. Kreutz at Kiel, via Harvard College Observatory, announced the discovery of a comet by Giacobini, at Nice, on January 15, together with a position secured at Nice on January 19. Captain C. M. Chester, Superintendent of the U.S. Naval Observatory, transmitted a position by Mr. Dinwiddie of January 21, which was circulated by telegraph to American astronomers, and a third position has been received from Professor Seares, Director of Laws Observatory, taken by himself. The latter was received via Harvard College Observatory. The positions and an orbit from Dr. Kreutz are here given:

1903 Gr. M.T.	α	δ	Observer
Jan. 19.2498	$22^h 57^m 48^s$	$+2° 12' 27''$	Nice
21.4915	23 0 6.5	2 47 46	Dinwiddie
25.5643	23 4 38.3	+3 48 16	Seares

Elements.

T = 1903 March 14.84

$$\left.\begin{aligned} \pi-\Omega &= 133° 37' \\ \Omega &= 2\ 32 \\ i &= 30\ 30 \end{aligned}\right\}\text{Mean Eq. 1903.0}$$

q = .4085

Ephemeris.

1903 Gr. Midnight	α	δ	Light
Jan. 25	$23^h 4^m 36^s$	$+3° 48'$	1.46
29	23 9 24	4 52	
Feb. 2	23 14 40	6 0	
6	23 20 20	+7 13	2.64

Computed from observations of January 19, 21 and 23. Light January 16 = 1.

ELEMENTS AND EPHEMERIS OF COMET a 1903 (GIACOBINI).

By H. R. MORGAN and ELEANOR A. LAMSON.

[Communicated by Captain C. M. Chester, U.S.N., Superintendent].

The following elements were deduced from three observations at Nice, Jan. 19, and at Washington, Jan. 21 and Jan. 23:

Elements.

T = 1903 April 6.4289 Gr.M.T.

$$\left.\begin{aligned} \pi &= 127° 57' 53'' \\ \Omega &= 359\ 39\ 10 \\ i &= 37\ 36\ 38 \end{aligned}\right\}\text{Ecliptic 1903.0}$$

$\log q$ = 9.72151

Residuals (O−C): $\Delta\lambda \cos\beta = -8''.1$

$\Delta\beta = -1.6$

Heliocentric Coordinates.

$$x = r[9.999997] \sin (218° 2' 13'' + v)$$
$$y = r[9.684735] \sin (127\ 39\ 12 + v)$$
$$z = r[9.942091] \sin (128\ 9\ 15 + v)$$

Ephemeris.

1903 Gr.M.T.	α	δ	log Δ	Light
Jan. 31.5	$23^h 11^m 52^s$	$+5° 22'.4$	0.3059	1.3
Feb. 4.5	23 17 14	6 30.9	0.2985	1.5
8.5	23 23 0	7 43.8	0.2901	1.7
12.5	23 29 12	9 1.2	0.2805	2.1
16.5	23 35 52	10 23.5	0.2698	2.4
20.5	23 43 5	+11 51.0	0.2577	2.9

Brightness on Jan. 19.5 is adopted as the unit.

No defined nucleus is seen as yet, but these elements indicate that the comet will be visible continuously east of the sun, becoming very much brighter, and passing the earth in May and June at about 0.5 of a unit's distance.

U.S. Naval Observatory, 1903 Jan. 27.

CONTENTS.

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POSITIONS AND MOTIONS OF 627 STANDARD STARS,

BY LEWIS BOSS.

An effort to ascertain the positions of the principal standard stars with exactness finds its justification, even though a relatively small improvement in our knowledge is thereby attained, since the results have fundamental bearings upon a variety of the more important problems which engage the attention of practical astronomers.

In the derivation of the position of a single star from past observations it may not be of very great importance whether the systematic corrections employed conform to one, or another, of the various standard catalogues. But in computing a large number of positions and motions of stars, as the basis of any investigation requiring high precision, the selection of the standard catalogue from which the systematic corrections of the catalogues of observation are to be ascertained predetermines the final result in a marked degree.

Thus, if the position of the solar apex be derived from proper motions of the stars between $+40°$ and $-30°$ of declination, computed from the various catalogues of observation by the aid of systematic corrections derived from the Standard Catalogue, B, of this paper, its declination will come out more than 10° further north, than it will if systematic corrections in conformity with the standard catalogue of the *Berliner Jahrbuch*, A, be employed. It will thus be seen that the most probable position of the solar apex is far more a matter of the standard catalogue upon which it is based than it is of the mathematical method employed. Any attempt to improve our knowledge of the direction of the sun's way, therefore, involves in the first line an improvement in our knowledge of the positions and motions of the standard stars.

In like manner, any question relating to a supposed rotation of the celestial sphere, or of any part of it, must depend in a very important degree upon the conclusions to be drawn relative to the accuracy of our standard catalogues upon which such computations must ultimately rest.

No great refinement in the computation of orbits of bodies within the solar system, which range over wide limits in declination, is possible without attention to the systematic errors of the star-positions upon which such computations must be based.

Scope of this Work and Acknowledgements.

The catalogue of 627 principal standard stars here presented is the result of an attempt to provide an improved basis for computations relating to real, or apparent, systematic motions of the stars. It is to be regarded as the exhibit of an intermediate stage of an investigation which is in progress and which it is not advisable to complete until further important series of meridian observations provide the necessary basis for a more definitive computation. This is especially pertinent as to the southern hemisphere, in respect to which our knowledge derived from observation is still scanty to a lamentable degree. It is to be hoped that the next few years will witness very important additions to the testimony of observation as to the positions of the brighter stars at various epochs. The plan of observation with the new transit-circle of the Cape observatory may be expected to double, in the systematic sense, the weight of determination of stellar motion for the southern sky; and it is to be hoped that, in the near future, other contributions of a similar character may be secured in respect to that region of sky. Generally we may hope that there will soon be a notable increase of contributions in this line through the re-reduction of older series of observations, through the reduction of recent observations already made, and by further and more precise observations in this field. The stimulus of increased interest in problems relating to the sidereal system seems sufficient to warrant this expectation.

The positions of the present catalogue are the result of successive approximations founded, in the first instance, upon the right-ascensions tabulated in a five-year ephemeris at the end of "NEWCOMB's *Standard and Zodiacal Stars,*" N_1, and upon the declinations of the principal stars contained in the writer's work entitled, "*Declinations of Fixed Stars*" (also declinations of the *American Ephemeris*, 1881 to 1899), B_6. As the right-ascensions now stand they are

supposed to represent essentially the equinox, N_1, of NEWCOMB's "*Equatorial Fundamental Stars*" in so far as terms in $\Delta\alpha_s$ are concerned, but in respect to terms, $\Delta\alpha_\delta$, varying with the declination they present a new and independent system differing decidedly from those of AUWERS and NEWCOMB. In declination a new system has resulted, though it is essentially identical with B_δ as to terms in $\Delta\alpha_s$.

With the exception of Albany observations and the last six hours of the Paris Catalogues, the results of unpublished observations are not included. The results for the last six hours of the Paris Catalogue, for the stars in this catalogue and for many others, were communicated in the most prompt and obliging manner by Director LOEWY of the National Observatory at Paris.

In this investigation were employed all star-catalogues found in the library of the Dudley Observatory which promise useful contributions to this purpose. Exceptions were made as to catalogues of annual results not yet compiled in the form of general catalogues and of some minor and partial catalogues, which were not included in the computations for the standard catalogue.

It may be well to remark that the positions of this catalogue have no dependence, in the systematic sense, upon any meridian observations of a date earlier than the Königsberg observations of 1820. It has also been assumed that in the computations for standard stars no special benefit (but some possible harm) could be anticipated from the employment, even in a differential sense, of observations like those of MAYER, PIAZZI and GROOMBRIDGE, however useful these may become in computations for the positions and motions of stars in general.

I am indebted to the courtesy of the Superintendent of the Naval Observatory, to Professor PICKERING, Director of the Harvard College Observatory, and to Dr. CHANDLER, Editor of the *Astronomical Journal*, for the loan of important star-catalogues not found in the library of this observatory. It may have happened that a few star-catalogues, that might have proved useful, have been overlooked in this necessity of gathering material from so many sources. The present work must be regarded, however, merely as one of the approximations aiming at a more definitive result which cannot be reached with good advantage for some years to come. It is merely supposed that, in this approximation, a stage has been reached such that further amendments to the positions and motions will be small, or, at least, not abruptly different for adjacent regions of sky. Meanwhile the catalogue in its present state is offered as a possible improvement on what has gone before, and as that which will serve as the temporary basis for various works in progress at this Observatory.

Moreover, this is an attempt to produce a consistent system of standard star-positions and motions, by considering simultaneously all the material of observation from pole to pole in a homogeneous investigation, in which each series of observations is designed to exert its due influence upon the result with careful reference to its relations to other series of observations.

The preparation for this research has been in progress at odd times for several years. Within the last two years the time of the Observatory staff has been almost wholly given up to this purpose. Most efficient aid, in the more responsible parts of the work as well as in the details of computation, has been rendered by Assistants ARTHUR J. ROY and WILLIAM B. VARNUM.

Throughout this and related investigations in progress here the work has been efficiently aided by liberal appropriations from the BACHE Fund of the National Academy of Sciences. Without such aid the work would not have been undertaken. Means for publication of the results in cooperation with the *Astronomical Journal* have also been accorded by the Directors of the BACHE Fund. All these grants have been made with a sympathy and readiness of appreciation which have enhanced their value, and for which I express my warmest thanks.

Comparison with the Standard Catalogues of Newcomb and Auwers.

Perhaps the general result of the present research can most readily be defined by comparison with the "*Catalogue of Fundamental Stars for 1875 and 1900,*" N_2, by Professor NEWCOMB, which is now serving as the basis for several astronomical ephemerides, and with the revised catalogues of Dr. AUWERS, as they appear in "*Vorläufige Verbesserung des Fundamental-Catalogs*" (*A.N.*, Bd. 147, p. 49 ff.), A_n; "*Fundamental-Catalog für Zonen-Beobachtungen am Süd-himmel*" (*A.N.*, Bd. 143, p. 361 ff.), A_s; and the revised positions of 303 stars, in an intermediate zone, as published in the *Berliner Jahrbuch* for 1901, A_i. The results of this comparison are presented in the following tables, in which the individual results were obtained by subtraction in the senses respectively indicated, and were then combined into regular groups with the use of weights printed in the catalogue. The epoch of the comparisons in R.A. is uniformly 1900.

RIGHT-ASCENSION; $\Delta\alpha_s$ AND 100 $\Delta\mu_s$.

Decl. +37°.5 to −22°.

	B − N_2			B − A_n		
α	No. ✱✱	$\Delta\alpha_s$	100 $\Delta\mu_s$	No. ✱✱	$\Delta\alpha_s$	$\Delta\mu_s$
h		s	s		s	s
0	25	−.005	−.005	15	.000	+.019
2	26	−.004	−.007	20	−.002	+.016
4	25	−.006	−.011	22	−.003	+.004
6	22	−.005	−.010	20	−.006	−.019
8	14	−.006	−.017	12	−.005	−.021
10	24	+.001	+.014	18	−.004	−.018
12	21	−.002	+.003	16	−.001	−.008
14	20	+.002	+.015	13	+.005	+.010
16	30	+.002	+.004	24	+.001	+.009
18	20	+.005	+.013	19	+.007	+.013
20	28	.000	+.004	21	+.005	−.004
22	31	−.002	+.005	25	.000	+.007

It should be remarked that the positions and motions of many stars were computed which are not included in the present catalogue, B,—especially in the sky south of $-22°$. These additional stars have been made use of in the preceding comparisons as well as in those which follow.

Following are the expressions for $\Delta\alpha_s$ and $\Delta\mu_s$ which result from the preceding table.

$$\begin{array}{llll} B-N_2\colon \ \Delta\alpha_s = & 0.\!\!^s000 & -0.\!\!^s0039 \sin\alpha & -0.\!\!^s0020 \cos\alpha \\ \quad 100\,\Delta\mu_s = & 0.000 & -0.010 & -0.005 \end{array}$$

$$\begin{array}{llll} B-A_n\colon \ \Delta\alpha_s = & +0.0270 & -0.0051 \sin\alpha & +0.0002 \cos\alpha \\ \quad 100\,\Delta\mu_s = & +0.088 & -0.010 & +0.010 \end{array}$$

The periodic terms in $\Delta\alpha_s$, though very small, are clearly indicated. The correction for A_n may safely be assumed to apply also to A_1 and A_s. The assumption of

$$+0^s.027 \ +0^s.088 \ \frac{T'-1900}{100}$$

as the equinox-correction of A_n is somewhat arbitrary; but any defect in this assumption is compensated in the values of $\Delta\alpha_\delta$ and $\Delta\mu_\delta$ which appear in the tables of correction.

In general, the determination of the equinox-correction is beset with minor difficulties due to uncertainties in the errors which depend upon the declination. These uncertainties are also inherent in the original determinations of the successive positions of the equinox. In view of this it may be assumed that the equinox, N_1, has been preserved in the present computation with such accuracy as the state of the case would readily allow.

The system, B, appears to satisfy very well the mean of the best modern determinations of absolute right-ascension in respect to terms in $\Delta\alpha_s$, as will hereafter appear; so that the testimony of these determinations does not point to a correction of B in this respect which would account for any important part of the differences, $B-N_2$ and $B-A_n$.

Right-Ascension; $\Delta\alpha_\delta$ and 100 $\Delta\mu_\delta$.

B — N_2

δ	No. **	$\Delta\alpha_\delta$	100 $\Delta\mu_\delta$	δ	No. **	$\Delta\alpha_\delta$	100 $\Delta\mu_\delta$
+87°	11	+.036	+.180	−5°	21	−.003	−.011
80	6	−.006	+.112	9	30	−.001	+.005
76	10	−.028	−.016	15	29	+.004	+.019
70	11	−.025	−.024	19	25	+.014	+.030
66	8	−.023	−.016	25	22	+.020	+.040
60	17	−.031	−.048	29	16	+.028	+.070
55	11	−.027	−.050	35	17	+.027	+.055
50	17	−.020	−.039	40	16	+.040	+.047
45	16	−.029	−.063	45	25	+.054	+.086
40	20	−.010	−.030	50	12	+.041	+.068
35	16	−.001	−.014	55	13	+.046	+.144
29	22	−.007	−.016	60	19	+.024	+.051
25	24	−.005	−.004	65	13	−.009	+.016
20	24	−.003	−.007	70	10	+.030	+.177
15	23	−.002	.000	75	4	+.019	+.137
10	27	−.002	−.001	79	9	+.055	+.214
+5	33	−.003	+.003	−87	15	−.018	−.113
0	17	−.001	+.006				

For the groups south of $-20°$ the individual differences are very irregular, as will be seen by reference to the individual comparisons printed in connection with the Catalogue.

Right-Ascension; $\Delta\alpha_\delta$ and 100 $\Delta\mu_\delta$.

δ	No. **	B — A_n $\Delta\alpha_\delta$	B — A_n 100 $\Delta\mu_\delta$	B — B.J. $\Delta\alpha_\delta$	B — B.J. 100 $\Delta\mu_\delta$
+87°	7	−.049	−.015	−.102	−.134
80	6	+.048	+.224	+.107	+.429
76	8	+.019	+.088	+.037	+.112
70	11	+.003	+.060	+.027	−.058
66	9	.000	+.045	+.017	+.116
60	17	−.018	−.001	−.010	−.079
55	10	−.031	−.031	−.021	−.016
50	17	−.026	−.021	−.031	−.036
45	16	−.029	−.026	−.028	−.017
40	20	−.019	−.004	−.019	+.014
35	16	−.021	−.012	−.024	−.014
29	20	−.017	−.022	−.023	−.033
25	22	−.015	−.023	−.009	−.005
20	18	−.007	−.030	−.007	−.014
15	22	−.002	−.005	.000	−.007
10	25	−.001	−.020	.000	−.014
+5	23	+.006	+.007	+.011	+.019
0	14	+.012	+.020	+.019	+.027
−4	14	+.006	+.024	+.017	+.032
9	21	+.010	+.018	+.006	+.016
15	22	+.013	+.022	+.018	+.063
19	12	+.011	+.036	+.001	+.017
25	13	+.016	+.041	+.025	+.046
−30	6	+.035	+.095	+.035	+.095

The meridians of right-ascension for 45° on either side of the equator, as defined by N_2 and A, seem to be inclined with reference to the meridian defined by B, both in the same direction and by nearly the same amounts. For both N_2 and A_s the discrepancy amounts to about

$$+0^s.045 \ +0^s.08 \ \frac{T'-1900}{100}$$

in the neighborhood of $-45°$ of declination; and to something like

$$-0^s.025 \ -0^s.02 \ \frac{T'-1900}{100}$$

for A_n, and

$$-0^s.017 \ -0^s.04 \ \frac{T'-1900}{100}$$

for N_2, at $+45°$. These discrepancies have received careful attention and, while the meridian south of the equator is still very uncertain, it does not seem probable that any considerable part of the differences is attributable to error in the meridian B, so far as weight of existing testimony is competent to decide.

The system of the "303 stars," A_1, and of the southern stars, A_s, should be conformable in right-ascension with A_n. The determination of $\Delta\alpha_\delta$ and $\Delta\mu_\delta$ will therefore suffice for these.

RIGHT-ASCENSION; $\Delta\alpha_s$ AND 100 $\Delta\mu_s$.

	B — A_i				B — A_s		
δ	No. ✱✱	$\Delta\alpha_s$	100 $\Delta\mu_s$	δ	No. ✱✱	$\Delta\alpha_s$	100 $\Delta\mu_s$
°		s	s	°		s	s
+ 2	24	+.009	+.013	−36	28	+.031	+.076
− 5	16	+.013	+.021	−40	23	+.041	+.103
−10	26	+.009	+.035	−45	29	+.045	+.099
−15	21	+.009	+.023	−50	15	+.042	+.081
−20	22	+.013	+.030	−54	18	+.025	+.062
−24	12	+.018	+.049	−60	20	+.010	+.065
				−65	17	+.013	+.081
	B — A_s			−70	13	+.013	+.151
−22	9	+.016	+.030	−75	5	+.012	+.058
−24	21	+.020	+.047	−79	8	+.010	+.130
−29	20	+.037	+.089	−87	13	−.211	−.591

As in A_n, in order to have the differences actually found, $+0^s.027$ should be added to the above values of $\Delta\alpha_s$, and $+0^s.088$ to those of 100 $\Delta\mu_s$, — these corrections corresponding to the difference of equinoxes, B — A.

Comparison of Declinations.

The subjoined tables exhibit the results of comparison for the declinations, corresponding to the epoch 1900, except for B_s, (*Declination of Fixed Stars* and *American Ephemeris*, 1881–1899, and its extension southward from −20°, as published in *Ast. Jour.*, No. 450), for which the epoch of comparison is 1875.

The values of $\Delta\delta_a$ and $\Delta\mu'_a$ are first cleared from the effect of terms in $\Delta\delta_s$ and $\Delta\mu'_s$.

DECLINATION; $\Delta\delta_a$ AND $\Delta\mu'_a$.

α	No. ✱✱	B — N_2 $\Delta\delta_a$	100 $\Delta\mu'_a$	No. ✱✱	B — A $\Delta\delta_a$	100 $\Delta\mu'_a$
		+80° to +40°				
h		″	″		″	″
0	8	+.05	+.09	8	+.15	+.19
2	10	−.07	−.08	9	−.01	−.05
4	10	−.03	+.06	10	−.03	+.03
6	5	+.02	+.08	5	+.03	+.22
8	4	.00	+.05	4	−.08	−.18
10	7	−.06	−.29	7	−.13	−.09
12	10	−.04	−.23	10	−.05	+.02
14	7	+.05	+.15	6	−.01	+.04
16	9	+.05	+.07	9	−.04	−.12
18	12	+.09	+.14	11	+.03	−.06
20	13	−.04	−.06	13	+.02	+.05
22	8	+.01	+.02	8	+.08	−.01
		+40° to −22°				
0	28	+.04	+.04	17	+.04	+.04
2	26	+.02	+.08	20	+.01	+.05
4	27	+.05	+.05	24	+.02	−.01
6	21	+.05	+.16	19	+.02	+.14
8	16	+.03	+.01	14	−.17	−.23
10	23	+.01	−.02	17	−.13	−.25
12	22	−.06	−.07	17	−.03	−.18
14	22	−.04	+.04	16	+.04	+.12
16	32	−.06	−.12	26	−.02	.00
18	21	−.01	−.06	20	+.01	+.06
20	28	−.05	−.12	22	+.06	−.03
22	32	+.04	+.10	25	+.13	+.19
		−22° to −70°				
h		″	″		″	″
0	13	+.16	+ .16	14	+.17	+.65
2	11	+.15	+ .28	15	+.18	+.56
4	16	−.10	− .09	19	−.06	−.03
6	15	+.01	− .08	19	−.06	−.32
8	13	−.11	− .28	14	−.10	−.11
10	16	+.29	+1.09	18	−.07	−.11
12	13	+.05	− .07	15	−.18	−.32
14	11	−.10	+ .07	15	+.12	+.58
16	18	−.08	− .13	26	+.08	+.27
18	25	−.05	− .12	26	.00	−.48
20	11	−.09	− .57	10	−.17	−.98
22	10	−.08	− .25	13	+.14	+.26

The periodic terms result as follows:

Limits		B — N_2, $\Delta\delta_a$		B — N_2, 100 $\Delta\mu'_a$	
+80°	+40°	−.033 sin α	−.002 cos α	−.04 sin α	+.05 cos α
+40	−22	+.040	+.032	+.07	+.04
−22	−70	+.056	+.001	+.18	−.15

Limits		B — A, $\Delta\delta_a$		B — A, 100 $\Delta\mu'_a$	
+80°	+40°	−.030 sin α	+.073 cos α	+.02 sin α	+.06 cos α
+40	−22	−.048	+.074	−.06	+.11
−22	−70	−.032	+.080	+.08	+.12

The consistency of the comparisons, B — A, in the several zones is worthy of special note. It seems to offer satisfactory evidence that the numerical accuracy of the computations for the positions and motions of the individual stars in each catalogue are practically above reproach.

From these, omitting NEWCOMB's stars south of −22°, we may assume the definitive corrections to be:

$$\text{B} - \text{N}_2 \qquad \Delta\delta_a = +0''.022 \sin\alpha \;\; +0''.024 \cos\alpha$$
$$100\,\Delta\mu'_a = +0.04 \qquad +0.04$$

$$\text{B} - \text{A} \qquad \Delta\delta_a = -0''.041 \sin\alpha \;\; +0''.075 \cos\alpha$$
$$100\,\Delta\mu'_a = -0.01 \qquad +0.10$$

The values of $\Delta\delta_a$ and 100 $\Delta\mu'_a$ for the *Berliner Jahrbuch*, 1883–1900 may safely be assumed to be the same as for A_n; and for B_s, 1875, practically the same as for the catalogue of the present investigation. In view of the uncertainties relating to our present knowledge of the laws of variation of latitude at different epochs, no highly critical investigation of the systematic terms in $\Delta\delta_a$ for the present catalogue has been undertaken. It is believed that this can be much more effectively done at a later time. It is notable, however, that for the modern series of observed declinations the discrepancies from the standard following the order of right-ascension are comparatively minute, notwithstanding differences of epoch and of the longitude of the observatories concerned. It is proposed to treat this matter more in detail in a subsequent chapter.

It should also be noted that the system, B_s, with which comparison is made, though practically identical in the systematic sense with that of "*Declinations of Fixed Stars,*" differs from it in that the individual positions and motions of 50 stars were first revised for the uses of the present computation.

Declination; $\varDelta\delta_s$ and 100 $\varDelta\mu'_s$ for $B-N_2$ and $B-B_s$.

$B-N_2$				$B-B_s$, 1875			
δ	No. **	$\varDelta\delta_s$	100 $\varDelta\mu'_s$	δ	No. **	$\varDelta\delta_s$	100 $\varDelta\mu'_s$
°		″	″	°		″	″
+87	11	−.08	− .09	–	–	–	–
80	6	−.01	+ .02	+78	11	+.12	+ .34
76	10	+.06	+ .28	–	–	–	–
70	11	−.01	− .06	69	19	+.15	+ .35
66	9	.00	+ .22	–	–	–	–
60	17	−.14	− .08	60	19	+.14	+ .49
55	11	−.17	− .06	55	10	+.16	+ .58
50	17	−.16	− .05	50	17	+.13	+ .48
45	16	−.19	− .09	45	14	+.10	+ .37
40	20	−.31	− .31	40	19	+.09	+ .32
35	16	−.30	− .24	34	14	+.06	+ .14
29	22	−.22	− .02	29	18	.00	+ .17
25	24	−.25	− .21	25	18	−.02	+ .14
20	24	−.22	− .15	20	16	−.04	+ .13
15	23	−.27	− .37	15	19	−.07	+ .14
10	27	−.22	+ .01	10	23	−.05	+ .26
+ 5	33	−.36	− .28	+ 5	21	−.04	+ .25
0	16	−.36	− .23	0	11	−.06	+ .14
− 5	21	−.31	− .27	− 6	13	.00	+ .26
9	30	−.31	− .17	10	15	.00	+ .39
15	29	−.35	− .16	14	18	+.03	+ .45
19	25	−.29	− .16	19	13	+.01	+ .54
25	23	−.26	+ .05	24	21	.00	+ .18
29	18	−.01	+ .40	27	17	+.01	+ .58
35	21	−.17	− .22	32	17	.00	+ .31
40	17	−.03	+ .36	40	15	−.04	+ .42
45	25	+.08	+ .67	44	29	−.01	+1.16
50	12	+.07	+ .69	48	25	−.04	+1.44
55	13	−.02	+ .55	54	21	+.01	+1.15
60	19	+.01	+ .78	59	19	−.02	+1.34
65	13	+.27	+1.41	63	22	−.02	+1.12
70	10	+.17	+ .87	66	15	.00	+1.15
75	4	−.10	− .28	70	11	−.05	+1.61
79	9	+.13	+ .22	−78	12	+.02	.00
−87	15	−.06	− .16	–	–	–	–

Down to the limit of stars which can be observed with advantage in high northern latitudes the systematic differences, $B-N_2$, for the proper motions, are very small, and are chiefly due to the weight which Professor Newcomb attributed to the results of planetary observation (*Ast. Pap. Am. Eph.*, Vol. VIII, Pt. II, p. 191). As to the region south of −30° the values of $\varDelta\mu'_s$ are intrinsically very uncertain on account of the small totality of weight of the observed declinations for the south polar regions; and also, in some degree, because of the defects of the Mural Circle at the Cape, used by Henderson and Maclear, — a matter which will be treated in some detail in a subsequent chapter.

Declination; $\varDelta\delta_s$ and 100 $\varDelta\mu'_s$ for $B-A_n$ and B−B.J.

		$B-A_n$		B−B.J.	
δ	No. **	$\varDelta\delta_s$	100 $\varDelta\mu'_s$	$\varDelta\delta_s$	100 $\varDelta\mu'_s$
°		″	″	″	″
+87	7	−.05	− .05	+.04	+ .49
80	6	+.02	+ .41	−.03	+ .39
76	8	+.21	+ .94	+.39	+1.22
70	11	+.16	+ .77	+.18	+ .82
66	9	+.22	+ .62	+.23	+ .67
60	17	+.15	+ .43	+.16	+ .48
55	10	+.13	+ .28	+.15	+ .23
50	17	+.08	− .10	+.14	+ .17
45	16	+.18	.00	+.21	.00
40	20	−.12	− .99	−.32	−1.51
35	16	−.28	−1.38	−.52	−2.16
29	20	−.19	−1.05	−.06	− .54
25	22	−.23	−1.17	−.22	−1.04
20	18	−.30	−1.38	−.28	−1.34
15	22	−.18	−1.20	−.14	−1.12
10	25	−.17	−1.18	−.15	−1.16
+ 5	23	−.12	−1.01	−.17	−1.07
0	14	−.04	− .84	+.05	− .68
− 5	14	.00	− .92	−.10	−1.09
9	21	+.07	− .99	+.02	−1.11
15	22	+.22	−1.05	+.24	−1.00
19	12	+.38	− .94	+.44	− .74
25	13	+.64	− .63	+.70	− .46
−30	6	+.75	− .82	+.65	− .99

There is, of course, a close general resemblance between the numbers, $B-A_n$ and B−B.J. But the latter, as might have been expected, show very much more clearly the systematic distortions produced by the dependence of the proper motions upon Bradley's declinations. In fact, the zones should be much less than five degrees wide in order properly to show the irregularities in the region +25° to +45°, wherein the defects of Bradley's quadrant are undoubtedly very great. In the tables of systematic differences, further on, it will be assumed that the systematic correction for B.J. (1883–1900) is the same as that for A_n, with the reservation that it would be difficult to assign any very exact corrections to the proper motions of the former in the zone of +25° to +45°.

Declinations; $\varDelta\delta_s$ and 100 $\varDelta\mu'_s$ for $B-A_1$ and $B-A_s$.

$B-A_1$				$B-A_s$			
δ	No. **	$\varDelta\delta_s$	100 $\varDelta\mu'_s$	δ	No. **	$\varDelta\delta_s$	100 $\varDelta\mu'_s$
°		″	″	°		″	″
+ 2	24	−.20		−29	20	−.24	− .65
− 5	16	−.21		35	28	−.02	+ .09
−10	26	−.21		40	23	−.07	− .18
−15	21	−.16		45	29	+.09	+ .33
−20	22	−.06		50	15	+.06	+ .33
−24	12	+.15		54	18	+.04	+ .14
				60	20	+.20	+ .77
				65	17	+.33	+1.02
				70	13	+.40	+ .98
		$B-A_s$		75	8	+.40	+ .52
−22	9	−.26	−.66	79	8	+.47	+ .28
−24	21	−.15	−.34	−87	13	−.05	− .25

Taking into account the smallness of the weights involved, the individual differences which make up the groups in the preceding table agree very well.

Tables of Systematic Correction for N_2 and A.

The results of the foregoing comparisons have been utilized to form tables of systematic corrections for N_2, A_n, A_1 and A_s. In right-ascension no distinction is necessary between the various catalogues published by Dr. AUWERS, beginning with the *Fundamental-Catalog;* but in declination the distinction between the northern, intermediate, and southern catalogues must be preserved, so far as $\Delta\delta_s$ is concerned. Through the use of these tables the positions and motions of many stars not included in the present catalogue can be brought into systematic harmony with it, and apparently without materially less accuracy for the individual stars than could be reached by special computations for these stars in conformity with the system of B. This is especially true of the star-places computed by Dr. AUWERS in the catalogues, A_1 and A_s. As will be seen by reference to the catalogue the positions and motions of south polar stars taken from N_2 agree better with the results of this investigation than do those taken from A_s, which, in turn, are quoted from the *Cape Catalogue for 1890.*

SYSTEMATIC CORRECTIONS: ORDER OF DECLINATIONS.

RIGHT-ASCENSIONS; CORRECTIONS, $\Delta\alpha_s$ AND 100 $\Delta\mu_s$.

δ	B — N_2 $\Delta\alpha_s$ (s)	B — N_2 100 $\Delta\mu_s$ (s)	B — A $\Delta\alpha_s$ (s)	B — A 100 $\Delta\mu_s$ (s)
+85°	0.000	+0.068	+0.029	+0.149
80	−0.017	+0.040	+0.026	+0.127
75	−0.025	+0.004	+0.017	+0.092
70	−0.028	−0.018	+0.007	+0.061
65	−0.027	−0.031	−0.005	+0.031
+60	−0.028	−0.040	−0.021	−0.002
55	−0.026	−0.047	−0.028	−0.019
50	−0.023	−0.048	−0.029	−0.022
45	−0.019	−0.042	−0.026	−0.020
40	−0.013	−0.033	−0.023	−0.016
+35	−0.007	−0.021	−0.020	−0.015
30	−0.005	−0.013	−0.017	−0.018
25	−0.004	−0.006	−0.014	−0.022
20	−0.003	−0.003	−0.010	−0.022
15	−0.002	0.000	−0.005	−0.018
+10	−0.002	0.000	0.000	−0.009
+ 5	−0.002	0.000	+0.005	+0.004
0	−0.002	−0.001	+0.009	+0.016
− 5	−0.002	0.000	+0.010	+0.023
10	0.000	+0.005	+0.009	+0.025
−15	+0.005	+0.014	+0.010	+0.028
20	+0.013	+0.031	+0.013	+0.036
25	+0.020	+0.044	+0.021	+0.054
30	+0.025	+0.054	+0.031	+0.077
35	+0.032	+0.063	+0.037	+0.090
−40	+0.040	+0.070	+0.042	+0.097
45	+0.045	+0.076	+0.043	+0.096
50	+0.045	+0.081	+0.038	+0.084
55	+0.037	+0.087	+0.026	+0.071
60	+0.019	+0.094	+0.016	+0.068
−65	+0.009	+0.097	+0.011	+0.080
70	+0.016	+0.096	+0.012	+0.093
75	+0.035	+0.092	+0.012	+0.100
80	+0.032	+0.069	+0.006	+0.086
−85	0.000	0.000	−0.034	−0.034

DECLINATIONS; CORRECTIONS, $\Delta\delta_s$ AND 100 $\Delta\mu'_s$.

δ	B — N_2 $\Delta\delta_s$ (″)	B — N_2 100 $\Delta\mu'_s$ (″)	B — A_n $\Delta\delta_s$ (″)	B — A_n 100 $\Delta\mu'_s$ (″)
+90°	0.00	0.00	0.00	0.00
85	0.00	0.00	0.00	0.00
80	0.00	0.00	+0.07	+0.44
75	0.00	+0.10	+0.14	+0.84
70	0.00	+0.14	+0.17	+0.79
+65	−0.04	+0.10	+0.18	+0.61
60	−0.09	+0.01	+0.17	+0.43
55	−0.14	−0.05	+0.15	+0.23
50	−0.19	−0.10	+0.12	−0.03
45	−0.23	−0.16	+0.04	−0.38
+40	−0.27	−0.20	−0.09	−0.83
35	−0.27	−0.20	−0.19	−1.12
30	−0.25	−0.17	−0.22	−1.20
25	−0.23	−0.16	−0.24	−1.22
20	−0.23	−0.18	−0.24	−1.24
+15	−0.25	−0.20	−0.21	−1.23
10	−0.28	−0.22	−0.17	−1.13
+ 5	−0.31	−0.23	−0.13	−0.99
0	−0.33	−0.24	−0.07	−0.89
− 5	−0.34	−0.23	+0.01	−0.92
−10	−0.34	−0.20	+0.11	−0.99
15	−0.33	−0.16	+0.26	−0.97
20	−0.28	−0.09	+0.43	−0.88
25	−0.19	+0.06	+0.61	−0.79
30	−0.12	+0.10	+0.83	−0.71
−35	−0.06	+0.16	–	–
40	0.00	+0.36	–	–
45	+0.03	+0.55	–	–
50	+0.04	+0.63	–	–
55	+0.03	+0.71	–	–
−60	+0.10	+0.93	–	–
65	+0.15	+0.97	–	–
70	+0.13	+0.67	–	–
75	+0.06	+0.18	–	–
80	0.00	0.00	–	–
−85	0.00	0.00	–	–
−90	0.00	0.00	–	–

δ	B — A_1 $\Delta\delta_s$ (″)	B — A_1 100 $\Delta\delta'_s$ (″)
+ 5°	−0.21	−0.99
0	−0.21	−0.89
− 5	−0.21	−0.92
10	−0.21	−0.99
15	−0.16	−0.97
20	−0.05	−0.88
−25	+0.17	−0.79

δ	B — A_s $\Delta\delta_s$ (″)	B — A_s 100 $\Delta\delta'_s$ (″)
−20°	−0.22	−0.73
25	−0.20	−0.55
30	−0.15	−0.35
35	−0.08	−0.15
40	−0.03	0.00
45	+0.03	+0.17
50	+0.05	+0.27
55	+0.10	+0.43
60	+0.20	+0.71
65	+0.33	+0.91
70	+0.40	+0.83
75	+0.40	+0.53
80	+0.29	+0.22
85	+0.10	−0.08
−90	0.00	−0.30

SYSTEMATIC CORRECTIONS: ORDER OF R. A.

	Right-Ascension				Declination			
	B — N_2		B — A		B — N_2		B — A	
	$\Delta\alpha$.	100 $\Delta\mu$.	$\Delta\alpha$.	100 $\Delta\mu$.	$\Delta\delta$.	100 $\Delta\mu'$.	$\Delta\delta$.	100 $\Delta\mu'$.
h	s	s	s	s	″	″	″	″
0	−.002	−.005	+.027	+.098	+.02	+.04	+.08	+.10
1	−.003	−.007	+.026	+.095	+.03	+.05	+.06	+.09
2	−.004	−.009	+.025	+.092	+.03	+.05	+.04	+.08
3	−.004	−.011	+.024	+.088	+.03	+.06	+.02	+.06
4	−.004	−.011	+.023	+.084	+.03	+.05	.00	+.04
5	−.004	−.011	+.022	+.081	+.03	+.05	−.02	+.02
6	−.004	−.010	+.022	+.078	+.02	+.04	−.04	−.01
7	−.003	−.008	+.022	+.076	+.02	+.03	−.06	−.04
8	−.002	−.006	+.022	+.074	+.01	+.01	−.07	−.06
9	−.001	−.004	+.023	+.074	.00	.00	−.08	−.08
10	.000	−.001	+.024	+.074	−.01	−.01	−.09	−.09
11	+.001	+.002	+.026	+.076	−.02	−.03	−.08	−.10
12	+.002	+.005	+.027	+.078	−.02	−.04	−.08	−.10
13	+.003	+.007	+.028	+.081	−.03	−.05	−.06	−.09
14	+.004	+.009	+.029	+.084	−.03	−.05	−.04	−.08
15	+.004	+.011	+.030	+.088	−.03	−.06	−.02	−.06
16	+.004	+.011	+.031	+.092	−.03	−.05	.00	−.04
17	+.004	+.011	+.032	+.095	−.03	−.05	+.02	−.02
18	+.004	+.010	+.032	+.098	−.02	−.04	+.04	+.01
19	+.003	+.008	+.032	+.100	−.02	−.03	+.06	+.04
20	+.002	+.006	+.032	+.102	−.01	−.01	+.07	+.06
21	+.001	+.004	+.031	+.102	.00	.00	+.08	+.08
22	.000	+.001	+.030	+.102	+.01	+.01	+.09	+.09
23	−.001	−.002	+.028	+.100	+.02	+.03	+.08	+.10
24	−.002	−.005	+.027	+.098	+.02	+.04	+.08	+.10

Notes Relating to the Catalogue.

The subjoined catalogue of 627 stars is divided into three sections, the limits of which are indicated in the respective captions. The selection of these stars was made with the idea that these would be best adapted to serve as a connecting link between the various catalogues of observation. In general, they are the stars whose positions can be computed for the early part of the nineteenth century with the greatest certainty. Suitability for this purpose rather than distribution, or previous use as standard stars, governed the choice. At the lower limit of precision it is doubtless true that other stars might have been introduced that would have been better adapted to the intended use than some stars which have been admitted. But it is believed that the number of these is not very great. Between the limits of −22° and −37°, however, some stars, otherwise suitable, have been omitted pending the definitive reduction of the Albany right-ascensions for 1898.

When the epoch of the position in the catalogue is much earlier than the general mean, it is an indication that a great improvement in the star as a standard would be effected by repeated modern observations; so that, by means of such observations, some of the stars for which the mean date of observation is now in the sixties could easily be placed in a relatively higher class than they now occupy.

One of the most essential qualifications of a standard star is the certainty with which its position can be predicted for future epochs. The observations needed for the present epoch are within the control of astronomers; but those of past epochs can only be improved through a new reduction of the older catalogues. It would therefore seem to be the part of wisdom to select, for the increase of our list of standards, those stars which have been well observed in the first sixty years of the nineteenth century, irrespective of the attention which they have received since that time.

The force of this is all the greater on account of the prevalent idea in regard to the supposed advantages, more imaginary than real, in adherence for a long term of years to the use of some one standard catalogue. If our standard catalogues were to be revised as often as they should be, the necessity for high weight as to the adopted proper motions, though still important, would not be of such vital consequence.

The names of several well known stars will be missed from the present collection. Nearly all of these are open to proof, or at least to well grounded suspicion, of periodic variation in proper motion. The duplicity of some of these stars also constitutes an objection to their use as standard stars. Among them are: *η Cassiopeae, α Can. Majoris, α Geminorum, α Can. min., γ Virginis, ζ Bootis, ζ Herculis,* 61 *Cygni, α Crucis,* and *α Centauri.* It is scarcely necessary to urge that these stars should still be included in observing lists where absolute determinations are intended, and also in those wherein differential determinations for the bright stars is the object in view.

From −20° to −40° of declination there is a rapid falling off in the precision with which the places of the principal stars are known; so that the mean weight of μ and μ' for far southern stars is scarcely one-fifth that for the northern. For this reason it appeared advisable to separate these southern from the northern stars in the catalogue.

It might also be remarked that the computed mean weight of 100 μ in the catalogue is never less than 0.22; so that, where the weight 0.2 is assigned the mean weight of 100μ is about 0.23 for such stars.

It remains to explain the numbers printed in the catalogue, so far as this seems to be required.

No. The stars in the three divisions of the catalogue are numbered together according to their order in right-ascension.

Magnitude. The magnitudes are adopted from the Harvard Photometry. For the northern stars the magnitudes are mostly copied from Newcomb's *Fundamental Catalogue*, these having also been taken from the Harvard Photometry.

Sec. Var. The secular variations are computed from Professor Newcomb's constants contained in his recent work upon the *Precessional Constant.* They are practically identical with those computed from the constants of Struve and Peters. The secular variations in R.A. are given to the fourth decimal place, and in declination to the third.

μ *and* μ'. The values of μ and μ' correspond strictly to the epoch 1900, and are for R.A. in units of the fourth decimal; for declination in units of the third.

The catalogue was first constructed with the use of STRUVE's precessions throughout. Since these secular variations are virtually identical with those computed from NEWCOMB's constants, the result for annual variation has been the same as it would have been if NEWCOMB's precessions had been used from the first. Accordingly, precessions computed from NEWCOMB's constants have been subtracted from the annual variations as printed in the catalogue, resulting in the values of μ and μ' there given. This course was decided on, in view of the intended investigations of which this catalogue forms a part; and because the precessions of NEWCOMB offer a more consistent basis for correction than that which is afforded by the use of the so-called STRUVE constants. Moreover, from the results of my "*Tentative Researches upon Precession*," etc. (*A.J.* 501), making all due allowances for the uncertainty due to the imperfection and fragmentary nature of the material of observation employed, it seems to the writer probable that, in the interests of further and more comprehensive computations relating to the solar motion, preliminary values of the proper motions corresponding to NEWCOMB's precessions would offer a more convenient and consistent basis.

100 $\Delta\mu$ and 100 $\Delta\mu'$ express, respectively, the computed change of the proper motion in R.A. and declination for one century, under the assumption that stellar proper motion is uniform in the arc of a great circle. The unit is

CATALOGUE OF 627 STANDARD STARS.

FIRST SECTION — (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 $\Delta\mu$		Ep. and Wt.			B — N		B — A	
					.0001	.0001	.0001	T	p_α	p_μ	$\Delta\alpha$ .001	$\Delta\mu$ .0001	$\Delta\alpha$ .001	$\Delta\mu$ .0001
		m	h m s	s	s	s	s				s	s	s	s
1	33 Piscium	4.6	0 0 13.010	+3.0709	− 14	− 13	0	70	32	1.3	−29	− 7	–	–
2	α Andromedae	2.1	0 3 13.022	3.0931	+ 185	+ 106	+ 1	66	160	7.1	− 8	− 2	+ 8	+ 7
3	β Cassiopeae	2.4	0 3 50.291	3.1765	+ 543	+ 675	−11	69	78	3.8	−51	− 4	+ 9	+ 9
5	γ Pegasi	2.9	0 8 5.123	3.0846	+ 102	0	0	68	170	6.9	−12	− 3	+20	+ 8
7	ι Ceti	3.8	0 14 19.981	3.0572	− 22	− 13	0	75	82	2.2	− 7	− 1	+32	+11
12	12 Ceti	6.2	0 24 56.127	+3.0612	+ 9	+ 3	0	75	95	1.9	−13	− 8	+43	+15
14	κ Cassiopeae	4.2	0 27 18.741	3.3777	+ 712	+ 17	0	67	61	2.4	−25	− 1	+27	+14
15	13 Ceti	5.2	0 30 6.043	3.0870	+ 14	+ 273	0	73	34	1.4	+ 7	+ 1	–	–
16	ζ Cassiopeae	3.8	0 31 23.796	3.3200	+ 497	+ 24	0	73	59	2.0	−61	−13	− 9	+ 5
17	π Andromedae	4.4	0 31 32.280	3.1939	+ 244	+ 17	0	76	41	1.3	− 5	− 2	+11	+ 9
18	ϵ Andromedae	4.6	0 33 16.166	+3.1612	+ 208	− 173	− 1	73	55	1.6	−15	− 1	+16	+10
19	δ Andromedae	3.5	0 33 58.726	3.1985	+ 224	+ 107	+ 1	75	41	1.4	−11	− 3	+15	+ 8
20	α Cassiopeae	2.4	0 34 49.739	3.3783	+ 561	+ 61	+ 1	66	122	5.5	−18	− 2	−11	+ 5
21	β Ceti	2.2	0 38 34.215	3.0133	− 54	+ 160	− 1	69	121	3.6	−11	0	+40	+16
22	ζ Andromedae	4.3	0 42 2.191	3.1722	+ 179	− 73	0	75	45	1.4	− 4	+ 1	+16	+ 9
23	δ Piscium	4.6	0 43 29.603	+3.1090	+ 80	+ 55	0	74	69	1.9	− 5	+ 1	+36	+15
24	20 Ceti	4.9	0 47 53.796	3.0638	+ 36	− 4	0	71	36	1.6	+ 8	+ 1	–	–
25	γ Cassiopeae	2.3	0 50 40.139	3.5876	+ 723	+ 41	+ 1	73	83	2.7	− 7	+ 5	+ 9	+10
26	μ Andromedae	3.9	0 51 12.016	3.3161	+ 309	+ 128	+ 1	75	82	1.7	−19	− 4	+16	+14
29	ϵ Piscium	4.5	0 57 45.145	3.1099	+ 88	− 54	0	74	144	3.1	− 4	0	+41	+13
30	μ Cassiopeae	5.2	1 1 36.828	+3.9608	+ 661	+3921	+39	63	34	2.0	+36	+ 9	–	–
32	80 Piscium	5.7	1 3 13.049	3.0868	+ 78	− 182	− 1	65	26	1.0	+ 5	− 2	–	–
33	η Ceti	3.6	1 3 33.548	3.0172	0	+ 141	0	73	35	1.1	−17	− 3	+53	+16
34	β Andromedae	2.4	1 4 7.837	3.3465	+ 289	+ 148	+ 1	75	118	3.9	+ 6	+ 1	− 3	+ 6
36	ζ Piscium	5.4	1 8 30.321	3.1299	+ 91	+ 89	0	71	35	1.4	−44	− 6	–	–
37	θ' Ceti	3.8	1 19 1.492	+2.9978	+ 18	− 54	0	70	136	4.0	+ 8	+ 3	+30	+11
38	δ Cassiopeae	2.8	1 19 16.186	3.8881	+ 793	+ 399	+ 6	67	63	3.7	−34	− 6	+20	+10
41	μ Piscium	5.2	1 24 56.668	3.1394	+ 91	+ 194	0	62	28	1.3	−19	− 5	–	–
42	η Piscium	3.7	1 26 7.861	3.2040	+ 142	+ 18	0	74	128	2.7	+ 9	+ 3	+26	+12
44	υ Persei	3.7	1 31 51.041	3.6600	+ 486	+ 61	0	70	67	2.9	−20	− 3	+ 3	+ 8

the fourth decimal for $100\,\varDelta\mu$, and the third decimal for $100\,\varDelta\mu'$.

Ep. and Wt. T is the mean epoch by weight of all the observations of the star, and p_α and p_δ the weights, respectively, of the R.A. and declination at those epochs. p_μ and p_μ' are, respectively, the weights of the computed centennial motions, $100\,\mu$ and $100\,\mu'$. The probable error of the unit of weight is intended to be, $\pm 0''.30 \sec\delta$, and $\pm 0''.30$, in R.A. and declination respectively. If the weight, p', be desired for any epoch, T', we shall have:

$$p_\alpha' = \frac{p_\alpha \times p_\mu \left(\frac{100}{T'-T}\right)^2}{p_\alpha + p_\mu \left(\frac{100}{T'-T}\right)^2}$$

and correspondingly for the declinations.

B—N *and* B—A. These signify, respectively, the individual comparisons with the catalogues of Newcomb and Auwers, from which the foregoing tables of comparisons have been constructed. The unit for $\varDelta\alpha$ is the third decimal; for $\varDelta\mu$, the fourth; for $\varDelta\delta$, the second; and for $\varDelta\mu'$, the third. In the first section of the catalogue the comparisons are invariably with A_n; and south of $-22°$ always with A_s.

Explanation of the manner in which the right-ascensions and declinations of the catalogue were formed, together with tables of adopted weights and systematic corrections for the catalogues of observation, are to appear in later sections of this paper.

CATALOGUE OF 627 STANDARD STARS.

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V. and Sec. V.		μ' and 100 $\varDelta\mu'$		Ep. and Wt.			B—N		B—A		
			.001	.001	.001	T	p_δ	$p_{\mu'}$	$\varDelta\delta$ .01	$\varDelta\mu'$ .001	$\varDelta\delta$ .01	$\varDelta\mu'$ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
1	− 6 16 1.22	+20.137	− 9	+ 90	0	67	35	1.9	− 13	0	–	–	
2	+28 32 17.86	19.884	− 15	− 161	0	64	169	7.6	− 15	+ 2	−14	− 9	
3	+58 35 53.45	19.863	− 17	− 181	0	69	80	4.8	− 17	− 1	+25	+ 6	
5	+14 37 39.16	20.021	− 24	− 13	0	66	165	7.1	− 31	− 4	−24	−14	
7	− 9 22 42.03	19.976	− 36	− 32	0	77	71	2.4	− 28	− 2	+ 6	−11	
12	− 4 30 35.61	+19.921	− 57	− 7	0	76	85	2.0	− 27	− 7	− 3	−10	
14	+62 22 47.67	19.905	− 67	0	0	69	57	3.2	+ 5	+ 3	+30	+ 5	
15	− 4 8 36.16	19.855	− 68	− 19	0	71	34	1.5	− 35	− 2	–	–	
16	+53 20 47.65	19.852	− 75	− 7	0	72	54	2.3	− 5	0	+17	+ 3	
17	+33 10 7.61	19.848	− 72	− 9	0	75	43	1.6	− 37	−10	−18	−19	
18	+28 46 7.60	+19.588	− 75	− 248	0	74	44	1.1	− 15	+ 6	− 3	− 4	
19	+30 18 49.54	19.741	− 78	− 86	0	73	46	1.9	+ 27	+11	−11	−11	
20	+55 59 20.15	19.785	− 83	− 31	0	63	140	7.6	− 5	+ 1	+23	+ 2	
21	−18 32 7.79	19.803	− 82	+ 39	0	68	100	3.4	− 37	− 2	+37	− 8	
22	+23 43 23.35	19.631	− 92	− 80	0	73	56	2.1	− 17	− 1	−17	−13	
23	+ 7 2 27.00	+19.643	− 94	− 44	0	75	75	2.2	− 19	0	− 4	− 8	
24	− 1 41 14.44	19.595	−101	− 16	0	66	41	2.0	− 79	−13	–	–	
25	+60 10 31.00	19.557	−123	− 2	0	69	78	3.2	0	+ 3	+42	+11	
26	+37 57 24.85	19.576	−116	+ 27	0	74	69	2.1	− 28	− 3	−44	−19	
29	+ 7 21 6.33	19.443	−121	+ 29	0	72	134	3.8	− 12	+ 3	−16	−12	
30	+54 25 47.33	+17.771	−185	−1556	−24	63	27	1.5	− 50	− 4	–	–	
32	+ 5 7 14.19	19.106	−1[illegible]0	− 183	+ 1	62	37	1.4	− 93	−12	–	–	e
33	−10 42 44.45	19.148	−129	− 133	− 1	77	37	1.8	− 34	− 7	+22	−11	
34	+35 5 25.40	19.152	−144	− 115	− 1	74	95	4.3	− 11	+ 2	−38	−15	
36	+ 7 2 47.50	19.106	−143	− 52	0	65	38	2.0	− 27	0	–	–	
37	− 8 41 57.79	+18.654	−156	− 213	0	69	119	4.0	− 25	+ 2	− 1	− 7	
38	+59 42 56.03	18.814	−202	− 46	− 2	63	52	3.4	− 54	− 9	+17	+ 5	
41	+ 5 37 41.94	18.641	−175	− 44	0	59	37	1.7	−114	−18	–	–	
42	+14 49 48.99	18.638	−180	− 10	0	72	118	3.5	− 37	− 7	−16	−13	
44	+48 7 17.80	18.346	−218	− 112	0	72	77	3.5	− 2	+ 7	+20	+ 1	

First Section — (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B — N		B — A	
					.0001	.0001	.0001	T	p_a	p_μ	Δα .001	Δμ .0001	Δα .001	Δμ .0001
		M	h m s	s		s	s				s	s	s	s
46	ν Piscium	4.7	1 36 13.587	+3.1184	+ 91	− 14	0	75	100	2.2	− 5	0	+30	+10
47	φ Persei	4.2	1 37 23.346	3.7361	+ 532	+ 28	0	72	50	1.8	−22	− 3	− 2	+ 8
48	τ Ceti	3.7	1 39 25.348	2.7868	+ 9	−1195	+ 6	73	33	1.4	+24	+ 4	+37	+15
49	o Piscium	4.5	1 40 6.713	3.1630	+ 112	+ 46	0	76	115	2.6	−11	− 3	+25	+ 7
52	ζ Ceti	3.9	1 46 31.450	2.9601	+ 24	+ 24	0	73	46	1.6	−15	+ 4	+35	+10
53	ε Cassiopeae	3.5	1 47 11.754	+4.2699	+1004	+ 58	+ 1	68	69	3.6	− 9	+ 6	+22	+14
54	α Trianguli	3.6	1 47 22.733	3.4093	+ 249	+ 12	− 1	75	44	1.7	−13	− 3	+10	+ 8
55	β Arietis	2.7	1 49 6.845	3.3059	+ 183	+ 67	0	73	134	3.6	+ 9	+ 3	+19	+ 9
57	50 Cassiopeae	4.1	1 54 53.214	5.0333	+1894	− 83	− 2	71	81	3.0	+14	+ 8	+39	+17
59	α Piscium	3.9	1 56 52.339	3.1011	+ 84	+ 28	0	67	33	1.5	+26	+ 4	–	–
60	γ Andromedae	2.2	1 57 45.484	+3.6649	+ 394	+ 42	0	70	92	3.9	−12	− 4	+ 4	+10
61	α Arietis	2.2	2 1 32.048	3.3728	+ 204	+ 137	0	67	173	7.2	−10	− 2	+14	+ 8
62	β Trianguli	3.1	2 3 35.438	3.5566	+ 305	+ 123	0	73	56	2.3	−16	− 3	+ 7	+10
63	65 Ceti	4.5	2 7 41.905	3.1747	+ 116	− 17	0	74	53	1.5	−10	− 5	–	–
65	o Ceti	Var.	2 14 17.649	3.0280	+ 63	0	− 1	74	57	1.8	−13	− 2	+41	+12
68	ι Cassiopeae	4.6	2 20 49.279	+4.8828	+1322	− 5	0	70	76	1.7	−27	− 2	+36	+17
69	ξ Ceti	4.3	2 22 50.461	3.1847	+ 116	+ 26	0	75	122	2.9	0	0	+25	+ 9
70	δ Ceti	4.0	2 34 21.356	3.0715	+ 82	+ 7	0	76	72	2.0	−17	− 4	+33	+12
71	θ Persei	4.3	2 37 21.963	4.0744	+ 513	+ 341	+ 2	72	61	2.4	−38	−12	−10	+ 3
73	γ Ceti	3.6	2 38 7.087	3.1044	+ 93	− 98	− 1	68	111	3.0	+ 1	− 1	+32	+11
74	μ Ceti	4.4	2 39 32.096	+3.2375	+ 125	+ 188	0	74	57	1.9	− 2	0	+26	+11
75	η Persei	3.9	2 43 23.880	4.3460	+ 678	+ 27	0	75	52	1.1	−61	−14	− 7	+ 3
76	41 Arietis	3.7	2 44 5.730	3.5213	+ 227	+ 49	0	74	63	1.9	− 5	− 1	+ 5	+ 6
78	η Eridani	4.1	2 51 32.511	2.9289	+ 50	+ 54	− 1	75	59	2.3	−10	− 6	+48	+14
79	ϵ^m Arietis	4.6	2 53 29.529	3.4225	+ 184	− 11	0	72	46	1.8	+ 5	− 1	–	–
81	α Ceti	2.8	2 57 3.057	+3.1317	+ 97	− 9	0	68	163	6.6	−12	0	+34	+11
82	γ Persei	3.1	2 57 32.992	4.3183	+ 593	+ 2	0	75	52	1.4	−33	− 8	− 1	+ 5
84	ρ Persei	Var.	2 58 45.943	3.8301	+ 331	+ 114	0	73	50	1.4	−11	− 2	+14	+12
85	β Persei	Var.	3 1 39.571	3.8878	+ 355	+ 5	0	71	73	2.9	− 4	− 3	+ 4	+ 9
86	ι Persei	4.2	3 1 50.797	4.3062	+ 516	+1290	+10	72	43	1.4	−34	−11	− 1	+ 8
88	δ Arietis	4.6	3 5 54.552	+3.4232	+ 171	+ 106	0	72	84	2.6	−11	− 4	+19	+ 8
90	ζ Arietis	5.0	3 9 9.113	3.4412	+ 176	− 17	0	72	38	1.4	+ 5	+ 2	–	–
92	α Persei	1.9	3 17 10.800	4.2615	+ 482	+ 27	0	67	123	5.4	−15	− 3	+ 8	+ 9
93	o Tauri	3.8	3 19 25.846	3.2239	+ 114	− 44	0	75	75	1.9	+ 2	+ 2	+27	+ 8
94	ξ Tauri	3.8	3 21 44.904	3.2466	+ 116	+ 40	0	71	50	1.5	−14	0	+31	+11
95	5 Tauri	4.3	3 25 21.047	+3.3066	+ 129	+ 11	0	76	71	1.8	−18	− 5	+26	+10
96	ε Eridani	3.8	3 28 13.128	2.8247	+ 56	− 657	+ 1	76	95	2.7	+ 9	+ 3	+43	+15
99	δ Persei	3.2	3 35 48.117	4.2533	+ 414	+ 31	0	70	84	3.6	−14	− 4	− 5	+ 6
100	δ Eridani	3.7	3 38 27.439	2.8719	+ 62	− 63	+ 4	73	56	1.9	−21	− 2	+50	+15
101	17 Tauri	3.8	3 38 56.131	3.5550	+ 177	+ 14	0	69	46	1.3	− 6	− 2	+ 9	+ 6
103	η Tauri	3.0	3 41 32.303	+3.5588	+ 175	+ 14	0	70	134	4.2	−12	− 2	+13	+ 7
105	27 Tauri	3.8	3 43 12.866	3.5601	+ 174	+ 14	0	69	49	1.7	− 4	+ 2	+13	+ 8
107	ζ Persei	3.0	3 47 50.650	3.7621	+ 220	+ 9	0	75	66	1.8	+ 4	− 2	+ 2	+ 5
110	ε Persei	3.0	3 51 8.459	4.0140	+ 286	+ 22	0	71	51	2.4	−32	− 9	0	+ 8
111	γ^1 Eridani	3.3	3 53 21.806	2.7976	+ 45	+ 45	− 1	71	109	3.0	−15	− 2	+40	+11
112	λ Tauri	3.5	3 55 8.332	+3.3193	+ 114	− 3	0	75	59	1.6	−17	− 5	+25	+ 8
114	ν Tauri	4.0	3 57 50.148	3.1877	+ 91	+ 2	0	80	50	1.2	−19	− 6	+24	+ 6
115	37 Tauri	4.5	3 58 46.908	3.5407	+ 151	+ 67	0	73	44	1.5	−12	− 3	–	–
116	48 Persei	4.0	4 1 23.959	4.3406	+ 362	+ 32	0	75	49	1.4	−25	−11	0	+ 4
118	o Eridani	4.1	4 6 59.019	2.9264	+ 59	+ 6	0	74	78	1.9	−12	0	+28	+ 8
119	40 Eridani	4.5	4 10 40.167	+2.7611	+ 16	−1484	−20	70	36	1.6	+ 7	− 1	–	–
123	γ Tauri	3.9	4 14 6.084	3.4097	+ 114	+ 82	0	74	96	2.6	− 9	− 2	+18	+ 7
124	δ^1 Tauri	3.9	4 17 9.998	3.4553	+ 118	+ 77	0	75	59	2.0	− 3	+ 2	+19	+ 8
127	ε Tauri	3.7	4 22 46.581	3.4988	+ 119	+ 81	0	72	115	3.5	− 6	− 1	+16	+ 7
128	α Tauri	1.1	4 30 10.893	3.4385	+ 102	+ 48	− 1	66	178	7.5	+ 3	+ 1	+25	+ 7

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ′ .001	100 Δμ′ .001	Ep. and Wt. T	p_δ	$p_{\mu'}$	B — N Δδ .01	B — N Δμ′ .001	B — A Δδ .01	B — A Δμ′ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
46	+ 4 58 53.64	+18.307	−193	+ 1	0	73	91	2.9	− 34	− 2	− 7	−10	
47	+50 11 5.96	18.248	−233	− 16	0	74	63	2.3	− 25	− 1	+19	0	
48	−16 27 50.72	19.046	−171	+ 856	+ 8	74	36	1.4	− 33	− 2	+48	− 5	
49	+ 8 39 16.09	18.215	−203	+ 50	0	74	105	3.2	− 10	+ 6	−11	− 9	
52	−10 49 44.63	17.889	−201	− 31	0	77	36	1.1	− 31	− 4	+14	−11	
53	+63 10 39.51	+17.877	−289	− 17	0	66	61	3.5	− 17	− 2	+ 9	+ 5	
54	+29 5 30.08	17.654	−232	− 232	0	74	48	1.8	− 23	− 2	− 9	−11	
55	+20 19 9.16	17.706	−229	− 111	0	73	125	3.8	− 20	− 1	−30	−13	
57	+71 56 14.85	17.603	−359	+ 23	+ 1	75	78	2.8	+ 3	+ 3	+20	+ 6	
59	+ 2 16 50.66	17.490	−228	− 6	0	67	33	2.0	− 18	0	−	−	
60	+41 50 59.62	+17.406	−270	− 52	0	68	89	4.8	− 30	− 2	− 9	− 9	
61	+22 59 22.60	17.148	−257	− 146	− 1	66	177	7.7	− 22	− 2	−13	−10	
62	+34 30 51.47	17.156	−275	− 46	− 1	73	56	2.0	− 23	− 2	−18	−13	
63	+ 8 22 39.64	17.008	−252	− 7	0	70	57	2.3	+ 16	+10	−	−	ξ^1
65	− 3 25 54.03	16.466	−251	− 237	0	76	48	1.5	− 51	− 8	+ 3	−12	
68	+66 57 10.57	+16.394	−417	+ 14	0	69	66	2.0	+ 10	+ 4	+27	+10	
69	+ 8 0 42.79	16.274	−278	− 4	0	75	112	3.3	− 20	+ 3	− 1	−11	ξ^2
70	− 0 6 10.13	15.670	−286	+ 1	0	79	57	2.1	− 34	− 4	+ 6	− 8	
71	+48 48 19.94	15.415	−387	− 89	− 3	70	68	4.2	− 29	− 2	+ 2	− 3	
73	+ 2 48 51.79	15.312	−294	− 150	+ 1	64	103	3.7	− 30	+ 1	−20	−11	
74	+ 9 41 31.10	+15.356	−311	− 27	− 2	72	60	2.5	− 35	− 2	−21	−12	
75	+55 28 49.84	15.151	−421	− 13	0	75	50	2.3	− 28	− 1	+10	+ 3	
76	+26 50 54.12	15.011	−344	− 113	0	74	64	2.0	− 32	− 2	−40	−12	
78	− 9 17 45.92	14.474	−298	− 215	− 1	75	51	2.5	− 17	− 2	+25	− 5	
79	+20 56 25.46	14.564	−349	− 8	0	70	48	2.1	− 16	+ 2	−	−	
81	+ 3 41 50.84	+14.279	−325	− 78	0	67	159	6.7	− 20	+ 1	− 6	−10	
82	+53 6 53.50	14.320	−447	− 6	0	76	58	2.8	− 42	− 2	− 9	− 4	
84	+38 27 10.22	14.143	−400	− 108	− 1	79	46	1.1	+ 2	+ 6	−28	−20	
85	+40 34 13.50	14.067	−410	− 5	0	68	75	3.6	− 32	− 3	+12	− 3	
86	+49 13 52.69	13.980	−468	− 80	−13	75	53	2.1	+ 62	+18	+13	+ 1	
88	+19 20 54.73	+13.799	−369	− 6	− 1	71	89	2.8	− 29	− 7	−36	−19	
90	+20 40 26.05	13.523	−374	− 75	0	68	51	2.2	+ 2	+ 7	−	−	
92	+49 30 19.18	13.046	−477	− 28	0	65	150	7.9	− 18	0	+ 9	− 2	
93	+ 8 40 37.08	12.846	−364	− 78	0	74	62	2.2	− 32	− 4	−10	−12	
94	+ 9 23 2.57	12.727	−371	− 41	0	71	53	1.8	+ 1	+ 5	− 5	− 7	
95	+12 35 38.43	+12.520	−382	− 4	0	74	64	2.2	− 29	− 6	+ 2	− 7	f
96	− 9 47 48.19	12.340	−323	+ 13	+ 8	77	81	2.4	− 46	−12	+16	−12	
99	+47 28 4.36	11.764	−507	− 33	0	70	96	4.9	− 11	+ 3	+32	+ 4	
100	−10 6 6.67	12.352	−346	+ 743	+ 1	70	43	1.9	+ 16	+13	−10	−22	
101	+23 47 56.22	11.525	−428	− 49	0	70	45	1.9	− 16	+ 1	−25	−14	
103	+23 47 45.44	+11.340	−432	− 48	0	67	134	5.0	− 15	+ 2	− 3	− 6	
105	+23 44 51.52	11.217	−434	− 50	0	70	46	1.4	− 28	− 2	−19	−13	
107	+31 35 12.02	10.913	−465	− 17	0	74	63	2.0	− 32	− 2	−26	−15	
110	+39 43 15.43	10.656	−500	− 31	0	74	47	2.0	− 30	− 4	−11	−11	
111	−13 47 34.66	10.410	−352	− 112	− 1	70	114	4.5	− 25	− 2	+15	−14	
112	+12 12 28.12	+10.375	−418	− 14	0	76	54	1.5	− 24	− 3	− 4	−10	
114	+ 5 42 42.80	10.180	−405	− 7	0	81	44	1.1	− 21	− 2	− 1	− 5	
115	+21 48 31.23	10.052	−451	− 63	− 1	69	43	2.1	− 40	− 6	−	−	
116	+47 26 44.06	9.886	−555	− 31	0	75	42	1.8	− 8	+ 1	+18	+ 2	c
118	− 7 5 53.89	9.571	−380	+ 81	0	75	63	1.6	− 40	− 5	− 2	−10	o^1
119	− 7 48 30.67	+ 5.768	−342	−3437	+19	71	43	1.7	− 32	− 2	−	−	o^2
123	+15 23 10.26	8.910	−450	− 27	− 1	72	93	3.4	− 22	− 1	− 4	− 9	
124	+17 18 28.76	8.662	−458	− 34	− 1	73	54	2.3	− 29	− 4	−21	− 9	
127	+18 57 31.22	8.213	−470	− 38	− 1	71	106	3.0	− 19	− 4	−11	− 9	
128	+16 18 29.77	7.466	−467	− 191	− 1	64	181	7.7	− 26	− 2	−17	−12	

First Section — (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.	.0001	μ and 100 Δμ .0001	.0001	Ep. and Wt. T	p_α	p_μ	B − N Δα .001	B − N Δμ .0001	B − A Δα .001	B − A Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
130	τ Tauri	4.3	4 36 14.523	+3.5965	+ 120	+ 4	0	74	53	2.0	− 6	− 3	+15	+ 7
133	μ Eridani	4.1	4 40 30.111	2.9981	+ 55	+ 13	0	75	76	2.0	− 7	+ 1	+32	+ 9
134	α Camelop.	4.4	4 44 6.327	5.9352	+ 677	+ 10	0	72	77	1.9	−66	−28	+36	+16
135	ι Aurigae	2.9	4 50 28.808	3.9016	+ 141	+ 7	0	74	102	2.2	+ 2	− 1	− 9	+ 2
136	β Camelop.	4.2	4 54 31.222	5.3204	+ 407	+ 3	0	75	64	1.4	+39	+ 8	+13	+ 5
137	ε Aurigae	3.2	4 54 47.487	+4.2975	+ 192	+ 4	0	66	51	2.8	−34	− 8	+ 5	+ 7
138	ζ Aurigae	3.9	4 55 29.189	4.1866	+ 172	+ 10	0	73	44	1.7	−10	− 4	+ 7	+10
139	ι Tauri	4.7	4 57 7.075	3.5829	+ 92	+ 52	0	68	37	1.7	− 2	− 4	+31	+ 8
140	η Aurigae	3.3	4 59 30.044	4.2004	+ 163	+ 26	− 1	69	57	2.3	−29	−13	+ 4	+ 3
142	β Eridani	2.9	5 2 55.999	2.9482	+ 43	− 59	− 1	77	56	2.0	−17	− 3	+39	+12
143	λ Eridani	4.3	5 4 21.630	+2.8698	+ 40	+ 2	0	73	37	1.1	− 7	− 4	+33	+12
144	19 H Camelop.	5.1	5 6 4.177	9.8036	+2022	− 274	− 9	78	40	1.1	−55	− 1	+98	+51
145	α Aurigae	0.2	5 9 18.012	4.4259	+ 157	+ 82	− 5	64	129	6.3	−23	− 5	− 8	+ 6
146	β Orionis	0.3	5 9 43.897	2.8817	+ 39	+ 1	0	67	169	7.4	0	0	+30	+ 8
147	γ Orionis	1.7	5 19 46.028	3.2163	+ 46	− 4	0	74	58	2.8	0	+ 1	+30	+ 9
148	β Tauri	1.8	5 19 58.189	+3.7900	+ 77	+ 24	− 1	66	164	7.4	− 3	− 1	+ 3	+ 6
149	Gr. 966	6.3	5 26 20.993	7.9978	+ 704	− 10	+ 1	74	76	1.6	+ 3	− 7	+54	+15
150	δ Orionis	2.5	5 26 53.844	3.0638	+ 37	+ 1	0	69	145	4.9	− 5	+ 1	+41	+11
152	α Leporis	2.7	5 28 19.173	2.6451	+ 29	+ 2	0	68	80	2.4	−10	− 2	+25	+ 7
154	ε Orionis	1.7	5 31 8.335	3.0430	+ 34	0	0	69	129	4.4	− 4	0	+39	+ 9
155	ζ Tauri	3.0	5 31 40.074	+3.5839	+ 51	+ 3	0	72	58	2.0	− 8	− 3	+16	+ 3
156	ζ Orionis	1.9	5 35 42.767	3.0265	+ 31	+ 5	0	66	57	3.3	− 5	0	–	–
158	κ Orionis	2.2	5 43 0.811	2.8445	+ 26	+ 1	0	74	72	2.6	−15	0	+29	+ 8
160	α Orionis	0.9	5 49 45.463	3.2474	+ 26	+ 19	0	66	176	7.6	− 9	− 1	+31	+ 8
161	β Aurigae	2.1	5 52 11.607	4.4007	+ 36	− 44	0	69	74	2.8	−30	− 6	− 3	+ 5
162	θ Aurigae	2.7	5 52 54.122	+4.0909	+ 28	+ 44	− 1	75	55	1.6	− 8	− 3	− 4	+ 4
165	η Geminorum	3.5	6 8 50.489	3.6221	+ 5	− 44	0	71	86	2.3	−24	− 5	+ 6	+ 3
168	μ Geminorum	3.2	6 16 54.659	3.6306	− 7	+ 44	− 1	69	128	4.1	− 4	− 2	+ 8	+ 2
169	β Can. Maj.	2.0	6 18 17.747	2.6415	+ 16	− 4	0	73	58	1.9	− 3	+ 2	+24	+ 9
171	ν Geminorum	4.1	6 23 1.540	3.5630	− 11	− 6	0	77	40	1.4	+ 6	− 1	–	–
173	γ Geminorum	1.9	6 31 56.119	+3.4671	− 17	+ 31	0	74	136	3.3	− 3	− 2	+20	+ 6
175	ε Geminorum	3.2	6 37 46.812	3.6935	− 38	0	0	72	71	3.1	+ 5	+ 1	+12	+ 6
176	ξ Geminorum	3.4	6 39 40.630	3.3686	− 21	− 77	− 1	77	63	2.1	− 1	− 1	+16	+ 5
182	ζ Geminorum	4.0	6 58 10.711	3.5613	− 52	− 3	0	73	91	2.7	0	− 1	+11	+ 3
183	γ Can. Maj.	4.1	6 59 14.060	2.7144	+ 4	0	0	71	73	1.6	− 9	− 3	+11	0
186	λ Geminorum	3.6	7 12 20.797	+3.4507	− 57	− 33	0	77	79	2.6	−14	− 4	+16	+ 4
188	δ Geminorum	3.6	7 14 9.097	3.5873	− 75	− 13	0	69	133	3.6	−10	− 3	+15	+ 4
190	ι Geminorum	3.9	7 19 31.006	3.7319	− 104	− 86	0	73	61	2.6	− 4	− 1	+ 5	+ 4
192	Gr. 1308	5.8	7 20 28.648	6.2855	− 855	− 5	− 2	72	61	1.2	−94	−24	+ 7	+ 5
193	β Can. min.	3.1	7 21 43.693	3.2559	− 43	− 34	0	76	98	2.6	− 9	− 2	+19	+ 5
195	κ Geminorum	3.7	7 38 24.696	+3.6278	− 111	− 18	0	74	54	1.5	−16	− 4	+18	+ 7
196	β Geminorum	1.2	7 39 11.869	3.6776	− 128	− 471	+ 1	66	167	7.7	+ 2	− 1	+15	+ 5
199	φ Geminorum	5.0	7 47 22.708	3.6785	− 132	− 21	0	68	30	1.4	−10	− 1	–	–
202	χ Geminorum	5.1	7 57 22.664	3.6921	− 150	− 16	0	75	76	1.6	−23	− 5	0	+ 3
206	β Cancri	3.7	8 11 5.565	3.2568	− 72	− 35	0	76	112	2.7	+ 4	0	+18	+ 3
209	o Ursae Maj.	3.5	8 21 57.602	+5.0230	− 768	− 167	− 2	70	78	3.6	−36	− 7	+18	+10
210	η Cancri	5.5	8 26 55.635	3.4762	− 132	− 26	0	74	85	1.5	+ 5	− 1	+ 6	+ 2
212	γ Cancri	4.8	8 37 30.025	3.4788	− 143	− 73	0	72	51	1.9	− 7	− 2	–	–
213	δ Cancri	4.2	8 39 0.194	3.4154	− 128	− 12	− 1	72	75	2.6	−13	− 3	+10	+ 5
214	ε Hydrae	3.5	8 41 28.871	3.1808	− 71	− 127	0	72	138	3.9	− 6	0	+33	+ 8
216	ι Ursae Maj.	3.1	8 52 21.821	+4.1292	− 445	− 437	+ 1	67	96	3.4	− 2	− 2	+ 2	+ 5
217	α² Cancri	4.3	8 53 1.148	3.2861	− 98	+ 25	0	73	82	2.4	+ 3	+ 1	+28	+ 8
219	κ Ursae Maj.	3.7	8 56 48.042	4.1169	− 434	− 30	0	73	60	1.8	−26	− 3	− 1	+ 8
220	σ² Ursae Maj.	4.8	9 1 35.984	5.3420	−1334	− 4	− 2	71	41	1.6	−35	− 1	+23	+17
221	κ Cancri	5.2	9 2 19.913	3.2539	− 94	− 14	0	75	79	2.0	− 5	− 2	–	–

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ′ .001	100 Δμ′ .001	T	p_δ	$p_{\mu'}$	B — N Δδ .01	B — N Δμ′ .001	B — A Δδ .01	B — A Δμ′ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
130	+22 45 54.31	+ 7.141	−493	− 23	0	72	54	2.1	− 28	− 4	−13	−10	
133	− 3 26 16.50	6.805	−414	− 10	0	78	60	1.6	− 29	− 1	+17	− 4	
134	+66 10 22.51	6.523	−821	+ 5	0	72	96	3.4	− 6	0	+ 9	+ 3	F 19
135	+33 0 27.84	5.961	−546	− 27	0	72	92	2.6	− 41	− 6	−23	−13	
136	+60 17 46.19	5.637	−746	− 13	0	72	58	2.3	− 12	− 2	+17	+ 4	F 10
137	+43 40 31.33	+ 5.612	−604	− 15	0	64	58	3.1	− 21	− 2	+ 4	− 3	
138	+40 55 47.52	5.539	−589	− 30	0	71	38	1.7	− 47	− 8	−25	−13	
139	+21 26 49.70	5.383	−506	− 49	− 1	67	36	1.7	+ 4	0	− 3	−13	
140	+41 5 57.36	5.156	−594	− 75	0	68	61	2.6	− 11	− 3	−11	−10	
142	− 5 12 56.44	4.861	−418	− 79	+ 1	80	58	2.2	− 30	− 5	+13	− 6	
143	− 8 52 56.39	+ 4.811	−408	− 8	0	74	30	1.1	− 23	0	− 7	−11	
144	+79 6 59.13	4.828	−1388	+ 154	+ 4	78	45	1.2	− 2	0	+ 7	+ 8	
145	+45 53 46.95	3.970	−633	− 429	− 1	63	151	7.7	− 14	0	+24	− 1	
146	− 8 19 1.67	4.361	−412	− 1	0	67	150	6.7	− 27	+ 1	+ 8	− 9	
147	+ 6 15 32.76	3.482	−463	− 19	0	75	65	3.2	− 25	− 2	+ 9	− 6	
148	+28 31 22.91	+ 3.307	−546	− 177	0	64	163	7.0	− 18	0	−14	− 9	
149	+74 58 40.00	2.955	−1155	+ 22	0	76	61	1.4	+ 5	+ 5	+31	+12	74 B Cam.
150	− 0 22 23.36	2.882	−443	− 3	0	67	128	4.8	− 30	− 1	−10	−11	
152	−17 53 37.76	2.765	−383	+ 3	0	66	64	2.5	− 11	+ 3	+41	− 8	
154	− 1 15 56.76	2.516	−441	− 2	0	69	106	3.9	− 37	− 3	+ 5	− 4	
155	+21 4 53.63	+ 2.444	−520	− 28	0	71	59	2.3	− 4	+ 4	−10	− 7	
156	− 1 59 43.72	2.113	−440	− 7	0	68	51	2.9	− 7	+ 7	−	−	
158	− 9 42 18.48	1.479	−414	− 5	0	78	64	2.1	− 24	− 2	+ 5	−10	
160	+ 7 23 18.41	0.904	−474	+ 8	0	64	169	7.2	− 21	− 1	−18	−12	
161	+44 56 14.48	0.678	−641	− 5	0	67	92	4.9	− 16	+ 1	+14	+ 1	
162	+37 12 20.19	+ 0.531	−597	− 90	0	73	56	2.3	− 22	+ 1	−19	−10	
165	+22 32 9.01	− 0.790	−527	− 17	0	70	79	2.8	− 9	0	−38	−14	
168	+22 33 53.90	1.591	−528	− 113	0	67	128	4.6	− 23	+ 1	−17	− 9	
169	−17 54 22.48	1.599	−383	0	0	71	41	1.6	− 47	− 4	+18	−10	
171	+20 16 31.57	2.032	−516	− 21	0	73	38	2.1	− 38	− 6	−	−	
173	+16 29 4.78	− 2.831	−500	− 47	0	72	127	4.2	− 16	+ 1	−22	−10	
175	+25 13 48.48	3.310	−530	− 20	0	69	78	3.7	− 47	− 2	−36	−13	
176	+13 0 12.40	3.653	−481	− 200	+ 1	77	59	1.6	− 31	− 7	− 9	− 8	
182	+20 43 1.27	5.042	−500	− 8	0	69	87	3.1	− 21	0	−35	−13	
183	−15 29 7.77	5.138	−380	− 14	0	73	49	1.1	− 30	− 5	+11	−12	
186	+16 43 14.83	− 6.272	−475	− 48	0	75	78	2.9	− 23	− 3	−30	−15	
188	+22 9 59.39	6.390	−493	− 16	0	65	127	4.5	− 27	− 2	−37	−15	
190	+27 59 48.65	6.907	−508	− 90	+ 1	73	70	2.8	− 26	− 3	−46	−17	
192	+68 40 12.23	6.939	−858	− 43	0	76	63	1.5	+ 11	+ 2	+21	+ 8	145 B Cam.
193	+ 8 29 27.19	7.040	−441	− 42	0	76	93	2.7	− 14	+ 5	−23	−13	
195	+24 38 16.07	− 8.408	−477	− 62	0	70	65	2.6	− 22	− 2	−39	−16	
196	+28 16 4.01	8.466	−477	− 58	+ 6	64	165	7.5	− 20	− 2	−23	−12	
199	+27 1 28.75	9.089	−474	− 36	0	66	38	1.6	− 43	−10	−	−	
202	+28 4 29.04	9.876	−465	− 52	0	75	70	1.8	− 15	+ 1	−34	−15	6 Cancri
206	+ 9 29 37.54	10.906	−394	− 54	0	74	101	3.3	− 33	− 2	−25	−12	
209	+61 3 9.21	−11.752	−589	− 114	+ 2	70	90	4.5	− 18	− 1	+ 3	+ 3	
210	+20 46 51.26	12.044	−402	− 54	0	72	74	2.2	− 20	0	−40	−15	
212	+21 49 41.23	12.768	−385	− 50	+ 1	70	53	2.3	− 43	− 7	−	−	
213	+18 31 18.81	13.058	−377	− 239	0	67	75	3.2	− 20	+ 1	−38	−15	
214	+ 6 47 8.64	13.038	−346	− 53	+ 1	71	130	4.0	− 38	− 4	−16	− 9	
216	+48 26 3.76	−13.944	−429	− 249	+ 5	66	117	5.6	− 15	0	+ 3	− 3	
217	+12 14 41.57	13.777	−343	− 40	0	73	78	2.6	− 9	+ 2	−25	−15	
219	+47 33 7.50	14.042	−424	− 66	0	74	69	2.9	− 15	+ 1	− 1	− 4	
220	+67 32 26.09	14.344	−541	− 70	0	70	44	1.7	0	− 4	− 4	+ 4	
221	+11 4 14.54	14.330	−326	− 11	0	74	75	2.7	− 5	+ 2	−	−	

FIRST SECTION — (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.	.0001	μ and 100 Δμ .0001	.0001	Ep. and Wt. T	p_α	p_μ	B − N Δα .001	Δμ .0001	B − A Δα .001	Δμ .0001
		m	h m s	s	s	s	s				s	s	s	s
223	θ Hydrae	3.8	9 9 9.738	+3.1244	− 60	+ 87	− 1	76	68	2.3	−14	− 1	+ 35	+ 7
226	83 Cancri	6.6	9 13 24.092	3.3553	− 135	− 78	0	74	81	1.5	−14	− 3	+ 19	+ 7
228	α Lyncis	3.4	9 14 57.897	3.6676	− 265	− 176	+ 1	72	64	2.6	+ 1	+ 1	+ 14	+11
231	α Hydrae	2.2	9 22 40.417	2.9488	− 13	− 11	0	67	171	7.0	− 5	0	+ 29	+ 8
232	1 H Draconis	4.5	9 22 51.276	8.9179	−7755	− 43	− 3	73	89	2.9	+20	+16	+121	+47
233	θ Ursae Maj.	3.3	9 26 10.340	+4.0388	− 550	−1025	+ 5	68	96	3.8	+ 2	+ 2	+ 8	+ 7
235	ο Leonis	3.8	9 35 48.869	3.2061	− 92	− 98	0	72	98	2.9	− 6	− 3	+ 26	+ 5
236	ε Leonis	3.1	9 40 10.590	3.4138	− 179	− 32	0	70	136	3.8	+ 3	+ 2	+ 10	+ 6
237	υ Ursae Maj.	3.9	9 43 52.979	4.3046	− 809	− 379	+ 3	71	69	2.5	0	+ 4	+ 20	+10
239	μ Leonis	4.1	9 47 4.673	3.4208	− 195	− 163	0	73	96	2.9	+30	+ 8	+ 14	+ 6
240	ν Leonis	5.2	9 52 50.676	+3.2318	− 105	− 21	0	67	21	1.2	+54	+ 8	–	–
242	π Leonis	4.8	9 54 55.787	3.1739	− 80	− 24	0	74	99	2.7	+ 4	+ 5	+ 20	+ 6
243	η Leonis	3.6	10 1 52.916	3.2768	− 129	− 1	0	75	57	2.1	+70	+22	+ 22	+ 8
244	α Leonis	1.3	10 3 2.838	3.1997	− 100	− 168	0	67	180	7.7	− 3	0	+ 19	+ 5
245	λ Hydrae	3.9	10 5 42.782	2.9244	+ 14	− 137	0	72	43	1.7	− 4	− 1	+ 26	+ 7
247	λ Ursae Maj.	3.6	10 11 4.094	+3.6361	− 381	− 148	+ 1	70	71	3.0	−33	− 7	+ 6	+ 8
249	γ^1 Leonis	2.3	10 14 27.628	3.3141	− 150	+ 215	− 1	70	116	3.8	+ 3	+ 3	–	–
250	μ Ursae Maj.	3.2	10 16 22.431	3.5908	− 358	− 73	+ 1	74	68	2.2	−15	− 5	+ 4	+ 5
252	μ Hydrae	4.1	10 21 15.233	2.9000	+ 40	− 88	0	75	56	1.6	+ 1	+ 1	+ 30	+ 7
253	β Leonis min.	4.4	10 22 6.175	3.4830	− 295	− 99	0	72	32	1.8	−25	− 5	+ 2	+ 9
256	9 H Draconis	5.0	10 26 36.313	+5.2255	−2725	− 79	− 1	73	70	1.9	− 1	+ 4	+ 80	+25
257	ρ Leonis	3.8	10 27 32.795	3.1627	− 79	− 4	0	72	121	2.7	− 8	− 1	+ 27	+ 6
258	34 Sextantis	6.6	10 37 27.679	3.1002	− 44	− 59	0	69	35	1.3	−18	0	–	–
262	53 Leonis	5.3	10 44 0.116	3.1574	− 80	− 1	0	75	101	2.2	− 2	− 2	+ 27	+ 6
263	ν Hydrae	3.3	10 44 41.422	2.9578	+ 54	+ 65	+ 1	70	37	1.6	+15	+ 4	+ 32	+10
264	46 Leonis min.	3.9	10 47 43.269	+3.3673	− 257	+ 73	− 1	74	55	1.6	+ 4	− 1	+ 2	+ 4
266	α Crateris	4.1	10 54 54.104	2.9197	+ 67	− 326	0	77	25	1.0	+11	0	–	–
267	58 Leonis	5.0	10 55 23.794	3.0999	− 37	+ 6	0	74	47	1.4	+ 4	+ 2	–	–
268	β Ursae Maj.	2.4	10 55 48.598	3.6493	− 624	+ 98	− 1	72	83	3.1	−37	− 7	− 11	+ 2
269	α Ursae Maj.	2.0	10 57 33.628	3.7402	− 804	− 169	+ 3	66	123	5.7	−30	− 4	+ 25	+12
270	χ Leonis	4.7	10 59 51.557	+3.0970	− 55	− 233	0	73	92	2.7	− 8	+ 1	+ 20	+ 6
271	ψ Ursae Maj.	3.2	11 4 2.623	3.3904	− 364	− 55	0	71	86	3.8	−28	− 2	+ 4	+11
273	δ Leonis	2.6	11 8 47.474	3.1970	− 132	+ 106	− 1	70	142	4.5	−16	− 2	+ 12	+ 4
274	θ Leonis	3.4	11 8 59.601	3.1523	− 98	− 45	0	77	44	1.2	+10	+ 4	+ 16	+ 5
275	ν Ursae Maj.	3.7	11 13 4.746	3.2513	− 225	− 18	0	72	46	2.1	−11	0	+ 1	+ 6
276	δ Crateris	3.8	11 14 20.439	+2.9967	+ 65	− 85	0	69	117	3.4	+12	+ 3	+ 44	+13
277	σ Leonis	4.1	11 15 58.835	3.0955	− 40	− 63	0	74	81	2.6	− 6	− 1	+ 29	+ 9
279	ι Leonis	4.0	11 18 42.709	3.1297	− 64	+ 105	0	72	57	2.2	+11	+ 1	+ 22	+ 5
280	γ Crateris	4.2	11 19 53.121	2.9931	+ 83	− 75	0	73	25	1.0	− 6	− 7	+ 27	+ 3
281	83 Leonis	6.1	11 21 41.645	3.0380	− 20	− 482	0	69	27	1.5	+44	+11	–	–
282	τ Leonis	5.2	11 22 47.704	+3.0866	− 20	+ 14	0	71	65	1.9	+ 8	+ 6	–	–
283	λ Draconis	4.1	11 25 28.345	3.6126	−1093	− 74	+ 2	70	93	3.8	−26	− 1	+ 37	+19
286	υ Leonis	4.5	11 31 49.728	3.0717	+ 4	+ 1	0	74	108	2.3	+ 3	+ 2	+ 41	+10
287	ν Virginis	4.2	11 40 43.189	3.0851	− 30	− 12	0	62	24	1.3	− 8	+ 2	–	–
288	χ Ursae Maj.	3.8	11 40 46.309	3.1845	− 352	− 137	+ 1	71	73	2.9	−40	− 8	0	+ 5
289	β Leonis	2.2	11 43 57.574	+3.0634	− 71	− 342	+ 1	67	169	7.2	−10	− 1	+ 26	+ 8
290	β Virginis	3.8	11 45 29.182	3.1253	− 1	+ 495	0	70	121	5.5	− 1	+ 1	+ 40	+11
291	γ Ursae Maj.	2.5	11 48 34.376	3.1756	− 431	+ 107	− 2	66	125	5.6	−40	− 8	− 5	+ 5
292	π Virginis	4.6	11 55 44.932	3.0752	− 21	− 3	0	74	58	1.7	+16	+ 6	–	–
293	ο Virginis	4.2	12 0 6.938	3.0575	− 30	− 147	0	76	95	2.6	+ 3	0	+ 30	+ 7
298	4 H Draconis	5.1	12 7 31.107	+2.8662	−1209	+ 32	− 1	71	76	2.6	− 1	+ 6	+ 53	+16
300	δ Ursae Maj.	3.4	12 10 28.746	2.9899	− 420	+ 138	− 2	69	67	2.8	−83	−12	− 2	+ 5
301	γ Corvi	2.7	12 10 39.738	3.0800	+ 116	− 113	0	72	42	1.4	+ 9	+ 1	+ 29	+10
304	η Virginis	4.0	12 14 47.375	3.0684	+ 28	− 41	0	73	141	3.4	−17	− 5	+ 40	+10
305	16 Virginis	5.1	12 15 16.234	3.0463	+ 8	− 200	0	69	15	0.8	–	–	–	–

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ′ .001	100 Δμ′ .001	Ep. and Wt. T	p_δ	$p_{\mu'}$	B − N Δδ .01	B − N Δμ′ .001	B − A Δδ .01	B − A Δμ′ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
223	+ 2 44 10.39	−15.042	−304	− 312	− 1	78	73	2.5	− 32	0	−20	−11	
226	+18 7 45.82	15.112	−318	− 132	+ 1	73	71	1.5	+ 6	+ 4	−26	−14	
228	+34 48 55.57	15.060	−344	+ 10	+ 2	74	66	2.2	− 31	− 2	−33	−13	
231	− 8 13 30.38	15.475	−266	+ 31	0	66	163	7.0	− 37	− 2	+14	− 6	
232	+81 46 6.91	15.541	−816	− 25	0	80	96	2.6	0	+ 2	+ 4	+ 1	
233	+52 7 59.46	−16.247	−351	− 549	+ 9	68	103	5.0	− 27	− 4	+ 9	0	
235	+10 20 50.27	16.248	−267	− 39	+ 1	71	100	4.1	− 48	− 6	−29	−14	
236	+24 14 4.93	16.454	−278	− 24	0	69	139	4.9	− 18	− 2	−32	−15	
237	+59 30 33.04	16.773	−341	− 159	+ 3	64	74	3.7	− 9	− 1	− 6	− 3	
239	+26 28 40.55	16.832	−265	− 63	+ 1	74	88	2.4	− 34	− 9	−50	−18	
240	+12 55 18.34	−17.069	−242	− 29	0	61	29	1.4	− 10	− 2	−	−	
242	+ 8 31 26.50	17.162	−233	− 27	0	74	92	3.1	− 28	0	−34	−17	
243	+17 15 1.00	17.455	−228	− 12	0	71	65	3.0	− 47	− 8	−46	−16	
244	+12 27 21.46	17.496	−219	− 3	+ 1	66	193	8.8	− 26	− 2	−21	−13	
245	−11 51 35.26	17.699	−195	− 93	+ 1	76	40	1.2	− 44	− 6	−10	−17	
247	+43 24 49.69	−17.870	−234	− 45	+ 1	67	80	3.8	− 28	− 7	+13	− 1	
249	+20 20 50.63	18.111	−208	− 153	− 1	68	112	4.4	− 25	− 1	−	−	
250	+42 0 8.77	18.012	−221	+ 20	0	72	71	3.1	− 38	− 7	−13	− 7	
252	−16 19 32.82	18.299	−168	− 84	+ 1	75	43	1.3	− 39	− 5	+31	− 7	
253	+37 13 10.63	18.358	−202	− 112	+ 1	71	41	1.7	− 16	0	−44	−18	
256	+76 13 41.43	−18.415	−293	− 10	0	77	75	1.9	− 8	0	− 5	+ 4	
257	+ 9 49 16.35	18.444	−173	− 6	0	70	117	3.7	− 33	− 3	−27	−15	
258	+ 4 6 20.37	18.738	−151	+ 23	0	67	32	1.3	− 31	− 5	−	−	
262	+11 4 27.56	18.989	−142	− 34	0	75	95	2.3	− 10	− 1	−34	−15	l
263	−15 40 13.30	18.781	−131	+ 193	0	71	32	1.2	− 80	−18	+ 1	−15	
264	+34 45 14.23	−19.348	−144	− 290	0	73	61	1.8	− 48	− 7	−62	−23	
266	−17 45 58.56	19.124	−110	+ 119	+ 1	74	24	1.3	+ 23	+12	−	−	
267	+ 4 9 15.83	19.277	−117	− 21	0	72	45	2.0	− 15	+ 1	−	−	d
268	+56 55 6.48	19.238	−139	+ 28	0	62	85	4.1	− 15	+ 2	0	+ 7	
269	+62 17 27.18	19.381	−138	− 74	+ 1	62	138	6.8	− 15	− 3	+11	+ 6	
270	+ 7 52 35.91	−19.407	−108	− 47	+ 1	72	90	3.6	− 45	− 6	−21	−11	
271	+45 2 27.88	19.490	−111	− 38	0	71	91	4.0	− 33	− 5	+ 3	− 2	
273	+21 4 17.57	19.693	− 95	− 145	0	68	147	5.6	− 29	− 4	−43	−19	
274	+15 58 34.05	19.638	− 93	− 86	0	75	58	2.2	− 23	− 1	−27	−15	
275	+33 38 23.53	19.613	− 88	+ 15	0	71	48	2.4	− 57	−11	−39	−16	
276	−14 14 14.58	−19.455	− 78	+ 195	0	70	113	3.7	− 24	0	+23	−10	
277	+ 6 34 38.48	19.693	− 78	− 15	0	71	80	3.2	− 47	− 2	−23	−12	
279	+11 4 48.15	19.807	− 74	− 85	0	76	64	2.4	− 32	− 1	−16	− 9	
280	−17 8 5.25	19.744	− 67	− 4	0	73	22	1.0	− 44	+ 2	+18	− 7	
281	+ 3 33 28.93	19.594	− 64	+ 173	+ 1	67	29	1.6	− 78	−13	−	−	
282	+ 3 24 25.01	−19.802	− 64	− 19	0	69	60	2.5	− 35	− 2	−	−	
283	+69 52 58.80	19.844	− 70	− 24	0	72	112	4.8	− 14	− 4	+ 5	+ 5	
286	− 0 16 18.10	19.861	− 46	+ 35	0	72	113	3.4	− 39	− 4	−19	−10	
287	+ 7 5 23.02	20.163	− 29	− 187	0	62	31	1.8	− 30	− 1	−	−	
288	+48 20 1.82	19.960	− 30	+ 16	0	72	83	3.6	− 22	− 4	+ 5	− 2	
289	+15 7 51.62	−20.121	− 22	− 123	0	65	166	7.5	− 31	− 5	−37	−16	
290	+ 2 19 41.51	20.286	− 21	− 279	0	68	120	5.9	− 46	− 5	−21	−14	
291	+54 15 2.72	20.019	− 15	+ 3	0	64	139	7.5	− 7	− 1	+16	+ 4	
292	+ 7 10 18.80	20.076	0	− 33	0	71	51	2.4	− 20	− 1	−	−	
293	+ 9 17 18.01	20.009	+ 9	+ 38	0	76	103	3.3	− 17	+ 6	−19	−13	
298	+78 10 18.82	−20.018	+ 22	+ 18	0	77	87	2.4	− 14	− 1	− 1	+ 4	
300	+57 35 17.61	20.023	+ 29	+ 3	0	67	85	4.7	− 20	− 2	+ 5	+ 1	
301	−16 59 12.30	20.014	+ 29	+ 11	0	70	38	1.9	− 57	− 5	−10	−17	
304	− 0 6 40.02	20.030	+ 37	− 25	0	73	138	4.4	− 21	+ 2	−11	−10	
305	+ 3 52 9.60	20.080	+ 38	− 78	0	62	23	1.6	−	−	−	−	

First Section — (Declination, +82° to −21° 50′).

No.	Name	Mag.	R.A. 1900 h m s	Ann. V. s	Sec. V. s .0001	μ s .0001	100 Δμ s .0001	Ep. T	Wt. p_α	Wt. p_μ	B—N Δα s .001	B—N Δμ s .0001	B—A Δα s .001	B—A Δμ s .0001
307	γ Comae Ber.	4.6	12 21 57.314	+2.9957	− 124	− 63	0	72	21	0.9	0	− 6	–	–
308	δ^2 Corvi	3.1	12 24 41.348	3.0990	+ 119	− 144	0	72	70	1.8	−10	− 4	+33	+ 9
310	β Canum Ven.	4.3	12 28 59.717	2.8582	− 193	− 628	+ 5	74	36	1.4	−41	−11	+ 2	+ 4
312	κ Draconis	3.9	12 29 13.002	2.5849	− 529	− 118	+ 3	71	88	2.7	−45	− 6	+26	+13
316	ε Ursae Maj.	1.8	12 49 37.886	2.6524	− 272	+ 140	− 2	71	74	3.1	− 1	+ 1	+ 6	+ 8
317	δ Virginis	3.7	12 50 33.958	+3.0205	+ 27	− 317	0	74	99	2.9	0	+ 1	+32	+ 7
318	α Canum Ven.	2.8	12 51 21.070	2.8132	− 147	− 199	+ 1	68	110	3.8	+10	+ 4	+15	+12
320	ε Virginis	3.0	12 57 11.941	2.9866	− 5	− 186	0	77	110	2.6	− 2	0	+27	+ 9
321	θ Virginis	4.4	13 4 46.299	3.1026	+ 79	− 24	0	74	115	2.7	+10	+ 6	+41	+10
322	53 Virginis	5.1	13 6 44.158	3.1862	+ 141	+ 63	+ 1	69	17	0.9	–	–	–	–
323	β Comae Ber.	4.3	13 7 12.449	+2.8032	− 76	− 604	+ 1	76	60	2.0	−18	− 5	+ 8	+ 5
324	61 Virginis	4.8	13 13 10.348	3.1313	+ 156	− 754	0	71	24	1.3	− 4	− 2	–	–
326	ζ Ursae Maj.	2.1	13 19 54.022	2.4242	− 172	+ 149	− 2	71	68	3.0	−32	− 5	+ 1	+ 9
327	α Virginis	1.2	13 19 55.426	3.1553	+ 116	− 28	0	66	174	7.6	− 7	0	+38	+11
330	ζ Virginis	3.4	13 29 35.826	3.0540	+ 64	− 191	0	74	146	3.7	+11	+ 4	+42	+13
332	82 Virginis	5.3	13 36 21.752	+3.1440	+ 108	− 69	0	75	60	1.6	+ 8	+ 4	–	–
333	τ Bootis	4.5	13 42 30.612	2.8510	− 5	− 340	+ 1	75	71	1.9	+ 6	+ 1	+24	+10
334	η Ursae Maj.	1.9	13 43 36.059	2.3690	− 100	− 122	+ 1	67	133	5.8	−16	− 4	+ 4	+ 7
336	η Bootis	2.8	13 49 55.396	2.8567	− 3	− 45	+ 1	71	153	4.7	− 7	− 1	+26	+ 7
337	τ Virginis	4.3	13 56 33.406	3.0506	+ 65	+ 13	0	74	112	2.8	+ 3	+ 3	+41	+11
340	α Draconis	3.6	14 1 40.899	+1.6228	+ 50	− 80	+ 1	69	107	4.4	−45	− 8	+21	+12
341	κ Virginis	4.3	14 7 33.618	3.1949	+ 123	+ 5	0	72	86	2.8	− 5	− 1	+41	+10
342	ι Virginis	4.2	14 10 46.194	3.1409	+ 106	− 12	+ 2	76	52	1.8	+12	0	+63	+12
344	α Bootis	0.3	14 11 5.999	2.7352	+ 25	− 781	+11	66	176	7.7	− 1	+ 1	+33	+ 8
345	λ Bootis	4.3	14 12 34.962	2.2833	− 49	− 177	0	73	53	2.0	−25	− 6	+ 5	+ 8
346	λ Virginis	4.5	14 13 41.853	+3.2396	+ 141	− 15	0	71	55	1.7	+19	+ 9	–	–
347	θ Bootis	4.2	14 21 47.566	2.0430	− 11	− 260	+ 7	72	78	3.0	−26	− 5	+ 2	+ 6
348	ρ Bootis	3.7	14 27 31.229	+2.5864	− 16	− 76	0	75	99	2.5	− 7	− 3	+14	+ 8
349	5 Ursae min.	4.4	14 27 43.907	−0.1791	+1174	+ 34	− 2	70	45	2.1	+33	+11	–	–
350	γ Bootis	3.0	14 28 3.088	+2.4172	− 28	− 95	0	73	58	2.3	−17	− 4	+ 9	+11
353	μ Virginis	3.9	14 37 47.359	+3.1571	+ 107	+ 71	+ 1	77	61	2.0	− 6	+ 1	+49	+15
355	ε Bootis	2.6	14 40 37.193	2.6202	+ 1	− 36	0	69	126	3.9	+ 3	0	–	–
356	8 Librae	5.3	14 45 9.261	3.3112	+ 155	− 70	0	66	66	3.0	+ 8	+ 3	+41	+13
357	α Librae	2.9	14 45 20.704	+3.3118	+ 155	− 74	0	67	150	5.7	+13	+ 4	+38	+12
358	β Ursae min.	2.2	14 50 59.588	−0.2205	+1005	− 74	+ 2	66	127	5.6	−61	− 9	+29	+16
359	ξ^2 Librae	5.7	14 51 20.446	+3.2489	+ 130	− 2	0	72	48	1.8	+ 9	+ 4	–	–
362	δ Librae	4.9	14 55 37.707	3.2001	+ 116	− 45	0	71	20	1.0	+22	+ 6	–	–
363	β Bootis	3.6	14 58 10.750	2.2597	+ 1	− 39	+ 1	74	82	2.0	−12	− 3	+15	+10
366	ψ Bootis	4.6	15 0 9.630	2.5700	+ 12	− 134	+ 1	74	66	1.5	−10	− 2	+ 4	+ 5
369	ι^1 Librae	4.7	15 6 31.196	3.4121	+ 171	− 26	0	72	49	1.4	+25	+ 5	+57	+17
371	δ Bootis	3.5	15 11 28.269	+2.4188	+ 11	+ 71	0	75	60	1.8	−10	− 3	+ 2	+ 5
372	β Librae	2.8	15 11 37.482	3.2230	+ 118	− 67	0	70	146	4.4	− 2	− 1	+33	+ 8
373	o^2 Librae	6.8	15 17 27.053	3.3397	+ 142	− 2	0	69	23	0.9	− 1	+ 3	–	–
374	η Coron. Bor.	5.0	15 19 4.398	2.4779	+ 18	+ 101	+ 1	71	21	1.4	+ 5	+ 4	–	–
376	μ^1 Bootis	4.3	15 20 42.747	2.2656	+ 14	− 126	0	68	54	1.8	−22	− 5	+ 8	+ 7
377	γ Ursae min.	3.1	15 20 53.046	−0.1273	+ 738	− 25	0	67	94	4.1	−62	− 6	+31	+16
378	ζ^1 Librae	6.0	15 22 36.948	+3.3770	+ 148	+ 11	0	74	55	1.5	+16	+ 5	–	–
379	ι Draconis	3.4	15 22 42.245	1.3293	+ 132	− 7	0	74	56	1.2	−95	−21	−12	+ 3
380	β Coron. Bor.	3.7	15 23 42.356	2.4732	+ 19	− 133	0	76	48	1.5	−14	− 3	+15	+ 9
383	37 Librae	4.9	15 28 42.690	3.2735	+ 119	+ 203	+ 1	70	22	1.2	–	–	–	–
384	γ Librae	4.1	15 29 55.874	+3.3503	+ 136	+ 47	0	72	47	1.5	− 7	0	+47	+13
385	α Coron. Bor.	2.3	15 30 27.223	2.5391	+ 25	+ 90	0	67	164	7.0	− 1	0	+18	+ 7
386	ζ Coron. Bor.	4.6	15 35 36.748	2.2594	+ 22	− 4	0	71	32	0.8	+ 7	+ 1	+37	+16
387	κ Librae	5.0	15 36 11.018	3.4492	+ 157	− 31	+ 1	73	28	1.4	+18	+ 4	–	–
388	η Librae	5.5	15 38 26.789	3.3689	+ 137	− 26	0	67	21	1.1	–	–	–	–

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ′ .001	100 Δμ′ .001	Ep. T	Wt. p_δ	Wt. $p_{\mu'}$	B — N Δδ .01	B — N Δμ′ .001	B — A Δδ .01	B — A Δμ′ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
307	+28 49 27.30	−20.042	+ 50	− 87	0	71	29	1.5	− 23	+ 1	–	–	
308	−15 57 31.46	20.074	+ 57	− 143	0	76	62	2.0	− 24	+ 5	+25	− 8	
310	+41 54 2.87	19.607	+ 60	+ 280	− 1	71	41	1.7	− 18	0	+ 5	− 1	F 8
312	+70 20 21.95	19.878	+ 56	+ 6	0	66	95	4.8	− 8	− 4	+11	+ 7	
316	+56 30 9.29	19.591	+ 92	− 12	0	71	74	3.4	− 9	0	+24	+ 7	
317	+ 3 56 26.80	−19.625	+104	− 64	− 1	72	86	3.5	− 40	− 4	− 7	− 9	
318	+38 51 29.86	19.503	+ 99	+ 43	− 1	68	122	4.4	− 43	− 6	−32	−14	F12
320	+11 29 47.55	19.409	+115	+ 17	− 1	76	95	3.2	− 23	+ 2	−19	−10	
321	− 5 0 18.83	19.294	+134	− 42	0	73	114	4.1	− 33	− 2	− 4	− 9	
322	−15 39 33.16	19.504	+142	− 301	0	69	20	0.9	–	–	–	–	
323	+28 23 5.71	−18.316	+124	+ 875	− 3	75	61	2.0	− 46	− 4	−27	−10	F 43
324	−17 45 18.49	20.118	+148	−1084	− 3	72	23	1.1	− 70	−14	–	–	
326	+55 26 51.03	18.871	+130	− 30	+ 1	65	67	3.9	− 26	0	− 6	− 1	
327	−10 38 22.04	18.876	+165	− 36	0	65	166	7.5	− 44	− 4	+ 7	−10	
330	− 0 5 5.11	18.500	+177	+ 34	− 1	73	126	3.8	− 50	− 5	−16	− 9	
332	− 8 11 54.50	−18.265	+194	+ 36	0	72	49	2.3	− 22	+ 3	–	–	m
333	+17 57 18.06	18.050	+185	+ 25	− 2	75	61	2.0	− 28	− 2	−27	−14	
334	+49 48 44.24	18.054	+158	− 21	− 1	65	160	8.3	− 5	+ 2	+11	+ 1	
336	+18 53 55.90	18.152	+200	− 367	0	70	152	5.2	− 30	− 4	−36	−15	
337	+ 2 1 42.10	17.535	+224	− 25	0	75	88	2.1	− 29	+ 4	+10	− 3	
340	+64 51 13.52	−17.272	+127	+ 15	− 1	70	125	5.4	0	+ 4	+24	+10	
341	− 9 48 30.13	16.891	+253	+ 130	0	74	80	3.0	− 29	− 2	− 4	−15	
342	− 5 31 24.44	17.298	+255	− 427	0	75	49	1.9	− 33	0	+10	− 6	
344	+19 42 10.43	18.858	+217	−2003	− 6	65	186	8.5	− 23	0	−29	−12	
345	+46 32 50.57	16.634	+188	+ 151	− 1	74	66	3.2	− 20	0	+ 9	− 1	
346	−12 54 39.13	−16.708	+267	+ 23	0	68	52	2.2	− 12	+ 3	–	–	
347	+52 18 46.32	16.737	+178	− 406	− 2	73	97	4.6	− 13	− 1	− 6	− 4	
348	+30 48 36.73	15.925	+233	+ 110	− 1	76	98	2.6	− 33	− 3	−32	−14	
349	+76 8 26.02	16.007	− 9	+ 17	0	65	51	2.7	− 18	− 4	–	–	
350	+38 44 43.92	15.864	+218	+ 143	− 1	68	62	3.1	− 33	− 2	− 9	− 4	
353	− 5 13 24.80	−15.802	+300	− 322	+ 1	77	51	1.8	− 21	0	+ 7	− 5	
355	+27 29 44.25	15.314	+253	+ 8	0	66	127	5.1	− 27	− 2	–	–	
356	−15 34 53.67	15.141	+324	− 78	− 1	68	44	2.1	− 49	− 4	+46	− 6	α^1
357	−15 37 34.85	15.129	+325	− 77	− 1	67	134	5.8	− 36	0	+38	− 7	α^2
358	+74 33 51.10	14.716	− 16	+ 5	− 1	65	161	7.7	+ 5	+ 2	+26	+11	
359	−11 0 22.34	−14.703	+328	− 2	0	70	41	2.1	− 27	− 1	–	–	
362	− 8 7 19.97	14.454	+329	− 11	0	69	23	1.1	− 15	+ 4	–	–	
363	+40 47 5.37	14.330	+237	− 43	0	73	82	2.4	− 30	− 3	+ 3	− 3	
366	+27 20 14.54	14.184	+270	− 19	− 1	73	61	1.7	− 34	− 5	−29	−13	
369	−19 24 48.10	13.815	+367	− 49	0	71	40	1.7	+ 8	+ 4	+40	−10	
371	+33 41 15.66	−13.575	+268	− 127	+ 1	74	67	1.9	− 29	− 1	−25	−12	
372	− 9 0 50.76	13.468	+354	− 30	− 1	70	152	5.9	− 38	− 6	+ 3	−10	
373	−14 46 37.98	13.053	+375	+ 3	0	71	20	0.8	− 21	0	–	–	
374	+30 38 55.33	13.147	+283	− 199	+ 1	65	23	1.4	− 28	− 1	–	–	
376	+37 43 39.65	12.760	+258	+ 78	− 1	68	60	2.2	− 35	− 3	−24	−11	
377	+72 11 23.27	−12.814	− 9	+ 12	0	68	94	5.4	− 6	− 1	+11	+10	
378	−16 22 4.69	12.753	+386	− 43	0	74	47	1.1	− 24	0	–	–	
379	+59 18 58.46	12.697	+156	+ 7	0	74	68	2.4	− 9	− 3	−12	− 4	
380	+29 27 0.84	12.560	+284	+ 76	− 2	76	55	1.6	− 32	− 2	− 6	− 5	
383	− 9 43 18.67	12.537	+385	− 244	+ 2	69	22	1.0	–	–	–	–	
384	−14 27 22.03	−12.210	+393	− 2	+ 1	72	42	1.8	− 59	− 8	−14	−16	
385	+27 3 3.74	12.274	+300	− 102	+ 1	67	171	8.4	− 26	− 2	−29	−13	
386	+36 57 37.09	11.819	+271	− 8	0	70	33	1.2	− 23	+ 3	− 2	− 3	
387	−19 21 17.39	11.889	+412	− 119	0	70	24	1.5	−123	−13	–	–	
388	−15 21 15.53	11.687	+405	− 78	0	66	22	1.0	–	–	–	–	

FIRST SECTION — (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.	.0001	μ and 100 Δμ .0001	.0001	Ep. and Wt. T	p_a	p_μ	B — N Δα .001	Δμ .0001	B — A Δα .001	Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
389	γ Coron. Bor.	3.9	15 38 32.595	+2.5188	+ 27	− 75	0	72	31	1.1	−32	− 4	+10	+ 7
390	α Serpentis	2.8	15 39 20.511	2.9522	+ 61	+ 90	0	68	181	7.2	+ 4	+ 1	+36	+ 8
391	β Serpentis	3.7	15 41 34.330	2.7673	+ 43	+ 49	0	73	52	1.8	−24	− 6	+25	+ 7
392	μ Serpentis	3.6	15 44 24.040	3.1270	+ 88	− 59	0	73	49	1.5	+ 7	− 1	+49	+14
393	ε Serpentis	3.8	15 45 49.835	2.9875	+ 65	+ 83	0	75	98	2.6	+ 5	+ 1	+35	+10
395	ζ Ursae min.	4.3	15 47 37.360	−2.2339	+2001	+ 79	− 2	66	107	4.0	−13	− 4	+78	+28
396	θ Librae	4.4	15 48 7.833	+3.4100	+ 134	+ 69	− 1	65	25	1.0	–	–	–	–
398	γ Serpentis	4.0	15 51 50.021	2.7688	+ 57	+ 210	+ 7	71	69	2.3	− 7	0	+29	+11
400	ε Coron. Bor.	4.2	15 53 26.824	2.4822	+ 31	− 62	+ 1	75	47	1.0	+11	+ 4	+27	+11
403	β^1 Scorpii	2.7	15 59 37.260	3.4817	+ 141	− 8	0	68	109	3.4	+24	+ 3	+33	+10
404	θ Draconis	4.1	16 0 0.888	+1.1188	+ 136	− 399	− 4	66	67	2.7	−36	− 8	+12	+ 8
405	ν^2 Scorpii	4.2	16 6 10.935	3.4807	+ 135	− 8	0	70	40	1.4	+42	+ 8	–	–
407	δ Ophiuchi	3.0	16 9 6.252	3.1401	+ 82	− 33	+ 1	71	153	4.4	− 8	− 2	+29	+ 8
409	ε Ophiuchi	3.3	16 13 1.754	3.1704	+ 82	+ 52	0	73	65	2.4	− 4	− 2	+44	+12
411	τ Herculis	3.9	16 16 44.074	1.8011	+ 51	− 12	0	74	70	1.5	−34	−13	+ 7	+ 5
412	γ Herculis	3.8	16 17 30.504	+2.6449	+ 38	− 34	0	73	77	2.4	− 4	0	+22	+10
414	ψ Ophiuchi	4.6	16 18 15.047	3.5056	+ 128	− 13	0	69	21	0.9	–	–	–	–
415	η Draconis	2.9	16 22 38.137	0.8046	+ 183	− 25	− 1	68	92	2.9	−40	− 5	+10	+ 7
418	φ Ophiuchi	4.4	16 25 24.852	3.4287	+ 109	− 38	0	71	24	1.1	–	–	–	–
419	λ Ophiuchi	3.8	16 25 52.151	3.0228	+ 63	− 24	0	72	56	1.6	− 4	− 2	+38	+11
420	β Herculis	2.8	16 25 55.232	+2.5768	+ 36	− 77	0	76	69	1.8	+13	− 1	+ 7	0
421	15 Draconis	5.0	16 28 10.580	−0.1355	+ 407	− 46	− 1	75	70	1.4	+ 6	+ 2	+26	+12
423	σ Herculis	4.3	16 30 52.731	+1.9323	+ 42	− 11	0	74	50	1.7	−20	− 6	−19	0
424	ζ Ophiuchi	2.7	16 31 39.087	3.2996	+ 86	+ 8	0	74	88	2.1	− 3	+ 1	+38	+12
425	24 Scorpii	5.1	16 35 47.312	3.4651	+ 103	− 17	0	72	29	1.2	+13	+ 1	–	–
427	η Herculis	3.7	16 39 28.042	+2.0550	+ 39	+ 28	+ 1	69	68	3.4	+ 2	− 2	+ 2	+ 6
434	κ Ophiuchi	3.4	16 52 56.072	2.8376	+ 43	− 199	0	74	142	3.3	+ 2	0	+39	+10
436	ε Herculis	3.9	16 56 27.800	2.2941	+ 31	− 36	0	73	78	2.2	0	0	+13	+ 7
437	η Ophiuchi	2.6	17 4 38.542	3.4372	+ 71	+ 25	− 1	70	91	2.9	+21	+ 8	+47	+15
439	ζ Draconis	3.2	17 8 29.784	0.1661	+ 190	− 21	− 1	74	64	2.1	− 8	0	+40	+17
440	α^1 Herculis	3.3	17 10 5.245	+2.7340	+ 34	− 8	0	67	156	6.7	− 3	0	+29	+ 8
441	δ Herculis	3.2	17 10 55.434	2.4628	+ 33	− 18	+ 1	75	45	1.9	+ 8	+ 1	+14	+ 3
442	π Herculis	3.3	17 11 33.827	2.0881	+ 33	− 23	0	75	57	1.6	+11	+ 2	+15	+ 8
443	ξ Ophiuchi	4.4	17 15 0.630	3.5928	+ 74	+ 172	+ 2	69	30	1.0	–	–	–	–
452	β Draconis	3.0	17 28 10.376	1.3537	+ 50	− 15	0	68	108	4.8	0	+ 2	− 2	+ 6
454	α Ophiuchi	2.1	17 30 17.535	+2.7833	+ 33	+ 80	+ 2	68	180	7.3	− 2	0	+39	+11
455	ξ Serpentis	3.7	17 31 51.606	3.4330	+ 46	− 31	0	74	31	0.8	+19	+ 8	+31	+12
457	ο Serpentis	4.7	17 35 47.650	3.3699	+ 40	− 49	0	67	23	1.1	+22	+ 6	–	–
458	ι Herculis	3.9	17 36 38.484	1.6917	+ 34	− 10	0	74	63	1.7	−56	−13	− 7	+ 5
459	58 Ophiuchi	4.8	17 37 26.264	3.5935	+ 45	− 64	0	66	22	0.8	–	–	–	–
460	ω Draconis	4.9	17 37 32.136	−0.3557	+ 107	+ 18	−16	73	42	1.3	−37	+ 1	− 6	+ 8
461	β Ophiuchi	2.9	17 38 31.939	+2.9623	+ 27	− 28	− 1	73	110	3.3	− 8	− 2	+39	+ 9
463	μ Herculis	3.5	17 42 32.660	2.3461	+ 38	− 243	+ 6	73	115	3.0	−17	− 4	+21	+ 6
465	ψ^1 Draconis	4.5	17 43 42.962	−1.0761	+ 195	+ 31	+18	73	75	2.4	+20	+11	+58	+22
466	ξ Draconis	3.9	17 51 47.975	+1.0366	+ 34	+ 122	− 2	71	56	2.2	−66	− 9	+ 7	+11
467	θ Herculis	4.0	17 52 49.398	+2.0562	+ 25	+ 1	0	75	43	1.1	−12	− 5	+12	+ 8
468	ν Ophiuchi	3.7	17 53 31.270	3.3014	+ 25	− 8	+ 1	76	55	1.6	+ 5	− 1	+47	+12
469	35 Draconis	5.0	17 53 55.547	−2.6894	+ 79	+ 141	−31	72	42	1.2	+ 6	+20	+55	+33
470	γ Draconis	2.4	17 54 17.028	+1.3917	+ 31	− 10	+ 1	65	124	6.1	−20	− 3	+ 3	+ 8
474	72 Ophiuchi	3.7	18 2 36.515	2.8433	+ 17	− 42	− 1	77	92	1.9	+ 1	+ 3	+24	+ 6
475	ο Herculis	3.8	18 3 38.486	+2.3392	+ 21	− 1	0	75	45	1.1	− 5	+ 1	+12	+ 7
478	μ Sagittarii	4.0	18 7 46.992	3.5876	+ 7	+ 3	0	67	106	3.0	+27	+ 7	+50	+16
480	36 Draconis	5.0	18 13 19.223	0.3452	− 7	+ 530	0	66	25	1.8	−52	− 5	−10	+ 4
482	η Serpentis	3.4	18 16 8.123	3.1030	+ 17	− 374	+ 4	75	103	2.7	+21	+ 5	+48	+13
485	χ Draconis	3.7	18 22 51.628	−1.0779	− 85	+1165	+30	70	73	3.2	−50	− 6	+41	+12

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ′ .001	100 Δμ′ .001	Ep. T	Wt. p_δ	Wt. $p_{\mu'}$	B — N Δδ .01	B — N Δμ′ .001	B — A Δδ .01	B — A Δμ′ .001
	° ′ ″	″	″	″	″				″	″	″	″
389	+26 36 44.00	−11.572	+304	+ 30	− 1	69	38	1.5	− 13	+ 2	−16	− 9
390	+ 6 44 24.14	11.508	+358	+ 38	+ 1	67	174	7.9	− 39	− 5	−27	−13
391	+15 44 4.48	11.442	+338	− 57	+ 1	74	51	1.9	− 20	− 1	−13	− 8
392	− 3 7 27.57	11.208	+382	− 27	− 1	78	38	0.9	− 19	+ 1	+10	− 4
393	+ 4 46 42.72	11.020	+369	+ 57	+ 1	75	85	2.5	− 73	−13	−18	−13
395	+78 6 7.92	−10.949	−267	− 3	+ 1	67	137	6.3	+ 2	+ 1	+ 6	+ 6
396	−16 26 9.43	10.788	+423	+ 121	+ 1	63	25	1.4	–	–	–	–
398	+15 59 15.99	11.933	+349	−1297	+ 3	72	55	2.2	− 47	− 8	−23	−13
400	+27 10 2.11	10.584	+312	− 68	− 1	76	56	1.2	− 25	− 1	−17	− 9
403	−19 31 54.72	10.081	+443	− 29	0	67	97	3.6	− 44	− 1	+47	− 8
404	+58 49 55.97	− 9.684	+141	+ 338	− 5	65	71	4.2	− 22	− 1	+ 6	+ 5
405	−19 12 3.32	9.584	+450	− 32	0	69	33	1.5	+ 16	+ 8	–	–
407	− 3 26 13.41	9.479	+409	− 153	0	70	138	4.8	− 60	− 9	− 9	−10
409	− 4 26 56.03	8.988	+417	+ 33	+ 1	75	59	2.0	− 32	− 4	+13	− 6
411	+46 33 4.93	8.699	+240	+ 31	0	73	81	2.9	− 9	+ 2	+22	+ 3
412	+19 23 15.98	− 8.630	+351	+ 39	0	72	67	2.3	− 14	+ 2	−22	−11
414	−19 48 12.65	8.674	+465	− 63	0	65	20	1.0	–	–	–	–
415	+61 44 25.81	8.204	+110	+ 59	0	66	101	4.7	+ 3	+ 1	+ 8	+ 4
418	−16 23 41.19	8.079	+461	− 38	0	69	23	1.1	–	–	–	–
419	+ 2 12 9.27	8.092	+407	− 88	0	72	54	1.7	− 49	− 9	− 9	− 7
420	+21 42 26.30	− 8.024	+347	− 24	−1	77	76	2.2	− 12	+ 1	−30	−15
421	+68 59 4.20	7.785	− 16	+ 34	−1	74	65	1.9	+ 3	− 2	+13	+ 7
423	+42 38 35.25	7.565	+264	+ 35	0	63	59	2.0	+ 6	+10	+15	− 3
424	−10 21 53.06	7.521	+449	+ 17	0	76	73	1.9	− 51	− 5	− 8	−14
425	−17 32 55.47	7.208	+474	− 7	0	69	28	1.5	− 47	− 3	–	–
427	+39 6 44.01	− 6.995	+285	− 95	0	68	72	3.7	− 23	− 2	−40	−18
434	+ 9 31 49.04	5.797	+396	− 14	− 3	74	132	3.5	− 28	− 3	−28	−13
436	+31 4 24.46	5.466	+324	+ 21	− 1	71	70	2.0	− 26	− 3	−33	−12
437	−15 36 4.36	4.709	+489	+ 86	0	73	73	2.6	− 38	− 5	+16	−13
439	+65 50 15.89	4.447	+ 25	+ 20	0	73	66	2.5	+ 1	+ 3	+ 8	+ 1
440	+14 30 14.87	− 4.304	+391	+ 27	0	65	157	7.4	− 16	− 2	−23	−12
441	+24 57 24.95	4.423	+352	− 163	0	74	47	1.8	− 26	− 5	−19	−11
442	+36 55 17.93	4.207	+299	− 2	0	77	54	1.3	− 28	− 1	−26	−12
443	−21 0 20.02	4.117	+518	− 207	+ 2	66	23	1.0	–	–	–	–
452	+52 22 31.00	2.769	+196	+ 6	0	68	118	5.9	− 20	− 3	+10	0
454	+12 37 57.46	− 2.826	+405	− 235	+ 1	67	180	7.8	− 14	0	− 5	− 8
455	−15 20 8.54	2.521	+497	− 66	0	74	25	0.9	− 59	− 6	+33	− 7
457	−12 49 18.95	2.168	+489	− 55	− 1	72	21	0.9	−106	−17	–	–
458	+46 3 33.67	2.042	+246	− 6	0	75	71	3.2	− 41	− 6	+20	+ 3
459	−21 38 4.78	2.025	+521	− 55	− 1	62	17	0.9	–	–	–	–
460	+68 48 15.17	− 1.635	− 50	+ 327	0	71	52	1.9	+ 14	+ 8	+25	+11
461	+ 4 36 31.88	1.724	+430	+ 151	0	76	93	3.0	− 50	− 7	−19	−11
463	+27 46 44.36	2.276	+338	− 751	− 3	72	101	2.8	− 23	− 2	−21	−11
465	+72 11 52.61	1.691	−156	− 268	0	74	71	2.5	− 5	0	+ 7	+ 7
466	+56 53 17.71	0.642	+153	+ 75	+ 2	67	48	2.5	− 20	− 1	+16	+ 3
467	+37 15 48.88	− 0.624	+300	+ 4	0	74	41	1.6	− 20	0	−19	−10
468	− 9 45 41.07	0.685	+481	− 118	0	79	45	1.3	− 11	+ 1	+13	− 6
469	+76 58 34.06	0.292	−390	+ 239	+ 3	77	40	1.8	− 12	− 3	+11	+ 6
470	+51 30 1.74	− 0.526	+203	− 26	0	65	133	7.1	− 23	− 2	− 1	− 3
474	+ 9 32 58.07	+ 0.310	+414	+ 92	− 1	77	73	1.9	− 39	− 5	− 3	− 5
475	+28 44 54.74	+ 0.320	+341	− 2	0	75	47	1.4	− 27	0	−13	− 8
478	−21 5 6.33	0.676	+522	− 5	0	69	71	2.4	− 19	− 3	+41	−10
480	+64 21 48.08	1.193	+ 57	+ 28	+ 8	64	35	1.9	+ 7	+ 4	+34	+ 8
482	− 2 55 29.86	0.711	+445	− 699	− 5	73	91	3.0	− 54	− 8	−16	−13
485	+72 41 22.27	1.627	−141	− 369	+17	74	78	3.5	+ 23	+ 5	+41	+ 9

FIRST SECTION — (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B — N		B — A	
					.0001	.0001	.0001	T	p_α	p_μ	Δα .001	Δμ .0001	Δα .001	Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
487	α Lyrae	0.1	18 33 33.146	+2.0309	+ 10	+ 174	− 3	67	159	7.2	−16	− 4	+17	+ 9
489	110 Herculis	4.3	18 41 21.471	2.5803	+ 17	− 18	+ 3	73	47	1.6	+14	+ 2	+ 7	+ 4
490	β Lyrae	Var.	18 46 23.277	2.2144	+ 14	+ 3	0	68	134	4.8	+ 7	− 1	+11	+ 7
492	50 Draconis	5.4	18 49 36.178	−1.9122	− 552	− 35	− 8	70	32	0.8	+15	− 4	−	−
493	o Draconis	4.7	18 49 43.560	+0.8877	− 46	+ 105	0	70	45	1.6	−67	−11	+ 2	+ 4
494	θ Serpentis	4.3	18 51 14.906	+2.9826	− 5	+ 31	0	71	47	1.9	+22	+ 3	+47	+13
495	ε Aquilae	4.2	18 55 5.022	2.7218	+ 6	− 44	+ 1	72	78	2.2	− 5	− 2	+27	+ 7
496	γ Lyrae	3.3	18 55 12.171	2.2437	+ 13	− 1	0	75	64	2.0	+11	+ 4	+15	+ 9
498	o Sagittarii	3.9	18 58 41.465	3.5970	− 55	+ 52	0	64	25	1.2	−	−	−	−
501	ζ Aquilae	3.0	19 0 48.836	2.7570	+ 4	− 6	+ 1	70	140	3.9	+ 5	+ 2	+32	+10
502	λ Aquilae	3.5	19 0 56.535	+3.1841	− 21	− 18	+ 1	77	66	2.1	+ 7	+ 2	+39	+ 9
504	π Sagittarii	3.0	19 3 49.038	3.5699	− 59	− 4	0	70	52	1.8	+ 3	+ 1	+36	+11
505	43 Sagittarii	5.0	19 11 47.094	3.5126	− 62	− 8	0	75	48	1.7	+29	+ 7	−	−
506	δ Draconis	3.2	19 12 31.998	0.0256	− 235	+ 173	− 2	66	95	4.5	−11	− 2	+42	+19
507	κ Cygni	4.0	19 14 47.526	1.3881	− 30	+ 70	− 2	71	72	2.5	− 7	− 2	+ 1	+ 5
508	ρ Sagittarii	4.0	19 15 52.423	+3.4822	− 63	− 17	0	67	29	1.4	−	−	−	−
509	τ Draconis	4.6	19 17 28.712	−1.1276	− 588	− 318	−11	72	76	3.0	−65	− 8	+36	+15
510	δ Aquilae	3.4	19 20 27.394	+3.0253	− 19	+ 169	0	70	151	4.5	+ 1	+ 1	+40	+12
512	β Cygni	3.1	19 26 41.306	2.4187	+ 10	− 2	0	72	72	2.7	0	0	+19	+ 8
513	ι Cygni	3.9	19 27 11.087	1.5134	− 25	+ 20	− 2	71	57	2.4	−20	− 3	− 4	+ 5
515	κ Aquilae	4.9	19 31 30.737	+3.2293	− 45	+ 3	0	73	42	1.3	− 6	− 2	−	−
516	θ Cygni	4.7	19 33 45.576	1.6086	− 23	− 30	− 4	71	43	2.3	−33	− 7	+ 5	+ 7
517	55 Sagittarii	5.0	19 36 47.980	3.4344	− 76	+ 42	0	64	25	1.2	0	+ 2	−	−
518	56 Sagittarii	5.1	19 40 31.786	3.5033	− 91	− 95	+ 1	70	26	1.3	+26	+ 4	−	−
519	γ Aquilae	2.8	19 41 30.336	2.8523	− 11	+ 9	0	67	164	7.5	+ 3	+ 2	+31	+10
520	δ Cygni	3.0	19 41 50.975	+1.8755	+ 1	+ 50	0	72	59	1.6	−30	− 5	+12	+11
521	α Aquilae	0.9	19 45 54.256	2.9274	− 19	+ 361	− 2	66	166	6.9	− 5	0	+25	+ 8
522	ε Draconis	4.0	19 48 30.849	−0.1819	− 441	+ 160	+ 1	70	72	1.8	−42	− 9	+37	+16
523	β Aquilae	3.9	19 50 24.073	+2.9469	− 15	+ 23	+ 3	67	167	7.6	− 5	− 1	+27	+ 8
525	θ Aquilae	3.4	20 6 8.744	3.0967	− 43	+ 21	0	75	106	3.0	+ 9	+ 1	+53	+13
526	31 Cygni	3.9	20 10 28.961	+1.8889	+ 4	+ 2	0	74	56	1.2	−39	−12	+ 2	+ 6
527	α^1 Capricorni	4.6	20 12 6.360	3.3283	− 85	+ 10	0	67	87	4.1	+ 5	+ 2	+37	+11
528	κ Cephei	4.4	20 12 15.751	−1.9397	−1676	+ 28	0	73	74	2.4	− 4	+ 4	+45	+16
529	α^2 Capricorni	3.8	20 12 30.424	+3.3318	− 86	+ 40	0	66	137	5.3	+ 5	+ 1	+30	+10
530	β Capricorni	3.2	20 15 23.620	3.3742	− 96	+ 24	0	70	68	2.4	−22	− 6	+40	+13
532	γ Cygni	2.3	20 18 38.348	+2.1522	+ 19	+ 2	0	72	103	4.7	− 7	− 2	+16	+10
533	π Capricorni	5.2	20 21 35.898	3.4385	− 116	+ 8	0	73	37	1.5	+14	+ 4	−	−
534	ρ Capricorni	4.9	20 23 9.466	3.4266	− 115	− 10	0	72	80	1.8	+ 7	+ 3	+37	+13
535	θ Cephei	4.3	20 27 54.285	1.0139	− 152	+ 64	+ 1	69	72	3.0	−15	− 1	+32	+13
536	ε Delphini	4.0	20 28 26.144	2.8665	− 12	+ 6	0	76	104	2.7	− 3	− 1	+30	+ 9
538	β Delphini	3.7	20 32 51.592	+2.8131	− 4	+ 74	0	72	46	1.9	−26	− 8	+32	+11
539	υ Capricorni	5.4	20 34 21.495	3.4200	− 122	− 17	0	71	24	1.0	+11	+ 1	+34	+11
540	α Delphini	3.9	20 34 59.603	2.7866	− 1	+ 45	0	70	75	2.4	− 9	− 2	+28	+10
542	α Cygni	1.3	20 38 1.344	2.0440	+ 22	0	0	66	132	6.8	−17	− 4	−10	+ 5
544	γ^2 Delphini	4.1	20 42 1.151	2.7832	+ 2	− 23	+ 1	69	34	1.1	+17	+ 1	+25	+ 8
545	ε Cygni	2.6	20 42 9.889	+2.4266	+ 28	+ 289	− 1	74	68	2.8	−16	− 5	+12	+ 8
546	ε Aquarii	3.9	20 42 15.806	3.2507	− 84	+ 19	0	75	83	2.0	− 1	+ 2	+49	+18
547	η Cephei	3.6	20 43 15.383	1.2266	− 145	+ 133	−16	66	79	3.4	−10	− 1	− 7	+ 6
549	μ Aquarii	4.8	20 47 15.657	3.2390	− 83	+ 25	0	75	60	1.9	+ 2	0	−	−
551	32 Vulpeculae	5.2	20 50 17.862	2.5555	+ 27	− 6	0	73	77	1.5	−22	− 4	+ 8	+ 4
552	ν Cygni	4.0	20 53 26.669	+2.2346	+ 38	+ 4	0	73	63	2.0	−20	− 4	−12	+ 3
553	η Capricorni	5.0	20 58 42.890	3.4199	− 142	− 30	0	66	22	1.3	−11	− 5	−	−
554	θ Capricorni	4.2	21 0 19.618	3.3779	− 127	+ 57	0	73	53	1.6	+12	+ 6	−	−
555	ξ Cygni	3.9	21 1 17.584	2.1803	+ 42	+ 6	0	72	57	1.6	−13	− 4	− 3	+ 3
556	ν Aquarii	4.5	21 4 8.873	3.2720	− 98	+ 62	0	74	41	1.4	+27	+ 6	+29	+ 9

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ′ .001	100 Δμ′ .001	Ep. T	Wt. p_δ	Wt. $p_{\mu'}$	B—N Δδ .01	B—N Δμ′ .001	B—A Δδ .01	B—A Δμ′ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
487	+38 41 25.44	+ 3.203	+294	+ 279	+ 3	64	168	7.8	− 27	− 1	− 7	− 9	
489	+20 27 1.43	3.254	+368	− 344	0	76	47	1.8	− 23	0	−18	−13	
490	+33 14 46.96	4.023	+315	− 7	0	65	138	5.7	− 27	− 2	−28	−13	
492	+75 18 58.33	4.382	−274	+ 77	− 1	71	25	0.8	+ 88	+26	−	−	
493	+59 15 57.66	4.339	+126	+ 23	+ 2	64	57	3.5	− 15	0	+24	+ 5	
494	+ 4 4 23.76	+ 4.472	+423	+ 26	0	72	46	1.9	− 35	− 1	−15	−12	
495	+14 55 56.34	4.695	+383	− 77	− 1	72	62	2.1	− 10	+ 4	−15	−11	
496	+32 33 7.78	4.775	+315	− 7	0	71	59	2.0	− 30	− 2	−33	−16	
498	−21 53 17.03	5.011	+506	− 67	+ 1	65	19	0.8	−	−	−	−	
501	+13 42 52.56	5.155	+385	− 102	0	69	143	5.1	− 32	− 3	−12	−11	
502	− 5 1 57.55	+ 5.178	+445	− 90	0	77	67	1.8	− 55	− 7	− 4	−11	
504	−21 10 57.67	5.470	+498	− 40	0	72	40	1.7	− 27	− 4	+47	−14	
505	−19 7 51.62	6.158	+484	− 19	0	74	49	1.8	− 32	− 2	−	−	d
506	+67 29 8.27	6.328	+ 3	+ 89	+ 2	67	114	6.1	− 2	+ 1	+30	+ 9	
507	+53 11 1.69	6.544	+190	+ 117	+ 1	71	77	3.9	− 23	− 4	+16	+ 1	
508	−18 2 8.12	+ 6.535	+477	+ 19	0	70	25	0.8	−	−	−	−	
509	+73 10 11.64	6.759	−162	+ 110	− 4	75	68	2.1	− 6	0	+19	+ 9	
510	+ 2 54 54.69	6.971	+414	+ 77	+ 2	70	143	5.2	− 35	− 4	−17	−11	
512	+27 44 58.03	7.394	+324	− 9	0	74	75	2.3	− 22	+ 1	−14	−11	
513	+51 30 59.54	7.566	+202	+ 122	0	71	71	3.9	− 36	− 6	+ 5	− 1	
515	− 7 14 59.66	+ 7.792	+431	− 2	0	74	42	1.2	− 38	− 3	−	−	
516	+49 59 21.57	8.221	+212	+ 247	0	67	68	4.2	− 36	− 4	+20	+ 3	
517	−16 21 30.48	8.201	+454	− 17	+ 1	65	21	0.7	− 30	− 2	−	−	
518	−20 0 6.13	8.417	+457	− 97	− 1	68	22	1.0	− 67	− 9	−	−	
519	+10 22 9.76	8.587	+372	− 4	0	65	167	7.9	− 19	− 1	−15	−12	
520	+44 53 11.40	+ 8.656	+244	+ 37	+ 1	68	75	3.5	− 22	− 7	+22	− 1	
521	+ 8 36 14.56	9.317	+383	+ 380	+ 5	65	172	8.1	− 12	+ 2	− 9	−11	
522	+70 0 47.67	9.172	− 25	+ 31	+ 2	70	66	3.1	+ 9	+ 4	+35	+11	
523	+ 6 9 24.45	8.805	+377	− 483	0	66	166	7.8	− 37	− 2	−22	−12	
525	− 1 7 5.73	10.488	+380	+ 3	0	76	86	2.7	− 36	− 3	0	−10	
526	+46 26 16.25	+10.808	+228	+ 1	0	73	64	2.2	− 33	− 4	+ 4	− 1	o[1] or o[2]
527	−12 49 2.55	10.932	+402	+ 6	0	66	65	3.5	− 24	+ 1	+22	− 9	
528	+77 24 37.15	10.964	−242	+ 26	0	73	77	2.5	− 1	+ 1	+ 6	+ 8	
529	−12 51 17.78	10.961	+403	+ 5	0	64	135	5.4	− 28	− 2	+23	−11	
530	−15 5 50.36	11.167	+404	+ 1	0	70	48	2.1	− 35	− 6	+ 8	−15	
532	+39 56 10.96	+11.398	+253	− 3	0	71	107	4.8	− 32	− 3	−21	−11	
533	−18 32 22.98	11.600	+404	− 13	0	75	36	1.4	− 74	−10	−	−	
534	−18 8 39.73	11.702	+400	− 22	0	71	63	1.8	− 18	− 2	+ 9	−17	
535	+62 39 28.31	12.041	+114	− 17	+ 1	71	71	3.8	− 9	+ 1	+ 9	0	
536	+10 57 47.51	12.069	+328	− 26	0	77	87	2.2	− 23	− 1	− 4	−10	
538	+14 14 49.48	+12.364	+318	− 37	+ 1	73	46	1.7	− 28	− 2	− 3	− 8	
539	−18 29 27.27	12.483	+384	− 21	0	72	27	1.3	− 81	−14	+37	− 9	
540	+15 33 32.65	12.539	+312	− 8	0	69	66	2.7	− 92	−25	−10	−12	
542	+44 55 22.17	12.752	+225	− 1	0	65	168	8.6	− 14	+ 2	+11	− 2	
544	+15 45 49.30	12.817	+303	− 204	0	69	39	1.4	− 43	− 8	−18	−12	
545	+33 35 43.67	+13.352	+267	+ 322	+ 3	71	75	3.2	− 37	− 4	−20	−14	
546	− 9 51 43.15	13.003	+355	− 34	0	75	69	2.4	− 36	− 4	+17	−11	
547	+61 27 0.99	13.923	+131	+ 820	+ 1	66	78	5.0	− 17	+ 1	+26	+ 7	
549	− 9 21 31.37	13.331	+346	− 35	0	72	61	2.4	− 13	+ 4	−	−	
551	+27 40 37.52	13.561	+269	− 2	0	73	70	1.5	− 36	− 6	−13	− 9	
552	+40 46 54.75	+13.740	+231	− 24	0	73	63	2.0	− 40	− 6	−16	−12	
553	−20 15 2.02	14.052	+348	− 44	0	64	21	1.3	+ 8	+ 3	−	−	
554	−17 37 49.39	14.129	+342	− 67	0	72	42	1.6	− 16	0	−	−	
555	+43 31 43.35	14.253	+218	− 2	0	73	56	2.2	− 53	−10	+28	− 2	
556	−11 46 36.29	14.417	+326	− 12	+ 1	74	50	2.2	− 60	− 6	+ 1	−12	

First Section — (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V. .0001		μ and 100 Δμ .0001	.0001	Ep. and Wt. T	p_a	p_δ	B − N Δα .001	Δμ .0001	B − A Δα .001	Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
557	ζ Cygni	3.5	21 8 40.792	+2.5515	+ 40	− 2	0	71	130	3.4	− 3	0	+10	+ 8
558	τ Cygni	3.9	21 10 47.914	2.3926	+ 46	+ 134	− 2	68	47	2.0	−29	− 7	− 7	+ 5
559	α Equulei	4.1	21 10 49.523	3.0001	− 27	+ 38	0	77	66	1.8	+ 7	+ 5	+29	+ 9
560	α Cephei	2.6	21 16 11.590	1.4353	− 69	+ 217	+ 2	66	131	5.4	−31	− 7	+15	+12
561	ι Capricorni	4.3	21 16 40.786	3.3460	− 129	+ 22	0	69	44	1.3	− 1	0	–	–
562	1 Pegasi	4.2	21 17 27.700	+2.7736	+ 19	+ 74	0	76	39	1.1	− 7	− 2	+82	+ 7
565	β Aquarii	3.3	21 26 17.711	3.1608	− 71	+ 10	0	70	148	4.6	− 5	− 2	+40	+11
566	β Cephei	3.4	21 27 22.310	0.7909	− 350	+ 22	0	76	123	5.6	−21	− 4	+38	+16
567	ε Capricorni	4.7	21 31 28.959	3.3652	− 148	+ 6	0	71	20	1.1	–	–	–	–
568	ξ Aquarii	4.8	21 32 25.757	3.1971	− 82	+ 77	0	74	63	1.8	+ 3	+ 2	–	–
569	γ Capricorni	3.8	21 34 33.100	+3.3294	− 131	+ 131	0	68	53	2.1	+ 1	+ 2	+33	+11
571	ε Pegasi	2.5	21 39 16.471	2.9464	− 5	+ 18	0	71	150	4.1	+ 3	+ 2	+29	+ 9
572	κ Pegasi	4.2	21 40 6.981	2.7145	+ 47	+ 24	0	77	37	0.8	−19	− 4	+15	+ 7
573	δ Capricorni	3.0	21 41 31.349	3.3163	− 125	+ 179	+ 1	68	64	2.4	+ 5	+ 3	+48	+14
574	μ Capricorni	5.2	21 47 50.713	3.2754	− 112	+ 211	0	72	47	2.0	+26	+ 7	–	–
576	α Aquarii	3.2	22 0 38.887	+3.0826	− 41	+ 9	0	67	168	6.6	− 9	0	+32	+10
577	ι Aquarii	4.3	22 1 2.237	3.2443	− 111	+ 23	0	71	52	1.5	+10	+ 1	+30	+12
579	ι Pegasi	3.9	22 2 21.313	2.7903	+ 63	+ 220	+1	73	55	1.7	−16	− 2	+16	+ 8
580	θ Pegasi	3.7	22 5 9.336	3.0266	− 11	+ 184	0	74	53	1.9	−14	− 3	+26	+ 8
581	ζ Cephei	3.6	22 7 23.014	2.0754	+ 115	+ 9	0	72	74	2.8	−37	− 9	− 9	+ 3
582	θ Aquarii	4.3	22 11 33.445	+3.1684	− 75	+ 74	0	71	108	3.1	+ 1	+ 1	+29	+11
585	γ Aquarii	3.9	22 16 29.491	3.0997	− 42	+ 81	0	73	108	3.4	− 5	0	+35	+ 9
586	β Lacertae	4.6	22 19 37.577	2.3522	+ 157	− 17	+ 1	69	40	2.4	−31	−10	+ 3	+ 4
587	σ Aquarii	4.8	22 25 21.362	3.1784	− 87	0	0	70	61	2.2	− 8	0	–	–
588	δ^2 Cephei	Var.	22 25 27.384	2.2194	+ 169	+ 13	0	64	55	2.7	−20	− 2	+ 1	+ 6
589	α Lacertae	3.8	22 27 10.220	+2.4643	+ 170	+ 145	+ 1	73	58	1.6	−42	−12	− 5	+ 5
590	η Aquarii	4.1	22 30 13.092	3.0840	− 30	+ 60	0	73	132	3.2	+ 6	+ 4	+36	+11
591	κ Aquarii	5.4	22 32 34.671	3.1084	− 49	− 52	0	68	22	1.2	−19	− 4	–	–
593	ζ Pegasi	3.6	22 36 28.467	2.9910	+ 24	+ 53	0	70	126	3.5	− 7	− 1	+28	+11
595	η Pegasi	3.1	22 38 18.805	2.8072	+ 110	+ 8	0	74	64	2.1	−17	− 3	+ 4	+ 7
596	λ Pegasi	4.2	22 41 42.817	+2.8860	+ 84	+ 41	0	74	56	2.0	+ 3	+ 3	+15	+ 8
598	τ^2 Aquarii	4.3	22 44 17.893	3.1803	− 98	− 11	0	71	48	1.5	− 8	− 3	+36	+11
599	μ Pegasi	3.7	22 45 10.554	2.8915	+ 92	+ 107	0	73	71	2.3	−20	− 4	+14	+ 6
600	ι Cephei	3.7	22 46 7.133	2.1245	+ 227	− 112	− 1	70	82	3.5	−24	− 2	+ 7	+ 8
601	λ Aquarii	3.9	22 47 23.882	3.1320	− 62	+ 3	0	74	105	3.0	+ 2	+ 1	+38	+11
602	δ Aquarii	3.5	22 49 20.633	+3.1880	− 109	− 33	0	67	42	2.2	+ 5	+ 1	+37	+13
604	ο Andromedae	3.6	22 57 19.112	2.7517	+ 190	+ 20	0	74	57	1.7	+ 3	0	− 2	+ 6
605	β Pegasi	2.7	22 58 55.530	2.9033	+ 120	+ 145	0	74	56	2.1	− 3	− 1	+18	+10
606	α Pegasi	2.6	22 59 46.740	2.9855	+ 58	+ 40	0	67	165	6.6	− 1	0	+23	+ 9
607	φ Aquarii	4.4	23 9 8.619	3.1081	− 43	+ 18	0	71	47	2.2	− 4	+ 3	–	–
608	ψ^1 Aquarii	4.5	23 10 39.175	+3.1456	− 61	+ 248	0	70	32	1.4	−22	− 2	–	–
610	γ Piscium	3.8	23 11 58.871	3.1093	+ 7	+ 502	0	72	120	3.3	− 2	+ 1	+33	+11
612	ψ^3 Aquarii	5.2	23 13 45.625	3.1235	− 61	+ 32	0	67	24	1.2	+ 4	+ 5	–	–
613	98 Aquarii	4.2	23 17 43.188	3.1560	− 121	− 87	0	68	19	0.8	+42	+12	–	–
614	κ Piscium	5.0	23 21 48.377	3.0753	+ 1	+ 57	0	75	99	2.4	− 4	+ 1	+33	+11
615	θ Piscium	4.4	23 22 53.707	+3.0415	+ 28	− 88	0	72	34	1.5	0	− 1	–	–
617	λ Andromedae	4.0	23 32 40.044	2.9232	+ 282	+ 150	+ 2	71	61	2.7	−37	− 7	− 1	+ 7
618	ι Piscium	4.3	23 34 48.388	3.0841	+ 32	+ 248	+ 1	71	133	3.7	− 6	+ 2	+28	+ 9
619	γ Cephei	3.4	23 35 14.439	2.4266	+ 753	− 178	−10	65	119	5.7	−45	− 7	+43	+20
620	20 Piscium	5.6	23 42 48.113	3.0844	− 8	+ 63	0	67	22	1.4	–	–	–	–
623	27 Piscium	5.0	23 53 33.217	+3.0712	− 6	− 38	0	66	25	1.3	− 1	− 4	–	–
624	ω Piscium	4.0	23 54 10.548	3.0787	+ 49	+ 101	0	74	144	3.4	− 3	− 1	+34	+12
625	29 Piscium	5.1	23 56 41.954	3.0743	− 2	+ 8	0	67	20	1.1	–	–	–	–
626	30 Piscium	4.7	23 56 49.886	3.0772	− 18	+ 27	0	72	33	1.4	−14	− 2	–	–
627	2 Ceti	4.6	23 58 37.052	3.0762	− 79	+ 13	0	74	51	1.6	+ 1	− 2	–	–

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V. and Sec. V.		μ' and 100 $\Delta\mu'$		Ep. and Wt.			B — N		B — A		
	° ′ ″	″	″ .001	″ .001	″ .001	T	p_δ	$p_{\mu'}$	$\Delta\delta$ ″ .01	$\Delta\mu'$ ″ .001	$\Delta\delta$ ″ .01	$\Delta\mu'$ ″ .001	
557	+29 48 59.59	+14.643	+247	− 59	0	69	122	3.5	− 19	+ 2	−16	−10	
558	+37 37 5.88	15.254	+230	+ 427	+ 1	70	51	2.0	− 47	− 7	−27	−13	
559	+ 4 50 3.27	14.742	+288	− 87	0	77	75	3.0	− 28	− 1	+ 4	− 8	
560	+62 9 42.48	15.190	+133	+ 49	+ 2	64	140	7.5	− 3	0	+ 8	+ 3	
561	−17 15 37.83	15.174	+313	+ 6	0	65	34	1.6	− 14	+ 2	−	−	
562	+19 22 35.24	+15.272	+258	+ 59	+ 1	75	51	1.8	− 38	− 5	−23	−11	
565	− 6 0 40.41	15.698	+280	− 7	0	67	146	5.1	− 9	+ 4	0	−12	
566	+70 7 18.04	15.768	+ 65	+ 5	0	63	144	6.5	− 2	− 1	+26	+ 8	
567	−19 54 51.19	15.981	+289	− 2	0	67	20	1.1	−	−	−	−	
568	− 8 18 10.08	16.009	+274	− 24	+ 1	73	60	2.5	− 24	0	−	−	
569	−17 6 50.75	+16.123	+282	− 20	+ 1	69	52	2.4	− 31	− 3	+23	−11	
571	+ 9 24 58.88	16.384	+241	− 1	0	70	143	4.4	− 22	0	− 6	−10	
572	+25 11 6.68	16.429	+220	− 2	0	75	45	1.5	− 38	−11	− 4	−10	
573	−16 34 52.17	16.202	+269	− 295	+ 1	67	56	2.5	− 12	+ 2	+36	−10	
574	−14 1 21.44	16.814	+255	+ 9	+ 2	71	53	1.9	+ 5	+ 8	−	−	
576	− 0 48 20.80	+17.383	+216	− 6	0	65	167	7.2	− 29	− 4	+ 3	− 8	
577	−14 21 17.82	17.346	+227	− 60	0	69	48	2.0	− 24	+ 2	+20	−12	
579	+24 51 23.37	17.481	+194	+ 18	+ 2	73	54	1.9	− 33	− 2	− 4	− 7	
580	+ 5 42 20.72	17.615	+206	+ 33	+ 1	75	53	1.9	− 32	− 3	+16	− 5	
581	+57 42 29.53	17.682	+135	+ 7	0	71	83	4.3	− 28	− 3	+15	+ 3	
582	− 8 16 52.77	+17.825	+203	− 19	0	71	106	3.6	− 31	− 1	+18	− 6	
585	− 1 53 28.90	18.046	+190	+ 9	+ 1	73	97	3.7	− 52	− 6	+16	− 5	
586	+51 43 40.47	17.965	+138	− 190	0	73	54	3.1	− 27	− 1	+20	+ 2	F 3
587	−11 11 23.30	18.333	+178	− 29	0	69	56	2.4	− 46	− 3	−	−	
588	+57 54 11.67	18.368	+122	+ 3	0	60	51	3.0	− 15	+ 2	+35	+ 7	
589	+49 46 5.67	+18.438	+135	+ 13	+ 1	72	71	3.3	− 12	0	+12	− 2	F 7
590	− 0 37 58.99	18.474	+164	− 54	0	73	119	3.3	− 38	− 1	+ 4	− 6	
591	− 4 44 38.12	18.491	+161	− 115	0	63	24	1.4	− 21	− 2	−	−	
593	+10 18 33.15	18.718	+148	− 12	0	68	126	4.4	− 13	+ 2	−10	−10	
595	+29 41 52.99	18.752	+135	− 35	0	75	63	1.8	− 13	+ 2	+16	− 2	
596	+23 2 21.34	+18.875	+133	− 14	0	75	66	2.4	− 29	− 5	−19	−14	
598	−14 7 13.82	18.927	+143	− 36	0	69	38	1.6	− 38	− 3	+15	−13	
599	+24 4 24.26	18.943	+128	− 45	+ 1	73	62	2.0	− 30	− 3	−22	−12	
600	+65 40 27.88	18.893	+ 89	− 121	− 1	69	97	4.8	+ 13	+ 5	+34	+ 9	
601	− 8 6 42.53	19.085	+134	+ 36	0	73	96	3.4	− 20	0	+10	− 9	
602	−16 21 9.77	+19.081	+133	− 21	0	69	38	1.9	− 18	+ 5	+25	−10	
604	+41 47 18.19	19.288	+100	− 13	0	73	54	1.6	− 40	− 4	+ 3	− 5	
605	+27 32 24.72	19.472	+104	+ 133	+ 1	69	67	2.9	− 30	− 2	−26	−14	
606	+14 40 1.47	19.314	+105	− 45	0	64	164	6.9	− 36	− 6	−21	−14	
607	− 6 35 17.26	19.365	+ 91	− 190	0	69	48	2.3	+ 7	+ 4	−	−	
608	− 9 37 57.58	+19.570	+ 91	− 14	+ 1	65	34	2.0	− 63	− 8	−	−	F 91
610	+ 2 44 8.83	19.627	+ 87	+ 19	+ 2	73	111	3.5	− 30	− 2	+ 6	− 6	
612	−10 9 27.05	19.642	+ 83	+ 2	0	63	29	1.6	+ 7	+ 3	−	−	F 95
613	−20 38 47.34	19.614	+ 76	− 93	0	72	15	0.5	+ 33	− 3	−	−	b¹
614	+ 0 42 29.14	19.679	+ 66	− 90	0	74	89	2.8	− 33	+ 2	+ 2	− 6	
615	+ 5 49 46.58	+19.742	+ 63	− 43	0	70	44	2.0	− 43	− 2	−	−	
617	+45 54 58.78	19.483	+ 42	− 422	0	70	67	3.7	− 14	− 1	+46	+ 3	
618	+ 5 5 3.09	19.487	+ 41	− 439	0	69	128	4.0	− 30	− 3	+ 2	− 7	
619	+77 4 27.30	20.088	+ 29	+ 158	0	66	156	7.6	0	+ 1	+29	+10	
620	− 3 19 3.30	19.996	+ 25	+ 6	0	66	26	1.4	−	−	−	−	
623	− 4 6 38.80	+19.972	+ 4	− 67	0	64	32	1.5	− 19	− 1	−	−	
624	+ 6 18 34.71	19.931	+ 3	− 109	0	74	144	4.6	− 35	− 2	− 9	− 9	
625	− 3 35 3.29	20.032	− 2	− 13	0	65	26	1.5	−	−	−	−	
626	− 6 34 11.45	20.011	− 2	− 34	0	68	32	1.8	− 12	+ 3	−	−	
627	−17 53 33.79	20.038	− 6	− 9	0	72	34	1.4	− 1	+ 5	−		

SECOND SECTION — (Declination, −21° 50′ to −82°).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 $\Delta\mu$		Ep. and Wt.			B — N		B — A	
					.0001	.0001	.0001	T	p_α	p_μ	$\Delta\alpha$ .001	$\Delta\mu$ .0001	$\Delta\alpha$ .001	$\Delta\mu$ .0001
		M	h m s	s		s	s				s	s	s	s
4	ε Phoenicis	3.8	0 4 20.246	+3.0569	− 288	+ 110	− 1	72	7	0.2	+ 60	+14	+ 65	+ 9
8	ζ Tucanae	4.3	0 14 51.687	3.1544	− 664	+2718	−55	72	8	0.3	−213	−32	+ 5	+20
9	ι Sculptoris	5.4	0 16 29.816	3.0217	− 135	+ 38	0	81	17	0.3	–	–	–	–
10	β Hydri	2.9	0 20 30.041	3.2218	−1481	+7025	−321	66	39	0.8	− 21	− 2	+ 65	+34
11	α Phoenicis	2.4	0 21 20.554	2.9750	− 228	+ 179	− 2	67	7	0.4	− 6	− 9	+ 75	+ 6
13	β¹ Tucanae	4.5	0 26 57.738	+2.7701	− 443	+ 125	− 3	68	6	0.2	+ 8	+ 8	–	–
27	α Sculptoris	4.3	0 53 47.311	2.8949	− 99	+ 13	0	79	27	0.9	+ 89	+30	+116	+40
31	β Phoenicis	3.4	1 1 37.337	2.6841	− 179	− 41	0	64	9	0.4	+107	+16	+ 79	+15
35	ζ Phoenicis	4.1	1 4 10.979	2.5304	− 219	+ 12	0	68	7	0.3	–	–	–	–
40	γ Phoenicis	3.3	1 24 1.414	2.6100	− 125	− 24	− 1	61	9	0.4	+ 44	+ 4	+ 82	+25
43	δ Phoenicis	3.9	1 27 5.334	+2.5034	− 133	+ 134	− 1	61	8	0.4	–	–	+ 85	+16
45	α Eridani	0.5	1 33 59.459	2.2395	− 129	+ 116	− 2	60	23	0.7	+ 40	+12	+ 82	+21
50	ε Sculptoris	5.4	1 40 57.761	2.8114	− 38	+ 115	− 1	79	18	0.5	+152	+63	+ 70	+25
56	χ Eridani	3.6	1 52 3.983	2.3382	− 97	+ 725	− 6	71	7	0.3	–	–	+128	+54
58	α Hydri	3.0	1 55 37.070	1.8896	− 33	+ 350	− 5	64	14	0.4	+220	+73	− 2	+ 7
64	φ Eridani	3.7	2 12 56.221	+2.1444	− 45	+ 87	− 1	67	7	0.3	+ 71	+25	+ 72	+18
66	κ Fornacis	5.4	2 17 58.022	2.7452	− 7	+ 141	− 1	83	17	0.2	+ 38	+ 3	+ 71	+16
67	δ Hydri	4.2	2 19 58.098	1.0544	+ 290	− 94	+ 3	65	7	0.2	+ 41	+ 4	+ 84	+27
72	ε Hydri	4.2	2 38 2.982	0.9089	+ 334	+ 170	− 3	65	7	0.3	–	–	+ 52	+25
77	β Fornacis	4.2	2 44 54.362	2.5114	− 4	+ 72	+ 1	85	16	0.4	+ 52	− 8	+ 66	+21
80	θ¹ Eridani	3.1	2 54 28.206	+2.2741	− 2	− 51	+ 1	67	9	0.3	− 54	−26	+134	+24
83	Br. 434	4.1	2 57 58.982	2.6443	+ 17	− 107	0	85	22	0.7	− 2	− 3	+ 59	+17
87	θ Hydri	5.4	3 2 2.845	0.0927	+ 715	+ 66	0	72	15	0.3	+145	+32	+100	+37
89	α Fornacis	4.0	3 7 49.396	2.5475	+ 18	+ 251	+ 3	75	24	0.8	+ 33	+11	+ 66	+17
91	Br. 469	4.0	3 15 4.113	2.6675	+ 26	+ 37	0	80	20	0.7	–	–	–	–
97	Br. 495	4.3	3 29 22.265	+2.6492	+ 30	+ 36	0	75	15	1.1	+ 81	+13	+ 57	+17
98	L 1161	4.5	3 33 30.417	2.1528	+ 23	− 1	0	70	7	0.3	–	–	+ 99	+25
102	L 1198	4.4	3 39 7.686	2.2237	+ 23	− 69	0	75	7	0.2	–	–	–	–
104	Br. 530	4.3	3 42 32.733	2.5801	+ 25	− 115	− 3	79	19	0.7	+ 6	− 1	+ 58	+15
106	L 1248	4.2	3 45 42.721	2.2447	+ 25	− 36	0	81	8	0.2	− 39	0	+ 25	+15
108	γ Hydri	3.2	3 48 47.092	−0.9783	+1070	+ 125	+ 6	67	31	0.6	+ 51	+30	+108	+48
109	L 1275	5.3	3 49 50.332	+2.2851	+ 26	+ 25	0	85	9	0.2	+ 92	+18	–	–
113	δ Reticuli	4.3	3 57 9.639	0.9390	+ 195	− 9	0	66	7	0.3	+ 44	+11	+ 59	+21
120	α Horologii	3.8	4 10 41.285	1.9867	+ 35	+ 39	− 2	68	6	0.3	− 5	− 2	+ 95	+31
121	α Reticuli	3.4	4 13 8.132	0.7623	+ 214	+ 53	+ 1	64	8	0.4	+ 42	+ 5	+ 58	+19
122	γ Doradus	4.3	4 13 24.362	+1.5680	+ 79	+ 103	+ 2	67	6	0.3	–	–	+ 87	+29
125	L 1441	3.9	4 20 16.846	2.2522	+ 33	+ 50	0	83	17	0.5	+ 2	− 2	+ 43	+11
126	η Reticuli	5.2	4 20 48.401	0.6381	+ 236	+ 123	+ 4	69	7	0.3	–	–	+ 49	+17
129	α Doradus	3.5	4 31 50.134	1.2931	+ 97	+ 66	0	64	8	0.4	+ 24	− 1	+ 61	+26
131	α Caeli	4.5	4 37 20.406	1.9310	+ 40	− 132	− 1	63	8	0.3	+ 86	+16	+ 68	+13
132	β Caeli	5.2	4 38 31.323	+2.1195	+ 39	+ 29	+ 2	77	7	0.3	–	–	–	–
141	ε Leporis	3.4	5 1 13.666	2.5383	+ 32	+ 16	0	73	54	1.3	+ 4	+ 4	+ 14	− 4
151	ε Columbae	3.9	5 27 39.784	2.1297	+ 28	+ 27	− 1	80	10	0.4	–	–	+ 46	+ 7
157	α Columbae	2.7	5 36 1.688	2.1720	+ 27	+ 5	0	68	48	1.5	+ 8	− 1	+ 65	+14
159	β Columbae	3.2	5 47 26.081	2.1137	+ 33	+ 41	+ 4	78	17	0.8	+ 61	+13	+ 67	+18
163	γ Columbae	4.5	5 53 59.505	+2.1266	+ 24	0	0	80	15	0.6	+ 4	+ 4	–	–
164	L 2137	6.4	6 1 35.771	1.7264	+ 30	− 78	+ 3	69	5	0.2	+ 24	+10	+ 44	+ 4
166	κ Columbae	4.5	6 12 59.674	2.1340	+ 21	− 3	+ 1	70	8	0.3	+ 24	7	+ 23	+10
167	ζ Can. Maj.	3.2	6 16 28.474	2.3030	+ 18	+ 9	0	79	21	0.9	+ 74	+14	+ 48	+11
170	α Carinae	−1.0	6 21 43.911	1.3315	+ 9	+ 19	0	60	27	0.7	− 13	− 3	+ 47	+ 7
172	Br. 972	4.6	6 30 51.913	+2.5142	+ 15	+ 9	0	78	18	0.7	− 7	−13	+ 23	+ 8
174	ν Puppis	3.2	6 34 42.144	1.8366	+ 12	+ 9	0	60	9	0.4	− 16	+ 1	+ 66	+16
177	κ Can. Maj.	3.9	6 46 6.354	2.2407	+ 15	− 8	0	76	16	0.6	–	–	+ 29	+ 7.
178	α Pictoris	3.3	6 47 9.946	0.6186	− 50	− 104	+ 7	65	9	0.3	− 24	+ 2	+ 43	+22
180	ε Can. Maj.	1.7	6 54 41.753	2.3578	+ 13	+ 4	0	69	72	1.9	+ 11	+ 6	+ 62	+14

Second Section — (Declination, −21° 50′ to −82°).

No.	Decl. 1900	Ann. V. and Sec. V.		μ' and 100 $\Delta\mu'$		Ep. and Wt.			B — N		B — A		
			.001	.001	.001	T	p_δ	$p_{\mu'}$	$\Delta\delta$ .01	$\Delta\mu'$ .001	$\Delta\delta$ .01	$\Delta\mu'$ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
4	−46 17 57.17	+19.856	− 17	− 187	0	73	9	0.3	− 2	+ 6	+31	+18	
8	−65 27 44.92	21.167	− 38	+1162	− 3	74	11	0.3	−103	−10	+66	+15	
9	−29 32 4.23	19.919	− 41	− 76	0	81	12	0.2	–	–	–	–	
10	−77 49 2.71	20.285	− 50	+ 318	+ 2	68	40	0.8	+ 15	0	+59	+ 7	
11	−42 50 56.80	19.559	− 49	− 401	0	63	11	0.6	− 20	+ 2	+ 6	+ 2	
13	−63 30 32.88	+19.852	− 56	− 56	0	69	9	0.3	− 4	− 1	–	–	
27	−29 53 52.63	19.493	−106	− 4	0	78	20	0.9	+ 15	+ 9	− 1	+11	
31	−47 15 15.71	19.313	−112	− 14	0	66	11	0.5	+ 32	+11	−14	− 5	
35	−55 46 49.47	19.284	−111	+ 18	0	72	9	0.3	–	–	–	–	
40	−43 49 50.03	18.499	−144	− 216	0	64	11	0.5	+ 32	+ 9	−29	−11	
43	−49 35 32.48	+18.769	−144	+ 152	− 1	67	8	0.3	–	–	− 5	− 6	
45	−57 44 40.94	18.358	−139	− 26	− 1	64	24	0.6	+ 37	+14	+35	+ 9	
50	−25 33 8.33	18.081	−183	− 52	− 1	71	18	0.5	− 38	− 1	+30	+14	
56	−52 6 24.10	17.977	−173	+ 279	− 5	75	8	0.2	–	–	+48	+19	
58	−62 3 22.76	17.591	−144	+ 41	− 2	65	15	0.5	+ 22	+15	+74	+27	
64	−51 58 30.15	+16.740	−179	− 28	0	67	10	0.4	+ 4	+ 1	+20	+ 6	
66	−24 16 14.35	16.460	−234	− 63	− 1	68	18	0.4	+ 19	+14	− 2	− 7	
67	−69 6 52.03	16.438	− 94	+ 15	+ 1	68	9	0.3	− 16	− 5	+25	+ 4	
72	−68 41 43.49	15.481	− 93	+ 15	− 2	68	8	0.3	–	–	+42	+13	
77	−32 49 33.60	15.227	−249	+ 149	− 1	80	14	0.5	− 44	− 6	−43	− 3	
80	−40 42 19.02	+14.543	−235	+ 30	+ 1	66	10	0.4	+ 11	+ 6	−20	− 4	
83	−24 0 59.09	14.257	−275	− 42	+ 1	69	19	0.8	− 39	+ 1	− 7	+ 4	τ^3 Eridani
87	−72 17 34.67	14.074	− 17	+ 26	− 1	74	18	0.3	+ 5	+12	+71	+25	
89	−29 22 52.43	14.330	−280	+ 647	− 3	74	21	0.9	+ 6	+10	−13	− 1	
91	−22 7 18.42	13.254	−297	+ 41	0	70	19	0.6	–	–	–	–	τ^4 Eridani
97	−21 58 5.34	+12.226	−311	− 21	0	77	14	0.9	+ 19	+18	−21	− 2	τ^5 Eridani
98	−40 36 9.57	11.931	−257	− 28	0	71	8	0.3	–	–	−50	−14	*y* Eridani
102	−37 37 45.04	11.476	−269	− 85	+ 1	69	8	0.3	–	–	–	–	*h* Eridani
104	−23 32 42.20	10.791	−314	− 524	+ 1	71	17	0.5	−179	−43	−36	− 9	τ^6 Eridani
106	−36 30 10.86	11.040	−277	− 46	0	77	9	0.3	− 40	−17	+20	+ 3	*g* Eridani
108	−74 32 43.81	+10.976	+113	+ 115	− 2	68	31	0.6	− 8	− 2	+59	+11	
109	−35 1 40.78	10.763	−286	− 20	0	75	9	0.3	− 17	− 3	–	–	*i* Eridani
113	−61 40 57.48	10.215	−122	− 23	0	70	8	0.3	−128	−20	−12	− 1	
120	−42 32 27.14	8.990	−262	− 213	− 1	68	8	0.4	+ 72	+17	+24	+10	
121	−62 43 26.75	9.066	−104	+ 54	− 1	68	11	0.4	+ 6	+10	+ 6	+ 4	
122	−51 44 19.89	+ 9.170	−210	+ 179	− 1	70	8	0.3	–	–	+36	+15	
125	−34 14 56.30	8.504	−302	+ 54	− 1	76	13	0.6	+ 21	+12	− 8	+ 2	δ Eridani
126	−63 37 25.34	8.579	− 90	+ 171	− 2	71	8	0.3	–	–	+49	+26	
129	−55 15 6.00	7.520	−179	− 3	− 1	67	9	0.4	+ 32	+ 8	+18	+ 5	
131	−42 3 16.82	6.986	−265	− 89	+ 2	68	8	0.3	+ 54	+18	−25	− 7	
132	−37 20 23.08	+ 7.162	−294	+ 184	0	68	6	0.2	–	–	–	–	
141	−22 30 18.90	5.020	−360	− 65	0	73	37	1.0	+ 17	− 1	+ 1	+ 3	
151	−35 32 37.81	2.770	−309	− 49	0	73	10	0.4	–	–	−19	− 2	
157	−34 7 38.59	2.056	−316	− 37	0	69	40	1.2	− 12	+ 1	− 2	− 3	
159	−35 48 21.19	1.494	−308	+ 395	− 1	71	15	0.8	+ 5	+ 7	−20	−12	
163	−35 17 38.33	+ 0.521	−310	− 5	0	74	13	0.6	− 20	−11	–	–	
164	−45 2 10.18	+ 0.093	−250	+ 233	+ 1	70	7	0.3	+ 71	+ 8	+ 1	− 2	1 G Pup.
166	−35 6 25.69	− 1.059	−310	+ 77	0	67	10	0.4	+ 51	+12	+42	+10	
167	−30 1 8.15	1.443	−334	− 3	0	79	20	0.9	+ 52	+20	−57	−20	
170	−52 38 27.56	− 1.884	−193	+ 14	0	63	24	0.7	+ 9	+ 5	+20	+ 7	Argus
172	−22 53 7.45	− 2.671	−362	+ 21	0	69	20	0.6	−128	−14	+12	+ 1	ξ^2 C. Maj.
174	−43 6 29.68	3.042	−264	− 18	0	62	11	0.5	− 22	+ 1	−14	− 2	Argus
177	−32 23 34.35	4.000	−318	+ 6	0	75	13	0.5	–	–	−39	−11	
178	−61 50 1.76	3.834	− 85	+ 262	+ 1	71	8	0.2	+ 42	+25	−38	−14	
180	−28 50 9.33	4.740	−332	− 1	0	70	50	1.3	− 23	− 4	− 6	+ 2	

Second Section — (Declination, −21° 50′ to −82°).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V		μ and 100 Δμ		Ep. and Wt.			B−N		B−A	
		M	h m s		.0001	.0001	.0001	T	p_α	p_μ	Δα .001	Δμ .0001	Δα .001	Δμ .0001
181	σ Can. Maj.	3.9	6 57 44.132	+2.3895	+ 12	− 7	0	80	16	0.8	+ 6	− 1	−	−
184	δ Can. Maj.	2.1	7 4 19.527	2.4393	+ 11	− 2	0	74	47	1.3	+ 43	+13	+ 32	+ 7
187	π Puppis	2.7	7 13 36.685	+2.1191	+ 10	− 5	0	71	17	0.9	+ 25	+ 3	+ 64	+11
189	δ Volantis	3.9	7 16 52.910	−0.0170	− 252	− 10	0	65	8	0.2	−350	−14	− 16	+ 4
191	η Can. Maj.	2.4	7 20 8.375	+2.3726	+ 11	− 7	0	74	27	1.3	− 43	−10	+ 20	− 2
194	σ Puppis	3.5	7 26 3.530	+1.9034	+ 8	− 55	+ 2	69	7	0.2	+ 60	+17	−	−
197	Br. 1120.	4.1	7 39 47.626	2.4079	+ 12	− 6	0	81	15	0.5	−	−	+ 36	+ 7
198	L 2958	3.4	7 41 41.496	2.1365	+ 12	− 21	0	75	8	0.3	−	−	+ 66	+20
201	χ Carinae	3.6	7 54 14.197	1.5271	− 30	− 36	0	64	8	0.5	− 3	+ 8	+ 60	+15
203	ζ Puppis	2.3	8 0 4.186	2.1081	+ 13	− 28	0	72	14	0.8	+ 26	+16	+ 30	+ 2
204	ρ Puppis	2.9	8 3 17.118	+2.5545	+ 10	− 65	0	70	83	2.3	+ 9	0	+ 20	0
205	γ Velorum	1.9	8 6 27.093	1.8501	0	0	0	61	12	0.6	− 7	+ 3	+ 67	+20
207	L 3259	4.5	8 14 48.730	2.2445	+ 21	− 96	0	71	9	0.5	−	−	+147	+37
208	ε Carinae	1.7	8 20 27.750	1.2360	− 90	− 34	0	58	12	0.5	+ 20	+ 9	+ 17	+ 7
211	β Pyxidis	3.9	8 36 11.270	2.3475	+ 28	+ 7	0	82	9	0.5	−	−	− 3	+ 6
215	δ Velorum	2.0	8 41 56.569	+1.6579	− 20	+ 23	− 1	65	10	0.5	+149	+58	+ 88	+18
218	L 3639	5.1	8 54 31.676	1.4712	− 53	− 12	0	63	8	0.3	+141	+22	−	−
222	λ Velorum	2.1	9 4 19.074	2.2049	+ 45	− 21	0	64	11	0.5	+ 24	− 6	+106	+30
225	β Carinae	2.0	9 12 6.251	0.6763	− 357	− 300	− 2	64	21	0.7	+ 16	+10	+ 35	+19
227	ι Carinae	2.2	9 14 24.768	1.6059	− 23	− 40	0	64	20	0.6	+ 49	+15	+ 33	+11
229	θ Pyxidis	5.1	9 17 3.953	+2.6541	+ 35	− 15	0	77	17	0.5	+139	+33	+ 61	+13
230	κ Velorum	2.6	9 19 1.011	1.8563	+ 27	− 18	0	71	8	0.4	+ 31	+15	+ 86	+27
234	ψ Velorum	3.5	9 26 45.692	2.3599	+ 64	− 165	− 1	78	12	0.4	+112	+15	+103	+25
238	υ Carinae	3.0	9 44 36.165	1.5019	− 47	− 21	− 1	65	11	0.4	− 5	+ 4	+ 63	+18
241	φ Velorum	3.7	9 53 21.087	2.1014	+ 94	− 21	0	73	9	0.3	+ 44	+12	+ 58	+15
246	L 4212	4.0	10 10 32.242	+2.5115	+ 118	− 149	− 2	68	7	0.3	+ 62	+ 4	+ 75	+23
248	ω Carinae	3.6	10 11 21.651	1.4316	− 73	− 55	0	72	10	0.3	−	−	− 50	−14
254	L 4319	4.0	10 22 24.645	1.1994	− 222	− 66	− 1	73	10	0.2	−	−	+ 53	+26
255	α Antliae	4.2	10 22 34.568	2.7418	+ 97	− 51	0	77	20	0.6	+ 63	+ 9	+ 90	+25
259	θ Carinae	3.0	10 39 23.280	2.1304	+ 201	− 32	− 1	60	9	0.5	+ 20	+10	+ 43	+18
260	η Carinae	Var.	10 41 10.840	+2.3180	+ 220	+ 4	0	61	18	0.5	+ 35	+ 5	+ 56	+17
261	μ Velorum	2.8	10 42 28.020	2.5692	+ 197	+ 52	0	71	9	0.4	− 10	−14	+ 14	− 3
265	L 4515	4.1	10 49 25.713	2.4219	+ 251	+ 60	+ 1	70	7	0.2	−	−	− 50	−10
272	β Crateris	4.5	11 6 44.348	2.9461	+ 99	− 1	0	76	32	1.5	+ 23	− 1	+ 40	+11
278	π Centauri	4.4	11 16 26.723	2.7219	+ 308	− 37	− 1	67	6	0.3	−	−	+ 47	+14
284	ξ Hydrae	3.6	11 28 4.967	+2.9440	+ 166	− 155	− 1	78	21	0.7	+ 30	+ 3	+ 37	+15
285	λ Centauri	3.3	11 31 10.039	2.7452	+ 451	− 54	− 1	71	8	0.4	+ 99	+18	+ 89	+29
294	δ Centauri	2.8	12 3 10.461	3.0904	+ 382	− 41	0	68	10	0.5	+ 51	+ 9	+ 48	+10
295	α Corvi	4.2	12 3 15.253	3.0869	+ 156	+ 60	+ 1	70	18	1.2	−	−	−	−
296	ε Corvi	3.2	12 4 58.846	3.0794	+ 143	− 47	0	71	65	1.2	+ 4	+ 4	+ 44	+14
297	ρ Centauri	4.2	12 6 25.422	+3.1153	+ 410	− 47	− 1	72	6	0.3	−	−	−	−
299	δ Crucis	3.1	12 9 49.966	3.1590	+ 532	− 58	− 1	68	9	0.4	−184	−79	+ 17	+ 8
302	β Chamael.	4.3	12 12 28.462	3.4221	+1865	− 160	− 8	66	28	0.6	+ 39	+27	− 35	+11
306	ε Crucis	3.5	12 15 57.695	3.2090	+ 585	− 234	− 4	71	5	0.3	−	−	+ 48	+13
309	γ Crucis	1.6	12 25 36.988	3.3001	+ 550	+ 21	+ 1	68	9	0.5	+ 58	+49	+ 17	− 2
311	β Corvi	2.7	12 29 7.981	+3.1437	+ 165	0	0	69	95	2.6	+ 30	+ 7	+ 43	+12
313	γ Centauri	2.4	12 35 59.977	3.2878	+ 417	− 201	− 2	68	10	0.5	− 13	− 6	+ 54	+12
314	β Crucis	1.5	12 41 52.511	3.4722	+ 660	− 66	− 1	60	12	0.6	− 9	− 2	− 2	+ 3
319	δ Muscae	3.6	12 55 23.208	4.0537	+1426	+ 520	+15	74	13	0.3	+ 28	+26	+ 15	+26
325	ι Centauri	3.0	13 14 58.440	3.3583	+ 303	− 282	− 2	75	16	0.9	+ 90	+12	+ 60	+19
329	L 5569	4.0	13 25 14.630	+3.4629	+ 342	− 12	0	70	9	0.3	−	−	+ 82	+24
331	ε Centauri	2.6	13 33 32.950	3.7716	+ 592	− 37	0	67	10	0.5	+ 20	+ 3	+ 36	+ 7
335	ζ Centauri	2.8	13 49 17.963	3.7194	+ 471	− 61	0	69	10	0.5	+ 23	+ 9	+ 77	+23
338	β Centauri	0.8	13 56 45.799	4.1931	+ 848	− 33	0	64	24	0.9	− 29	+ 1	+ 36	+22
339	θ Centauri	2.2	14 0 47.742	3.5155	+ 318	− 431	0	75	18	0.8	+ 22	+ 7	+ 57	+23

Second Section — (Declination, −21° 50′ to −82°).

No.	Decl. 1900	Ann. V. and Sec. V.		μ' and 100 $\Delta\mu'$		Ep. and Wt.			B — N		B — A		
	° ′ ″	″	.001 ″	.001 ″	.001 ″	T	p_δ	$p_{\mu'}$	$\Delta\delta$ .01 ″	$\Delta\mu'$ .001 ″	$\Delta\delta$ .01 ″	$\Delta\mu'$ .001 ″	
181	−27 47 29.62	− 5.000	−335	− 3	0	79	16	0.7	− 43	− 5	–	–	
184	−26 14 3.91	5.555	−339	− 2	0	76	43	1.3	− 37	− 5	−30	− 5	
187	−36 55 4.29	6.328	−290	+ 1	0	67	16	0.8	+ 49	+10	+ 1	− 4	Argus
189	−67 46 26.76	6.600	+ 5	0	0	71	9	0.2	+ 28	+ 6	+33	+15	
191	−29 6 28.78	6.865	−322	+ 3	0	74	22	1.1	− 27	− 4	−21	− 6	
194	−43 5 56.19	− 7.174	−254	+178	+ 1	67	10	0.4	− 39	− 2	–	–	Argus
197	−28 42 56.60	8.466	−315	− 10	0	81	14	0.4	–	–	−24	− 7	*l* Puppis
198	−37 43 33.01	8.616	−277	− 10	0	70	10	0.4	–	–	− 1	− 6	ε Puppis
201	−52 42 50.60	9.566	−191	+ 18	0	68	10	0.5	+ 22	+12	+16	+ 4	Argus
203	−39 43 16.97	10.021	−261	+ 8	0	68	14	0.8	+ 24	+13	− 8	− 2	Argus
204	−24 0 57.37	−10.226	−315	+ 45	+ 1	71	64	1.8	− 36	− 7	−37	− 5	Argus
205	−47 2 30.54	10.507	−225	+ 1	0	63	15	0.7	+ 8	+12	+41	+14	Argus
207	−36 20 57.69	11.035	−266	+ 89	+ 1	71	11	0.4	–	–	+22	+ 7	*q* Puppis
208	−59 11 15.50	11.519	−142	+ 13	0	62	13	0.6	− 5	+ 5	+11	+11	Argus
211	−34 57 12.04	12.649	−261	− 20	0	73	8	0.4	–	–	+ 7	+ 7	
215	−54 20 31.77	−13.109	−179	− 94	0	69	13	0.5	− 51	+ 6	+24	+ 4	Argus
218	−58 50 35.77	13.836	−149	− 3	0	66	9	0.3	+ 20	+16	–	–	b^1 Carinae
222	−43 1 43.71	14.435	−217	+ 5	0	65	14	0.6	+ 19	+11	−18	− 1	Argus
225	−69 18 18.89	14.804	− 57	+100	+ 3	67	23	0.8	+ 8	+ 7	+34	+10	Argus
227	−58 51 20.27	15.034	−148	+ 4	0	67	19	0.6	− 46	− 2	−10	+ 8	Argus
229	−25 32 23.19	−15.199	−246	− 9	0	74	17	0.6	+109	+23	0	+ 3	
230	−54 35 0.82	15.301	−168	0	0	70	11	0.5	+ 38	+19	−30	−11	Argus
234	−40 1 44.14	15.674	−205	+ 57	+ 1	76	12	0.4	+ 28	+18	−58	−23	Argus
238	−64 36 28.87	16.644	−115	+ 5	0	67	16	0.5	+ 36	+22	+ 2	− 2	Argus
241	−54 5 30.36	17.066	−153	− 3	0	71	12	0.4	+ 11	+18	− 3	+ 1	Argus
246	−41 37 35.32	−17.771	−160	+ 32	+ 1	67	9	0.3	− 38	0	−70	−23	*q* Velor.
248	−69 32 28.70	17.838	− 87	− 2	0	74	14	0.4	–	–	+ 6	− 4	Argus
254	−73 31 21.33	18.276	− 64	− 19	+ 1	74	14	0.3	–	–	+23	+ 8	I Carinae
255	−30 33 31.27	18.270	−157	− 7	0	75	18	0.7	+ 20	+16	−56	−13	
259	−63 52 13.63	18.804	− 99	+ 16	0	67	11	0.5	+154	+42	+42	+19	Argus
260	−59 9 31.20	−18.869	−106	+ 4	0	63	19	0.6	+ 23	+13	−18	− 9	Argus
261	−48 53 30.48	18.972	−117	− 61	0	73	11	0.4	+ 43	+20	+ 8	− 3	Argus
265	−58 19 19.23	19.079	− 99	+ 25	0	69	9	0.3	–	–	−12	+ 1	*u* Carinae
272	−22 16 47.40	19.606	− 91	− 98	0	76	24	1.0	+ 43	+ 8	−18	− 3	
278	−53 56 34.97	19.700	− 67	− 14	0	67	9	0.4	–	–	−16	+ 1	
284	−31 18 15.71	−19.911	− 51	− 58	0	76	17	0.7	− 19	− 4	−49	−15	
285	−62 27 59.21	19.901	− 42	− 13	0	75	10	0.4	+ 5	+14	+48	+12	
294	−50 9 55.65	20.061	+ 15	− 16	0	69	13	0.6	+ 29	+14	+14	+10	
295	−24 10 15.97	20.094	+ 15	− 49	0	70	14	0.9	–	–	–	–	
296	−22 3 49.07	20.039	+ 18	+ 3	0	70	40	0.9	− 12	0	−64	−17	
297	−51 48 41.94	−20.060	+ 22	− 21	0	74	8	0.2	–	–	–	–	
299	−58 11 33.83	20.054	+ 28	− 26	0	69	12	0.5	+ 40	+13	−35	−13	
302	−78 45 25.04	20.006	+ 36	+ 11	0	69	29	0.7	− 6	− 6	+24	− 3	
306	−59 50 54.63	19.909	+ 41	+ 89	0	72	7	0.3	–	–	+42	+ 9	
309	−56 33 11.85	20.195	+ 62	−273	0	69	12	0.5	− 62	−13	+10	− 2	
311	−22 50 37.48	−19.945	+ 67	− 60	0	70	69	2.1	− 7	+ 2	−22	− 3	
313	−48 24 38.25	19.818	+ 83	− 18	0	69	12	0.6	+ 2	+ 2	+ 6	+ 7	
314	−59 8 31.51	19.737	+100	− 24	0	65	15	0.7	+ 9	+ 9	+ 4	+ 6	
319	−71 0 33.96	19.491	+152	− 27	+ 2	74	16	0.4	0	+ 5	+21	+ 4	
325	−36 11 5.65	19.080	+164	− 96	− 1	74	16	0.8	− 37	0	+ 1	+ 3	
329	−38 53 27.13	−18.703	+192	− 27	0	69	10	0.4	–	–	− 9	− 9	δ Centauri
331	−52 57 28.79	18.428	+226	− 28	0	70	11	0.5	− 2	+11	+ 2	+ 8	
335	−46 47 45.73	17.861	+256	− 51	0	70	11	0.5	+ 18	+13	− 9	− 3	
338	−59 53 26.04	17.530	+306	− 29	0	69	28	0.8	− 15	+ 4	+53	+24	
339	−35 52 41.00	17.853	+262	−527	− 3	73	15	0.8	− 24	− 3	−10	+ 1	

Second Section — (Declination, −21° 50′ to −82°).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B − N		B − A	
		M	h m s	s	.0001 s	.0001 s	.0001 s	T	p_a	p_μ	Δα .001 s	Δμ .0001 s	Δα .001 s	Δμ .0001 s
351	η Centauri	2.5	14 29 9.335	+3.7915	+390	−27	0	66	8	0.4	+15	+5	+26	+10
352	α Lupi	2.5	14 35 16.616	3.9677	+473	−18	0	68	7	0.4	+56	+23	+49	+10
360	β Lupi	2.7	14 51 58.778	3.9092	+393	−50	0	65	8	0.5	+78	+20	+46	+8
361	κ Centauri	3.4	14 52 39.282	3.8863	+378	−7	0	65	6	0.3	−122	−11	+79	+28
364	σ Librae	3.6	14 58 12.925	3.5018	+209	−56	0	73	32	1.1	−28	+1	+35	+15
365	π Lupi	3.8	14 58 18.435	+4.0637	+451	−27	0	73	9	0.3	–	–	+53	+14
367	κ Lupi	4.2	15 4 58.749	4.1463	+475	−117	0	71	7	0.3	+49	+6	–	–
368	ζ Lupi	3.5	15 5 5.933	4.2841	+548	−120	0	71	7	0.3	+13	+6	+100	+22
370	γ Tri. Austr.	3.0	15 9 34.190	5.5348	+1397	−106	−1	66	10	0.4	+70	+31	+34	+21
381	ε Tri. Austr.	4.1	15 27 33.854	5.4354	+1125	+33	+3	67	7	0.3	–	–	−12	+10
382	γ Lupi	3.0	15 28 28.501	+3.9822	+331	−16	0	67	10	0.5	+11	+3	+85	+32
394	β Tri. Austr.	3.1	15 46 19.726	5.2448	+873	−284	+7	67	8	0.4	−4	+7	+32	+8
397	ρ Scorpii	4.0	15 50 42.526	3.6952	+199	−12	0	73	12	0.7	–	–	+51	+18
399	π Scorpii	3.0	15 52 48.065	3.6210	+178	−11	0	76	22	1.1	+7	−1	+50	+14
401	η Lupi	3.8	15 53 29.588	3.9627	+267	−23	0	74	9	0.4	–	–	–	–
402	δ Scorpii	2.7	15 54 25.147	+3.5403	+158	−8	0	71	60	2.0	+17	+4	+26	+9
406	δ Tri. Austr.	4.0	16 6 20.020	5.4240	+783	+13	+1	69	4	0.2	–	–	+56	+28
408	L 6764	4.2	16 12 21.328	4.4701	+373	−180	0	73	8	0.3	+118	+36	+49	+12
410	σ Scorpii	3.0	16 15 6.539	3.6396	+154	−8	0	73	41	1.4	+8	+3	+44	+14
413	γ Apodis	3.9	16 18 6.283	9.0568	+3206	−386	+4	71	15	0.4	+47	+23	+151	+54
416	α Scorpii	1.3	16 23 16.483	+3.6719	+149	−6	0	68	93	3.4	+4	0	–	–
417	L 6859	4.4	16 24 50.794	3.9122	+192	−2	0	82	11	0.4	+34	+5	+94	+41
422	τ Scorpii	2.8	16 29 39.363	3.7279	+150	−8	0	72	36	1.4	+8	+5	+46	+17
426	α Tri. Austr.	1.9	16 38 4.380	6.3115	+889	+40	+3	64	32	0.7	+27	+12	+18	+18
428	η Arae	3.6	16 41 8.890	5.1591	+446	+42	+1	73	7	0.3	–	–	+65	+30
429	ε Scorpii	2.3	16 43 41.120	+3.8783	+161	−496	+1	75	19	1.1	+30	+10	+48	+9
430	μ^1 Scorpii	3.3	16 45 5.720	4.0557	+177	−11	0	73	11	0.6	+130	−7	−20	+3
431	μ^2 Scorpii	3.7	16 45 33.669	4.0547	+176	−18	0	78	7	0.3	–	–	−2	+6
432	ζ Arae	3.0	16 50 20.588	4.9485	+342	−25	+1	71	8	0.3	+328	−10	+64	+32
433	ε Arae	4.2	16 51 36.701	4.7670	+294	−9	0	70	8	0.3	+31	+2	+60	+19
438	η Scorpii	3.4	17 4 59.403	+4.2891	+170	+17	+4	68	10	0.4	+33	−6	+34	+4
444	θ Ophiuchi	3.3	17 15 52.058	3.6811	+78	−1	0	71	70	1.7	+19	+5	+58	+18
445	γ Arae	3.4	17 16 58.542	5.0393	+225	−5	0	65	8	0.4	–	–	+33	+17
446	β Arae	2.7	17 16 59.153	4.9765	+217	−16	+1	67	8	0.3	−19	−12	+4	+2
447	Br. 2198	4.1	17 20 15.755	3.6607	+72	0	+1	75	36	1.1	+34	+9	+51	+19
448	υ Scorpii	2.8	17 23 57.872	+4.0747	+94	−1	0	82	11	0.4	+102	+23	+69	+20
449	α Arae	2.9	17 24 6.648	4.6309	+146	−34	+1	62	10	0.5	+18	+2	+51	+16
450	Br. 2209	4.8	17 25 18.851	3.6577	+64	+3	0	72	25	1.3	–	–	+70	+18
451	λ Scorpii	1.8	17 26 49.052	4.0697	+87	−3	0	76	16	0.8	+16	0	+56	+19
453	θ Scorpii	2.0	17 30 7.967	4.3062	+96	+10	0	67	7	0.3	+62	+19	+41	+19
456	κ Scorpii	2.6	17 35 34.177	+4.1471	+72	−5	0	73	12	0.6	−33	−10	+61	+20
462	ι^1 Scorpii	3.1	17 40 35.426	4.1936	+62	+4	0	67	10	0.5	−23	−2	+45	+11
464	L 7449	3.2	17 43 3.053	4.0819	+51	+48	0	77	7	0.3	–	–	+23	+21
472	θ Arae	3.8	17 58 50.846	4.6704	+17	−2	+1	69	7	0.3	+66	+8	–	–
473	γ Sagittarii	3.0	17 59 23.039	3.8530	+20	−42	+2	79	38	0.9	+38	+13	+60	+13
476	ε Telescopii	4.5	18 3 48.418	+4.4529	+2	−18	0	73	8	0.4	+248	+49	+77	+19
479	η Sagittarii	3.1	18 10 51.676	4.0600	−6	−107	+2	81	14	0.4	−4	+2	+83	+25
481	δ Sagittarii	2.8	18 14 35.547	3.8415	−8	+31	0	76	26	1.0	+15	+8	+46	+14
483	ε Sagittarii	1.9	18 17 32.080	3.9823	−18	−35	+1	73	19	1.1	+12	+6	+35	+6
484	λ Sagittarii	2.9	18 21 47.970	3.7027	−12	−36	+2	74	50	1.9	+6	−2	+47	+12
486	ζ Pavonis	4.0	18 31 21.117	+7.0287	−431	−37	+10	71	14	0.4	+71	+21	+53	+25
488	φ Sagittarii	3.3	18 39 24.563	3.7499	−44	+30	0	75	32	1.3	+32	+5	–	–
491	σ Sagittarii	2.1	18 49 3.903	3.7219	−55	+7	+1	71	51	1.8	+36	+10	+56	+17
497	ζ Sagittarii	2.7	18 56 15.013	3.8202	−78	−13	0	76	26	1.0	+43	+11	+71	+20
500	τ Sagittarii	3.5	19 0 41.858	3.7485	−72	−44	+2	78	20	1.0	−3	+2	+70	+19

Second Section — (Declination, −21° 50′ to −82°).

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ′ .001	100 Δμ′ .001	Ep. T	p_δ	$p_{\mu'}$	B−N Δδ .01	B−N Δμ′ .001	B−A Δδ .01	B−A Δμ′ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
351	−41 43 6.99	−15.979	+ 342	− 30	0	68	10	0.5	− 25	+ 2	−10	+ 6	
352	−46 57 32.18	15.643	+ 369	− 24	0	67	9	0.5	+ 9	+11	+21	+ 9	
360	−42 43 52.12	14.714	+ 395	− 51	0	67	10	0.5	+ 18	+11	+ 6	+10	
361	−41 42 10.60	14.656	+ 394	− 34	0	67	8	0.4	− 57	− 8	− 7	−12	
364	−24 53 20.61	14.341	+ 364	− 56	− 1	73	28	1.1	− 55	− 8	−10	− 2	ν Scorpii
365	−46 39 35.55	−14.314	+ 422	− 35	0	72	12	0.4	−	−	+ 4	+ 3	
367	−48 21 27.18	13.924	+ 441	− 60	− 1	74	9	0.2	− 9	+ 2	−	−	
368	−51 43 7.07	13.920	+ 456	− 64	− 1	72	9	0.4	− 45	+ 2	+18	+ 9	
370	−68 18 36.08	13.591	+ 599	− 20	− 1	70	14	0.4	+ 54	+22	+60	+22	
381	−65 58 50.09	12.440	+ 630	− 68	0	70	8	0.3	−	−	+64	+20	
382	−40 49 50.06	−12.342	+ 463	− 33	0	66	12	0.5	+ 55	+16	+10	+ 8	
394	−63 7 18.73	11.430	+ 640	−389	− 3	69	11	0.4	+ 50	+18	+41	+17	
397	−28 55 19.66	10.750	+ 460	− 31	0	73	13	0.6	−	−	−26	− 3	
399	−25 49 34.59	10.600	+ 453	− 36	0	74	20	1.2	− 24	+12	+17	+ 3	
401	−38 6 39.04	10.546	+ 496	− 34	0	72	10	0.4	−	−	−	−	
402	−22 20 14.05	−10.480	+ 445	− 37	0	73	53	2.0	− 34	− 2	− 6	+ 2	
406	−63 25 48.07	9.556	+ 700	− 16	0	72	6	0.3	−	−	+13	+ 2	
408	−49 54 36.62	9.126	+ 583	− 52	− 2	71	10	0.4	+ 17	+11	+27	+12	γ² Normae
410	−25 21 10.53	8.891	+ 480	− 33	0	72	34	1.4	− 26	+ 6	−25	− 6	
413	−78 40 20.94	8.700	+1.196	− 78	− 5	74	18	0.3	+ 25	+ 4	+35	+ 1	
416	−26 12 36.85	− 8.247	+ 492	− 35	0	70	70	2.4	− 46	− 7	−	−	
417	−34 29 11.58	8.108	+ 525	− 22	0	76	11	0.4	+ 10	+ 6	+ 7	+ 6	N Scorpii
422	−28 0 31.18	7.736	+ 505	− 37	0	75	24	1.0	− 28	− 2	−26	− 7	
426	−68 50 38.13	7.043	+ 866	− 28	+ 1	68	31	0.6	+ 50	+21	+77	+20	
428	−58 51 46.16	6.809	+ 712	− 47	+ 1	73	10	0.3	−	−	−12	− 4	
429	−34 6 42.39	− 6.811	+ 529	−258	− 7	72	17	0.9	+ 23	+ 4	+14	+ 7	
430	−37 52 32.82	6.466	+ 563	− 30	0	69	12	0.6	− 28	− 6	+12	+ 2	
431	−37 50 49.39	6.429	+ 563	− 32	0	69	8	0.4	−	−	− 4	+ 3	
432	−55 49 56.09	6.046	+ 690	− 46	0	71	11	0.4	− 4	+ 2	+15	− 1	
433	−53 0 24.28	5.898	+ 667	− 4	0	70	10	0.4	+ 17	+12	−19	− 8	ε¹
438	−43 6 26.42	− 5.062	+ 610	−296	0	67	14	0.5	+ 39	+10	− 3	− 6	
444	−24 53 59.27	3.866	+ 528	− 30	0	72	48	1.4	− 20	+ 6	− 5	− 3	
445	−56 17 0.20	3.751	+ 724	− 10	0	67	11	0.4	−	−	+11	+ 1	
446	−55 26 7.03	3.775	+ 714	− 35	0	70	11	0.4	− 26	− 7	− 1	−12	
447	−24 5 0.43	3.589	+ 527	−130	0	73	36	1.1	− 12	+ 7	0	− 6	44 Ophiu.
448	−37 12 57.67	− 3.181	+ 588	− 42	0	76	9	0.4	− 10	− 7	+ 5	− 4	
449	−49 47 48.78	3.216	+ 668	− 89	0	66	12	0.6	− 17	− 6	−20	−10	
450	−23 53 7.46	3.058	+ 529	− 36	0	74	19	0.9	−	−	−36	− 6	51 Ophiu.
451	−37 1 51.23	2.926	+ 588	− 34	0	73	14	0.8	− 41	− 6	− 2	+ 5	
453	−42 56 3.08	2.620	+ 623	− 15	0	67	9	0.4	− 25	− 6	+ 2	− 2	
456	−38 58 42.20	− 2.159	+ 602	− 26	0	71	13	0.6	− 68	−11	−19	−14	
462	−40 5 17.83	1.708	+ 610	− 12	0	72	11	0.4	− 18	− 5	−56	−13	
464	−37 0 40.87	1.458	+ 595	+ 23	+ 1	74	8	0.3	−	−	− 5	+ 1	G Scorpii
472	−50 5 52.88	0.136	+ 681	− 35	0	70	10	0.3	+ 32	+15	−	−	
473	−30 25 31.16	− 0.250	+ 561	−196	− 1	80	34	0.8	− 7	+ 2	−24	−10	
476	−45 58 17.91	+ 0.296	+ 649	− 37	0	72	10	0.5	+ 16	+ 8	+11	− 1	
479	−36 47 30.32	0.781	+ 589	−169	− 2	74	12	0.5	− 77	−16	−16	− 8	
481	−29 52 14.43	1.237	+ 559	− 39	0	75	21	0.8	− 18	− 5	−20	− 7	
483	−34 25 54.70	1.405	+ 578	−127	− 1	71	18	1.1	− 34	− 5	+24	+11	
484	−25 28 37.41	1.710	+ 536	−194	0	72	37	1.6	− 1	+ 5	−36	−11	
486	−71 30 49.23	+ 2.573	+1014	−161	− 1	75	17	0.2	+ 11	+ 4	+36	+ 8	
488	−27 5 37.04	3.426	+ 538	− 4	0	73	28	1.2	− 26	+ 2	−	−	
491	−26 25 15.74	4.191	+ 528	− 68	0	73	47	1.6	− 19	+ 7	+ 2	− 2	
497	−30 1 23.07	4.868	+ 538	− 3	0	76	22	0.8	+ 45	+16	−31	−12	
500	−27 49 0.10	4.984	+ 524	−263	− 1	76	18	0.8	− 30	−10	−41	−13	

Second Section — (Declination, −21° 50′ to −82°).

No.	Name and Magnitude		R.A. 1900	Ann. V.	Sec. V. .0001	μ .0001	100 Δμ .0001	Ep. and Wt. T	pα	pμ	B−N Δα .001	B−N Δμ .0001	B−A Δα .001	B−A Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
503	α Coron. Austr.	4.2	19 2 40.222	+4.0874	− 122	+ 75	+ 1	77	8	0.3	+ 92	+24	+ 84	+28
514	Br. 2478	4.5	19 30 37.384	3.6555	− 104	+ 53	0	71	56	1.2	+ 39	+ 9	+ 81	+26
524	Br. 2549	4.6	19 56 30.641	3.6956	− 148	+ 28	0	76	41	1.5	+ 30	+ 5	+ 72	+25
531	α Pavonis	2.0	20 17 44.337	4.7743	− 595	+ 10	+ 2	62	19	0.5	+ 58	+10	+ 50	+16
537	α Indi	3.2	20 30 32.117	4.2372	− 402	+ 41	− 1	65	8	0.4	+107	+14	+ 48	+11
541	β Pavonis	3.5	20 35 57.031	+5.4611	−1164	− 75	0	65	11	0.4	+ 13	+ 5	+ 19	+14
543	ψ Capricorni	4.2	20 40 10.569	3.5591	− 167	− 41	+ 1	77	20	1.0	− 1	0	+ 39	+15
548	β Indi	3.7	20 46 59.780	4.7220	− 734	+ 12	0	71	7	0.3	− 20	− 5	+ 53	+21
563	γ Pavonis	4.2	21 18 10.679	5.0169	−1240	+ 130	−23	65	10	0.4	− 41	−25	+ 36	+14
564	ζ Capricorni	4.1	21 20 57.554	3.4324	− 166	0	0	74	33	1.3	− 5	− 4	+ 67	+20
575	γ Gruis	3.2	21 47 52.555	+3.6472	− 310	+ 91	0	76	14	0.6	+ 75	+14	+ 95	+31
578	α Gruis	1.9	22 1 55.985	3.8018	− 455	+ 123	0	64	21	0.6	+ 52	+13	+ 53	+12
583	α Tucanae	2.9	22 11 39.181	4.1486	− 846	− 107	+ 2	61	11	0.5	+ 71	+11	+ 27	+14
592	β Octantis	4.4	22 35 50.958	6.4162	−6264	− 295	+19	67	20	0.4	+106	+ 9	+ 75	+28
594	β Gruis	2.1	22 36 41.854	3.6015	− 435	+ 123	− 1	67	8	0.3	− 6	−10	+ 28	− 1
597	ε Gruis	3.7	22 42 30.950	+3.6466	− 516	+ 101	− 1	68	8	0.3	+ 20	+ 8	+ 87	+22
603	α Pisc. Austr.	1.3	22 52 7.579	3.3240	− 211	+ 251	− 1	68	69	1.9	+ 9	− 1	−	−
609	γ Tucanae	4.0	23 11 35.675	3.5292	− 636	− 55	0	66	9	0.4	+ 25	+ 2	+ 62	+26
616	β Sculptoris	4.6	23 27 36.649	3.2291	− 258	+ 77	0	79	11	0.4	+ 41	+ 6	+ 84	+26
621	δ Sculptoris	4.6	23 43 43.107	3.1322	− 160	+ 80	0	73	45	0.8	+ 56	+21	+108	+36

Third Section — (Declinations north of +82° and south of −82°).

No.	Name and Magnitude		R.A. 1900	Ann. V.	Sec. V.	μ .0001	100 Δμ .0001	Ep. and Wt. T	pα	pμ	B−N Δα .001	B−N Δμ .0001	B−A Δα .001	B−A Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
28	43 H Cephei	4.5	0 55 1.569	+ 7.3964	+ 1.4817	+ 769	+ 97	74	62	3.3	+ 98	+ 55	− 9	− 13
39	α Ursae min.	2.1	1 22 33.276	+ 25.2219	+20.130	+1377	+589	68	138	7.7	+ 86	+ 10	−239	− 3
117	Gr. 750	6.1	4 5 5.719	+ 17.3154	+ 1.7966	+ 145	+ 37	75	61	1.5	+ 72	+ 21	+ 62	− 7
153	Gr. 944	6.4	5 29 54.630	+ 18.6966	+ 0.5114	+ 184	+ 4	73	30	1.1	+290	+ 56	−	−
179	51 H Cephei	5.2	6 53 44.292	+ 29.6574	− 2.6345	− 469	− 79	72	118	3.7	+302	+ 91	+ 16	+ 24
185	25 H Camelop.	5.3	7 10 3.203	+ 12.8946	− 0.5208	+ 31	− 16	71	50	2.3	−470	−104	−	−
251	30 H Camelop.	5.3	10 18 54.857	+ 7.7197	− 0.8984	− 450	+ 37	78	48	1.0	− 57	+ 19	+ 81	+ 39
303	6 B Ursae min.	6.3	12 14 23.174	+ 0.2482	+ 0.8188	− 737	+208	74	45	3.0	+ 69	+ 14	−	−
315	32² H Camelop.	5.2	12 48 23.221	+ 0.4091	+ 0.2077	− 179	+ 15	69	40	2.9	+ 40	+ 8	−	−
435	ε Ursae min.	4.4	16 56 12.208	− 6.3039	+ 0.3152	+ 72	− 1	66	100	4.2	− 14	+ 15	+ 28	+ 9
477	δ Ursae min.	4.4	18 4 32.798	− 19.4859	− 0.1434	+ 209	− 88	69	142	6.7	+ 78	+ 19	+ 91	+ 36
511	λ Ursae min.	6.6	19 22 29.27	− 67.822	−26.888	−1033	−414	71	122	4.0	−310	+ 3	−829	−203
550	76 Draconis	5.7	20 49 50.647	− 4.0715	− 0.5374	+ 177	+ 2	78	44	1.6	+ 78	+ 46	+119	+ 50
6	o Octantis	7.2	0 12 29.31	− 0.7813	+ 2.3873	+ 49	− 24	69	24	0.7	−159	− 13	−639	−356
51	L 634	5.6	1 43 7.95	− 3.9460	+ 1.1741	+ 143	− 8	77	20	0.2	+ 98	+ 57	− 82	− 41
200	L 3911	7.8	7 53 1.62	− 44.248	−16.884	− 445	− 8	68	22	0.8	− 61	− 43	−361	−176
224	ζ Octantis	5.5	9 11 14.28	− 7.8696	− 1.6294	−1075	− 69	75	21	0.3	+119	+ 32	−461	−180
328	κ Octantis	5.7	13 24 42.09	+ 8.8359	+ 1.6060	− 753	− 73	75	23	0.3	+ 40	− 2	−180	− 73
343	δ Octantis	4.1	14 10 51.76	+ 9.0873	+ 1.0431	− 514	− 32	68	21	0.5	− 18	− 8	−	−
354	L 5823	6.5	14 38 59.75	+ 24.5623	+ 8.7590	−1814	−181	68	28	0.8	− 49	− 17	− 69	− 27
375	ρ Octantis	5.7	15 20 11.45	+ 13.1258	+ 1.4044	+ 852	+ 16	74	25	0.4	+ 8	+ 12	− 82	− 28
471	χ Octantis	5.2	17 56 4.25	+ 35.718	+ 0.3694	−1151	+494	75	22	0.3	− 92	− 98	−322	−140
499	σ Octantis	5.5	18 59 43.48	+102.433	−38.853	+1108	− 65	70	30	0.9	+273	+ 9	−217	− 55
570	L 6460	6.6	21 37 38.63	+ 68.373	−88.557	+ 56	+814	70	31	0.8	+117	+ 2	−163	−121
584	υ Octantis	5.7	22 12 34.94	+ 12.8332	− 3.1995	− 426	+ 7	69	31	0.8	− 45	− 23	+ 5	− 5
611	τ Octantis	5.6	23 13 9.30	+ 10.9791	− 5.2362	+ 147	− 57	69	34	0.8	−183	− 52	−183	− 78
622	γ¹ Octantis	5.1	23 46 14.49	+ 3.6585	− 0.3151	− 291	+ 22	67	20	0.5	−120	− 40	−	−

Second Section — (Declination, −21° 50′ to −82°).

No.	Decl. 1900	Ann. V. and Sec. V.		μ' and 100 $\Delta\mu'$		Ep. and Wt.			B—N		B — A		
			.001	.001	.001	T	p_δ	$p_{\mu'}$	$\Delta\delta$ .01	$\Delta\mu'$ .001	$\Delta\delta$ .01	$\Delta\mu'$ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
503	−38 3 36.95	+ 5.302	+572	−112	+ 1	70	8	0.4	+ 24	+ 6	+26	+ 7	
514	−25 6 15.78	7.696	+489	− 26	+ 1	70	39	1.2	− 7	+ 1	−40	−13	*h* Sagitt.
524	−27 59 16.59	9.770	+467	+ 12	0	79	31	1.0	− 16	− 1	−29	−15	*c* Sagitt.
531	−57 3 19.72	11.251	+569	− 85	0	64	18	0.6	+ 11	+ 7	−17	− 8	
537	−47 38 24.67	12.305	+484	+ 64	0	66	12	0.5	+ 26	+11	−26	−14	
541	−66 33 44.55	+12.630	+613	+ 18	− 1	68	14	0.4	+ 59	+20	+88	+27	
543	−25 37 49.43	12.740	+391	−158	− 1	75	21	1.0	− 93	−10	−25	− 4	
548	−58 49 52.94	13.325	+508	− 24	0	74	9	0.3	− 44	−15	−32	−15	
563	−65 49 6.94	16.061	+469	+807	+ 1	65	13	0.5	+ 64	+23	+47	+14	
564	−22 50 40.27	15.436	+314	+ 25	0	74	30	1.5	− 10	+ 6	− 2	0	
575	−37 50 6.94	+16.790	+283	− 17	+ 1	71	15	0.7	+ 1	+ 4	+23	+ 8	
578	−47 26 43.45	17.276	+266	−169	+ 1	66	23	0.7	− 4	+ 6	+ 7	+ 2	
583	−60 45 29.03	17.810	+267	− 38	− 1	66	12	0.5	− 77	− 4	+43	+12	
592	−81 54 20.88	18.713	+326	+ 2	− 2	68	27	0.6	+ 4	0	+65	+ 7	
594	−47 24 27.19	18.721	+179	− 16	+ 1	69	11	0.4	+ 12	+10	+21	0	
597	−51 50 33.87	+18.853	+169	− 59	+ 1	71	11	0.4	− 35	0	+18	+ 7	
603	−30 9 8.29	19.005	+134	−169	+ 1	70	50	1.4	− 7	+ 2	−	−	
609	−58 47 2.42	19.684	+ 99	+ 83	0	68	11	0.4	+ 73	+22	+ 3	+ 3	
616	−38 22 16.82	19.861	+ 58	+ 14	0	76	10	0.3	+ 16	+ 8	+57	+25	
621	−28 40 59.91	19.894	+ 24	−102	0	72	28	0.7	+ 68	+31	+ 9	+ 3	

Third Section — (Declinations north of +82° and south of −82°).

No.	Decl. 1900	Ann. V. and Sec. V.		μ' and 100 $\Delta\mu'$		Ep. and Wt.			B — N		B — A		
				.001	.001	T	p_δ	$p_{\mu'}$	$\Delta\delta$ .01	$\Delta\mu'$ .001	$\Delta\delta$ .01	$\Delta\mu'$ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
28	+85 43 14.50	+19.467	−0.268	− 5	− 3	80	65	2.2	− 24	− 1	− 1	+ 1	
39	+88 46 26.49	+18.761	−1.311	+ 1	− 7	68	256	14.2	− 12	− 3	−11	− 3	
117	+85 17 28.72	+ 9.668	−2.219	+ 33	− 2	82	66	1.3	− 34	−10	+11	+ 4	151 H Ceph.
153	+85 8 49.74	+ 2.626	−2.706	+ 1	− 3	74	17	0.6	+ 14	+ 5	−	−	158 H Ceph.
179	+87 12 20.38	− 4.696	−4.197	− 38	+ 7	76	148	4.3	− 15	− 3	+11	+ 7	
185	+82 36 16.00	− 6.077	−1.790	− 44	0	70	36	2.2	− 6	+ 3	−	−	
251	+83 4 2.96	−18.105	−0.470	+ 23	+ 3	79	52	1.0	+ 18	+15	−13	0	
303	+88 15 15.19	−19.947	+0.010	+ 60	− 1	76	36	2.1	− 1	+ 2	−	−	
315	+83 57 23.45	−19.585	+0.020	+ 17	− 1	70	32	1.8	+ 6	0	−	−	
435	+82 12 7.67	− 5.511	−0.880	− 3	+ 1	74	130	4.6	− 1	− 2	− 7	0	
477	+86 36 47.61	+ 0.446	−2.837	+ 48	+ 3	74	185	7.8	− 10	+ 1	−14	− 2	
511	+88 59 15.83	+ 7.071	−9.271	+ 11	−14	72	151	4.8	+ 2	+ 2	− 6	+ 1	
550	+82 9 40.11	+13.560	−0.442	+ 27	+ 2	70	52	1.8	+ 10	+ 2	+14	+ 6	
6	−88 55 8.30	+20.024	−0.002	+ 7	0	69	27	0.9	+ 1	+ 1	− 1	+ 2	
51	−85 16 29.30	+18.077	+0.242	+ 16	− 1	78	26	0.2	− 16	− 2	−24	− 9	4 G Oct.
200	−88 34 24.89	− 9.483	+5.691	+ 8	+ 6	69	28	0.8	+ 3	0	−25	−10	A Oct.
224	−85 15 46.87	−14.816	+0.787	+ 37	+10	77	28	0.3	− 6	− 4	−17	− 9	
328	−85 16 24.78	−18.721	+0.470	− 28	− 4	76	31	0.4	− 4	− 5	+20	+ 2	
343	−83 12 35.20	−16.882	+0.720	− 15	− 4	67	24	0.6	− 9	− 2	−	−	
354	−87 44 30.61	−15.478	+2.280	− 62	−17	68	31	1.0	+ 4	0	− 7	− 1	20 G Oct.
375	−84 7 55.06	−12.798	+1.482	+ 75	+ 9	73	30	0.4	− 2	− 4	+13	− 1	
471	−87 39 51.46	− 0.473	+5.190	−129	−17	74	29	0.4	− 8	− 5	− 7	− 4	
499	−89 15 16.46	+ 5.161	+14.44	− 4	+16	69	42	1.1	− 1	0	0	0	
570	−89 19 3.16	+16.270	+5.795	− 32	0	70	37	0.9	− 4	− 2	− 4	− 1	B Oct.
584	−86 28 33.87	+17.954	+0.835	+ 69	− 3	71	37	1.0	− 24	− 6	−13	− 1	
611	−88 1 52.83	+19.645	+0.315	+ 15	− 4	69	40	1.1	+ 2	0	− 7	0	
622	−82 34 28.59	+19.998	+0.023	− 17	0	70	25	0.7	− 15	− 1	−	−	

OBSERVATIONS OF COMET *b* 1902 (PERRINE),

MADE WITH THE 11-INCH EQUATORIAL AT THE SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS.,

BY ABBY E. TUCKER.

[Communicated by the Director, MARY E. BYRD.]

1902 Greenwich M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Oct. 8 15 51 56	1	12, 7	−0 26.57	−4 51.4	19 47 32.91	+41 8 26.3	9.719	0.423	+2.37	+33.5
21 13 58 16	3	8, 8	+0 29.18	+2 37.4	17 48 59.16	+ 6 10 9.1	9.633	0.758	+2.02	+16.1
25 12 55 46	4	12, 10	+0 20.61	+1 6.9	17 34 45.73	+ 0 42 9.4	9.614	0.768	+2.05	+13.1
Nov. 2 12 22 48	5	12, 6	+1 32.72	−2 2.3	17 13 29.64	− 6 42 31.6	9.628	0.781	+2.02	+ 9.2
7 11 8 18	6	8, 6	−4 4.71	+0 56.2	17 2 5.35	− 9 47 19.7	9.602	0.798	+2.06	+ 7.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	19 47 57.11	+41 12 44.2	Micro. Comp. with *2	4	17 34 23.07	+ 0 40 49.4	Nicolajew, A.G. 4375
2	19 48 31.71	+41 5 46.3	Bonn, A.G. 13479	5	17 11 54.90	− 6 40 38.5	Ottakring, A.G. Zones
3	17 48 27.96	+ 6 7 15.6	Leipzig II, A.G. 8141	6	17 6 8.00	− 9 48 23.7	Paris III, 21734

ELEMENTS AND EPHEMERIS OF COMET *a* 1903 (GIACOBINI),

BY H. R. MORGAN AND ELEANOR A. LAMSON.

[Communicated by Captain COLBY M. CHESTER, U.S.N., Superintendent.]

The following elements were deduced from three normal places derived from observations made at Washington on Jan. 21, 22, 23; 30; and Feb. 5 and 6.

ELEMENTS.

T = 1903 March 15.4644 Gr. M.T.

$$\left.\begin{array}{l}\pi = 136^\circ\ 5'\ 50''\\ \Omega = 2\ \ 21\ \ \ 6\\ i = 30\ \ 40\ \ 20\end{array}\right\}\text{Ecliptic 1903.0}$$

$q = .409204$

Residuals (O−C): $\cos\beta\,\Delta\lambda = +1''.6$, $\Delta\beta = -1''.5$

HELIOCENTRIC COORDINATES.

$$x = r[9.999905]\ \sin(225^\circ\ 46'\ \ 7''+v)$$
$$y = r[9.768323]\ \sin(137\ \ 25\ \ 31+v)$$
$$z = r[9.908572]\ \sin(134\ \ 54\ \ \ 2+v)$$

U.S. Naval Observatory, 1903 *Feb.* 10.

EPHEMERIS.

1903 Gr. M.T.	α	δ	log Δ	Light
	h m s	° ′		
Feb. 20.5	23 43 42	+12 3.8	0.1174	5.9
22.5	23 47 29	12 48.8	0.1038	
24.5	23 51 21	13 33.9	0.0891	8.4
26.5	23 55 17	14 18.4	0.0730	
28.5	23 59 17	15 1.3	0.0554	12.1
Mar. 2.5	0 3 17	15 41.4	0.0361	
4.5	0 7 13	16 16.8	0.0150	18.9
6.5	0 11 2	16 45.1	9.9917	
8.5	0 14 36	17 2.7	9.9663	29.4
10.5	0 17 46	17 5.5	9.9386	
12.5	0 20 25	16 48.2	9.9088	44.5
14.5	0 22 22	16 4.9	9.8771	
16.5	0 23 31	14 50.2	9.8442	62.4
18.5	0 23 49	12 59.3	9.8109	
20.5	0 23 18	10 29.5	9.7784	76.2
22.5	0 22 4	7 20.7	9.7477	
24.5	0 20 17	+ 3 35.4	9.7200	82.1

Brightness on Jan. 19.5 is adopted as the unit.

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THE ASTRONOMICAL JOURNAL.—SUPPLEMENT TO 531-532.

OBSERVATIONS OF COMET *b* 1902 (PERRINE),

MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY,

BY J. P. McCALLIE.

1902 Charl. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 10 12h 7m 1s	1	5,10	−1m 8s.71	+4′ 30″.1	3h 3m 53s.42	+39° 18′ 55″.1	n0.867	0.221	+4s.43	+2″.1
27 14 39 24	2	8,6	−1 34.68	+2 36.8	0 46 57.21	+55 15 52.0	0.836	n0.215	+6.01	+18.5
28 16 13 0	3	10,8	+1 19.01	+4 43.9	0 22 42.45	+56 11 0.5	1.052	9.736	+5.92	+21.8
29 15 41 59	4	10,8	−1 33.84	−3 38.0	23 57 21.79	+56 46 42.9	1.058	9.643	+5.80	+24.3
Oct. 1 14 39 10	5	10,8	+1 22.01	+4 18.4	23 0 14.05	+56 52 11.8	0.987	n9.904	+5.11	+30.2
7 13 2 31	6	8,8	−1 18.53	−3 59.1	20 6 0.70	+44 16 13.7	0.986	0.587	+2.55	+34.6
8 12 52 36	7	10,8	+1 43.29	+2 38.4	19 45 45.69	+40 48 14.5	0.961	0.640	+2.33	+33.2
14 10 10 18	8	d14,12	−0 0.38	+4 22.8	18 29 54.36	+21 12 0.2	0.862	0.653	+2.02	+23.9
17 8 50 30	9	8,8	+1 10.29	−1 13.0	18 8 54.27	+13 45 8.2	0.815	0.660	+1.99	+20.0
21 7 54 15	10	10,8	+0 37.83	+6 0.8	17 49 7.82	+ 6 13 32.1	0.785	0.698	+2.03	+16.1
22 7 55 48	11	d 8,8	−0 28.84	−1 40.1	17 45 3.1	+ 4 39.9	0.793	0.709	+2.04	+15.4
24 7 31 3	12	8,8	−0 54.20	−6 50.7	17 38 1.89	+ 1 56 31.8	0.782	0.722	+2.05	+14.0
28 7 6 33	13	d14,12	−0 16.88	−2 15.6	17 26 5.95	− 2 29 45.1	0.786	0.743	+2.04	+11.4
29 6 30 39	14	6,5	−2 55.86	+3 35.5	17 23 30.1	− 3 24.8	0.751	0.750	+2.05	+11.1
31 6 11 3	15	4,4	−3 1.40	−8 6.0	17 18 26.64	− 5 7 54.0	0.742	0.759	+2.04	+10.4
Nov. 1 6 9 42	16	8,8	+1 13.08	−7 2.3	17 15 59.89	− 5 55 27.5	0.750	0.762	+2.02	+ 9.6

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	3h 4m 57s.70	+39° 14′ 22″.9	Lund, A.G. 1634	9	18h 7m 41s.99	+13° 46′ 1″.2	Leipzig I, A.G. 6476
2	0 48 25.88	+55 12 56.7	Camb., (U.S.) A.G. 400	10	17 48 27.98	+ 6 7 15.6	Leipzig II, A.G. 8141
3	0 21 17.52	+56 5 54.8	Hels.-Gotha, A.G. 322	11	17 45 29.9	+ 4 41.3	DM. +4°3523
4	23 58 49.83	+56 49 56.6	Hels.-Gotha, A.G.14633	12	17 38 54.02	+ 2 3 7.7	Albany, A.G. 5893
5	22 58 46.93	+56 47 23.2	Hels.-Gotha, A.G.13683	13	17 26 20.78	− 2 27 41.2	Paris III, 22273
6	20 7 16.68	+44 19 38.0	Bonn, A.G. 13855	14	17 26 23.9	− 3 28.6	DM. −3°4120
7	19 44 0.07	+40 45 3.0	Bonn, A.G. 13394	15	17 21 25.97	− 4 59 59.0	Paris III, 22116
8	18 29 52.74	+21 7 13.8	Berlin B, A.G. 6540	16	17 14 44.76	− 5 48 35.4	Paris III, 21919

NOTES. Those comparisons marked *d* were taken directly by micrometrical measurements. — Corrections for refraction have been made. Oct. 7. Seeing 2. Images fuzzy. — Oct. 17. Seeing 1. Comet very faint. Hazy and full moon. — Oct. 21. Tail extends across 5-inch finder. — Oct. 28. Seeing 3. Slightly tremulous.

OBSERVATIONS OF COMET *b* 1902 (PERRINE),

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY.

[Communicated by Captain COLBY M. CHESTER, U.S.N., Superintendent.]

These observations give a correction of +5s in α and −1′.2 in δ to the ephemeris of ELIS STRÖMGREN in A.N. 3821.

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Feb. 5 11h 0m 8s	1	23,5	+1m 37s.18	−5′ 24″.2	8h 3m 49s.73	−33° 23′ 32″.7	n7.890	0.922	+2s.26	−17″.7
" 5 11 16 47	1	20,4	+1 32.90	−4 32.7	8 3 45.45	−33 22 41.2	8.477	0.922	+2.26	−17.7

Mean Place of Comparison-Star for the beginning of the year.

*	α	δ	Authority
1	8h 2m 10s.29	−33° 17′ 50″.8	C.G.C. 10728

Δα determined by transits. First observation by FREDERICK, second by DINWIDDIE.

THE

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FOUNDED BY B. A. GOULD.

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ON THE CONVERGENCY OF THE SERIES USED IN THE DETERMINATION OF THE ELEMENTS OF PARABOLIC ORBITS,

By WILLIAM ALBERT HAMILTON.

1. *Introductory.* — The elements of the orbit of a comet are usually determined by means of the data obtained at three separate, complete observations; and it often becomes a question of importance to the astronomer as to the suitability, or perhaps one might say the sufficiency, of such a set of observations to determine with accuracy the required elements. He is confronted on the one hand with a set of formulas somewhat complex in their nature, and which are subject to limitations in their application owing to the properties of the various functions involved in their construction; on the other hand, the data of observation are subject to limitations owing to unavoidable inaccuracies in the construction of the telescope and the multitude of items which fall under the class known as errors. It is thus both a mathematical and a physical problem with which he has to deal, and it becomes important, first that a careful analysis be made of the properties of the formulas and the conditions under which they may be applied, and secondly, that the errors which present themselves in the observations, in spite of the greatest care and skill on the observer's part, may not be allowed to become obscured in the final results of the computation. It has been the purpose of the study of which this paper is a partial result to investigate the formulas for computing cometary orbits from each of these two stand-points. In pursuance of this plan we have investigated among others the nature of the functions usually known as the "ratios of the triangles;" and have found the conditions under which they may be developed into power-series of the time-intervals between the observations. This discussion is given in the first part of this paper. In another part of the investigation, which is not included in the present paper, we have found the effects of the errors of the observations upon the computed elements of the orbit of a comet, using OLBERS's method as a basis of the study. To this is added the results of a computation, by use of the formulas so deduced, of the differentials of error in an actual case.

We proceed to discuss the ratios of the triangles and convergency of the series, using the following notation.

2. *Notation.* — Let t_1, t_2, t_3 denote the first, second, and third times of observation respectively. And if k^2 denote the Gaussian constant, and m the mass of the comet in terms of the mass of the sun taken as unity, then the differential equations of motion of the comet referred to the sun's center as origin of coordinates are

$$\left.\begin{aligned}\frac{d^2x}{dt^2} &= -\frac{k^2(1+m)\,x}{r^3}\\ \frac{d^2y}{dt^2} &= -\frac{k^2(1+m)\,y}{r^3}\\ \frac{d^2z}{dt^2} &= -\frac{k^2(1+m)\,z}{r^3}\end{aligned}\right\} \qquad (1)$$

where r is the heliocentric distance of the comet, and x, y, z are its rectangular cartesian coordinates. In all practical cases m will be infinitesimal in comparison with the mass of the sun, and therefore may be neglected. Furthermore, if we so change the unit of time that the new unit shall be equal to the old when the latter has been multiplied by k, and denote the time when expressed in the new units by $t-t_0$, where t_0 is any particular epoch, we may express these equations of motion very simply thus:

$$\left.\begin{aligned}\frac{d^2x}{dt^2} &= -\frac{x}{r^3}\\ \frac{d^2y}{dt^2} &= -\frac{y}{r^3}\\ \frac{d^2z}{dt^2} &= -\frac{z}{r^3}\end{aligned}\right\} \qquad (2)$$

In these equations the attractions of all the bodies of the solar system are neglected except that of the sun.

3. *Preliminary Notions.*—Suppose, now, the coordinates of the comet at the time t_0 to be x_0, y_0, z_0, and its velocities to be

$$\frac{dx_0}{dt}, \frac{dy_0}{dt}, \frac{dz_0}{dt}$$

then, at any other time, the coordinates and velocities are functions of these initial conditions and $t-t_0$; or, as we may write,

$$x = f\left(x_0, y_0, z_0, \frac{dx_0}{dt}, \frac{dy_0}{dt}, \frac{dz_0}{dt}, t-t_0\right)$$

with similar expressions for the other coordinates and the velocities.

Now, it is known from the theory of differential equations* that the coordinates and velocities are expansible into power-series in $t-t_0$ of the form

$$x = f\left(x_0 \ldots \frac{dx_0}{dt} \ldots 0\right) + \left[\frac{\partial f}{\partial t}\right]_0 (t-t_0) + \left[\frac{\partial^2 f}{\partial t^2}\right]_0 \frac{(t-t_0)^2}{2!} + \ldots \tag{3}$$

which have finite radii of convergency, if r does not vanish for $t-t_0 = 0$.

In the partial derivatives above, $t-t_0$ is to be placed equal to zero after differentiation. Hence from (3) it follows that

$$\left[\frac{\partial f}{\partial t}\right]_0 = \frac{dx_0}{dt}, \ldots \quad \left[\frac{\partial^n f}{\partial t^n}\right]_0 = \frac{d^n x_0}{dt^n} \ldots \tag{4}$$

From equations (2) we obtain

$$\left\{\begin{aligned} \frac{d^2x_0}{dt^2} &= -\frac{x^0}{r_0^3} \\ \frac{d^3x_0}{dt^3} &= \frac{3x_0}{r_0^4}\frac{dr_0}{dt} - \frac{1}{r_0^3}\frac{dx_0}{dt} \\ &\ldots\ldots\ldots\ldots \end{aligned}\right. \tag{5}$$

Equations (5) enable us to find the coefficients for the developments of the type (3), by means of which the coordinates and velocities of the comet at any time t are expressed as power-series of the time-intervals $t-t_0$, the coefficients depending only upon the coordinates and velocities at the initial time t_0. By means of these developments of the coordinates, the so-called ratios of the triangles are built up in the form of series which depend upon particular time-intervals selected from those determined by the three observations. It is of these latter series that we wish to find the conditions of convergency; and it is at once evident that their convergency will depend upon the convergency of the series of the type (3), since the ratios of the triangles are simple functions of the coordinates alone.

4. *Convergency of Series.*—From well known theorems of the theory of functions it follows that any expansions whatever of the ratios of the triangles into power-series for given time-intervals and initial conditions cannot have greater radii of convergency than the values which are determined by the positions of the poles and branch-points of the expressions of those ratios as functions of the time-intervals. First, however, we study the nature of the functions which express x, y, z in terms of t; and from these find the true radii of convergency.

5. *Coordinates as Functions of the Time.*—From the geometrical relations of the orbit of the comet we have the relations

$$\left.\begin{aligned} x &= r[\cos(v+\omega)\cos\Omega - \sin(v+\omega)\sin\Omega\cos i] \\ y &= r[\cos(v+\omega)\sin\Omega + \sin(v+\omega)\cos\Omega\cos i] \\ z &= r[\sin(v+\omega)\sin i] \end{aligned}\right\} \tag{6}$$

where v is the true anomaly, ω is the argument of the latitude of the perihelion, Ω is the longitude of the node, and i is the inclination of the orbit to the ecliptic. The last three quantities are independent of the time; while v and r are expressible in terms of t by means of the relations

$$\left.\begin{aligned} r &= \frac{p}{1+\cos v} \\ \tan\frac{v}{2} + \frac{1}{3}\tan^3\frac{v}{2} &= \frac{2}{p^{\frac{3}{2}}}(t-\Pi) \end{aligned}\right\} \tag{7}$$

where p is the *latus rectum* of the parabolic path of the comet and Π is the time of perihelion passage. Π and t are thought of as expressed in the units described in section 2 above—a usage which we shall continue throughout this paper.

6. *The Solution of the Cubic.*—By means of equations (7), we are enabled to express x, y and z in terms of the time-intervals $t-\Pi$. In order to do this we introduce the auxiliaries

$$\left.\begin{aligned} \tau &= \frac{3(t-\Pi)}{p^{\frac{3}{2}}} \\ \varphi &= \tan\frac{v}{2} \end{aligned}\right\} \tag{8}$$

Then the second equation of (7) becomes

$$\varphi^3 + 3\varphi - 2\tau = 0 \tag{9}$$

This is the so-called normal form of the cubic in the quantity φ. Its solutions by CARDAN's formula are

$$\left.\begin{aligned} \varphi_1 &= q_1 + q_2 \\ \varphi_2 &= \epsilon q_1 + \epsilon^2 q_2 \\ \varphi_3 &= \epsilon^2 q_1 + \epsilon q_2 \end{aligned}\right\} \tag{10}$$

where $q_1 = (\tau + \sqrt{1+\tau^2})^{\frac{1}{3}}$; $q_2 = (\tau - \sqrt{1+\tau^2})^{\frac{1}{3}}$, and ϵ, ϵ^2 are cube roots of unity (see BURNSIDE & PANTON's *Theory of Equations*, p. 108).

* JORDAN's *Cours d'Analyse*, Vol. III, p. 99.

7. *Branch-Points.* We want to express φ as a power-series in τ, and must therefore find the branch-points and poles of the function. At once we have the branch-points $\tau = i$ and $\tau = -i$, where $i = \sqrt{-1}$. Also $\tau = \infty$ is a branch-point, as is easily seen by putting $\tau = \frac{1}{\tau'}$, and letting τ' approach zero. This is the same as putting $\tau = \infty$, and we easily find that all three solutions have the same value at this point. If now we consider a RIEMANN-surface of three sheets whose branch-points are at $\tau = i$, $\tau = -i$ and $\tau = \infty$, then by the theory of functions of a complex variable we know that when the proper cross-cuts are introduced the quantity φ is a uniform function of position on this surface.

8. *Connection of the Sheets.* In order to get a clear idea of the surface it is necessary to find what sheets pass into each other at the two branch-points which are in the finite part of the plane. To do this, we need to follow only the purely imaginary values of τ; for the two branch-points in question are on the axis of pure imaginaries. Indeed, we may also consider the branch-point $\tau = \infty$ to be on this same axis.

In order to simplify matters, and at the same time to render the reasoning clearer, we make the transformation,

$$\tau = i\cos\theta \tag{11}$$

where θ is real or complex. Then q_1 and q_2 become

$$\begin{cases} q_1 = [i(\cos\theta - i\sin\theta)]^{\frac{1}{3}} = -ie^{-\frac{i\theta}{3}} \\ q_1 = [i(\cos\theta + i\sin\theta)]^{\frac{1}{3}} = -ie^{-\frac{i\theta}{3}} \end{cases}$$

And, since we may write $\epsilon = e^{\frac{2\pi i}{3}}$, $\epsilon^2 = e^{-\frac{2\pi i}{3}}$, we obtain from (10),

$$(12)\begin{cases} \varphi_1 = -i(e^{\frac{i\theta}{3}} + e^{-\frac{i\theta}{3}}) = -2i\cos\frac{\theta}{3} \\ \varphi_2 = -i(e^{\frac{i}{3}(\theta-2\pi)} + e^{-\frac{i}{3}(\theta-2\pi)}) = -2i\cos\left(\frac{\theta-2\pi}{3}\right) \\ \varphi_3 = -i(e^{\frac{i}{3}(\theta+2\pi)} + e^{-\frac{i}{3}(\theta+2\pi)}) = -2i\cos\left(\frac{\theta+2\pi}{3}\right) \end{cases}$$

Now, from (11), if θ takes real values, τ is purely imaginary, and takes values between $\tau = i$ and $\tau = -i$; while if θ is a pure imaginary, τ takes pure imaginary values with moduli greater than unity. Only when θ is complex does τ take real or complex values. Hence, for our purpose, we need consider only imaginary values of θ, or real values of θ, in order to find the connection of the sheets.

We must notice, also, that τ is a periodic function of θ; hence, when we wish τ to trace the line between the two branch-points but once, we take the primitive period and consider this alone. Now, in order that τ may take only pure imaginary values while passing from $\tau = i$ to $\tau = -i$, θ must take the real values between 0 and π, and therefore $\frac{\theta}{3}$ will take the real values between 0 and $\frac{\pi}{3}$. We get the following correspondence for θ, τ, φ_1, φ_2, φ_3:

θ	τ	φ_1	φ_2	φ_3
0	i	$-2i$	i	i
$\frac{\pi}{2}$	0	$-i\sqrt{3}$	0	$i\sqrt{3}$
π	$-i$	$-i$	$-i$	$2i$

Denote the branch-points $\tau = i$ and $\tau = -i$ by A and A' respectively. Then, from the table above we find, according to the period selected, that the two values φ_2 and φ_3 become equal when τ approaches A; but when τ arrives at A' along the path selected this does not repeat itself; but instead we have $\varphi_1 = \varphi_2$. Hence sheets φ_2 and φ_3 are connected at $\tau = i$; while φ_2 and φ_1 are connected at $\tau = -i$. It follows that if we start at $\tau = O$ in the τ-surface and make a complete circuit once around A, then O and $i\sqrt{3}$ will change places. If we draw branch-cuts from A to infinity and from A' to negative infinity, the continuation of the sheets when crossing these cuts will be

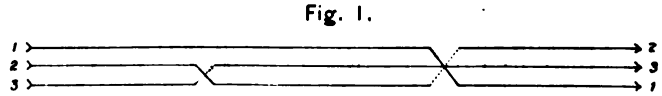
$$\text{Along } A \text{ to } \infty, \ldots \frac{1.2.3}{1.3.2}$$

$$\text{Along } A' \text{ to } -\infty, \ldots \frac{1.2.3}{2.1.3}$$

All three sheets are connected at $\tau = \infty$. A section along the axis of pure imaginaries will appear as in Fig. 1.

It will be of importance for what follows to notice that in the θ-plane the portion which is bounded by the axis of pure imaginaries and the line $\theta = \pi$ is a conform representation of the whole τ-plane — each sheet being represented once in the fundamental region.

Fig. 1.



9. *Poles of φ in the Sheets.* It is well known that, where z is a complex variable, the function e^z can become infinite only for infinite values of z. Hence it follows from (12), that φ cannot become infinite except for infinite values of θ. Moreover, owing to the periodicity of the function e^z which makes $e^{z+2\pi i} = e^z$, the above infinite value of θ must be either purely imaginary, or perhaps complex with the imaginary part of the complex expression infinitely great. But, from (11), such a value of θ gives τ infinite. Consequently, it follows that φ cannot become infinite

except for infinite values of τ. Hence, there are no poles of φ in the finite parts of the sheets of the RIEMANN-surface.

10. *Zeroes in the Sheets.* By use of (12) we are also enabled to find at once the zeroes of φ in the τ-surface. The general condition for the vanishing of φ is given by either of the equations

$$\left\{\begin{array}{l} \cos\dfrac{\theta}{3} = 0 \\ \cos\left(\dfrac{\theta\pm 2\pi}{3}\right) = 0 \end{array}\right.$$

These are virtually the same, since we may get the one from the other by putting $\theta = \theta' \pm 2\pi$. It is then only necessary to find the value of θ for which $\cos\frac{\theta}{3} = 0$. Now, by methods well known in the theory of trigonometry (see CHRYSTAL'S *Algebra*, Vol. II, Chap. 29), it is readily proved that the only values of θ (real or complex) which satisfy this condition are

$$\theta = \frac{2n+1}{2}\pi$$

where n is a positive or negative integer or zero. It follows that only one zero of φ is to be found in each fundamental region of the θ-plane for each of the sheets of the RIEMANN-surface. Thus, for the first sheet, it is that value which corresponds to the value of θ which gives, by use of (11), $\tau = 0$. We have already seen in the table following (12) that only one branch of φ vanishes at this point; and that the particular one which vanishes thus is dependent entirely upon the sheet of the RIEMANN-surface in which τ is found.

11. *Résumé.* We note here the following summary of results as to critical points upon the RIEMANN-surface upon which φ is a function of position:

(13)

$$\left\{\begin{array}{l} \text{Zeroes at} \left\{\begin{array}{l} \tau = 0 \text{ in the } \tau\text{-surface} \\ \theta = \dfrac{\pi}{2} \text{ in the fundamental region of } \theta\text{-plane} \end{array}\right. \\ \text{Poles — None in the finite part of the plane} \\ \text{Branch-points} \left\{\begin{array}{l} \tau = i \\ \tau = -i \\ \tau = \infty \end{array}\right. \end{array}\right.$$

12. *Rational Functions of φ and τ.* At this place we state the following theorem which will be useful for later work.

Every rational function of φ and τ is a uniform function of position on the same RIEMANN-*surface as that which describes φ as a function of τ, and its branch-points are at the same places.* (See FORSYTHE, *Theory of Functions*, page 369). It is to be remembered, however, that this theorem does not apply to the *zeroes* and *poles* of such a rational function of φ and τ. These may be located differently, as described in (13).

13. *Coordinates x and y as functions of τ.* We may write the first two equations of (6) as follows:

$$\left.\begin{array}{l} x = [c\cos v - s\sin v]\left(1+\tan^2\dfrac{v}{2}\right) \\ y = [c_1\cos v - s_1\sin v]\left(1+\tan^2\dfrac{v}{2}\right) \end{array}\right\} \quad (14)$$

where $p = r(1+\cos v)$; and

$$\left.\begin{array}{l} \dfrac{2c}{p} = \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i \\ \dfrac{2s}{p} = \sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i \\ \dfrac{2c_1}{p} = \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i \\ \dfrac{2s_1}{p} = \sin\omega\sin\Omega - \cos\omega\cos\Omega\cos i \end{array}\right\} \quad (15$$

Using the relations

$$\cos v = \frac{1-\tan^2\dfrac{v}{2}}{1+\tan^2\dfrac{v}{2}} \quad \text{and} \quad \sin v = \frac{2\tan\dfrac{v}{2}}{1+\tan^2\dfrac{v}{2}}$$

we may write (14), where we put $\tan\frac{v}{2} = \varphi$, in the form

$$\left.\begin{array}{l} x = c\ -2s\varphi - c\varphi^2 \\ y = c_1 - 2s_1\varphi - c_1\varphi^2 \end{array}\right\} \quad (16)$$

Now, in the equations (16) c, c_1, s and s_1 are constants, independent of τ; and, by the theorem given in the last article, x and y, considered as functions of τ, are functions of position on the same RIEMANN-surface which defines φ as a function of τ, and the branch-points of x and y are $\varphi = i$, $\varphi = -i$ and $\varphi = \infty$. Moreover, since, c, c_1, s and s_1 are constants and never infinite, x and y cannot become infinite except where φ becomes infinite, viz.: at $\tau = \infty$. Hence we have

THEOREM: *x and y have poles in the* RIEMANN-*sheets only at $\tau = \infty$; and they have branch-points at $\tau = i$, $\tau = -i$ and $\tau = \infty$.*

It follows from the above that x and y are holomorphic functions of τ in the sheets of the RIEMANN-surface, except at the points $\tau = i$, $\tau = -i$ and $\tau = \infty$. Therefore, they may each be expanded into power-series with argument $\tau - \tau_0$ in the vicinity of any point $\tau = \tau_0$. These series will be convergent inside of a circle with center τ_0 and radius reaching from τ_0 to the nearest of the points $\tau = i$ or $\tau = -i$.

14. *Radius of Convergency.* If in (3) we replace t and t_0 by their corresponding values in τ by relation (8), we

have just such an expansion as described in the last article. If we should at the same time take $\tau_0 = 0$, the expansion in x becomes of the form

$$x = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3 + \ldots$$

where a_0, a_1, &c., are constants. This series will be convergent inside a circle whose center is $\tau_0 = 0$, and with radius unity reaching up to the branch-points $\tau = i$ and $\tau = -i$. Hence, the true radius of convergency in this case would be $|\tau| = 1$; or from (8)

$$(17) \qquad t - \Pi = \frac{p^{\frac{3}{2}}}{3}$$

Suppose in (17) we give to p any value, say 1, which would correspond to a perihelion distance of 0.5; then if we make $k = \frac{1}{60}$, which is its approximate value, the case under supposition would give, as the limit for the time-interval for which the expansion of x into power-series would be convergent, the value 20 days. The same period would hold for the corresponding expansion of y.

If τ_0 were any finite point not equal to zero, say some point on the real axis of the τ-plane, then the radius of the true circle of convergency would be larger than that given above. In this case the radius would be $R_T = \sqrt{1+\tau^2}$, which holds for both the x and y series. The radius of convergency of the corresponding series in t is at once deducible from the series in τ through the relations (8). The relation is always

$$(18) \qquad R_t = \frac{p^{\frac{3}{2}}}{3} R_T$$

where the subscripts denote the argument of the series.

Since in an absolutely convergent series we are at liberty to change the order of the terms at will, we may express (3) and the corresponding equation in y by use of coefficients of the kind given in (5), as follows:

$$(19) \qquad \left\{ \begin{aligned} x &= A x_0 + B \frac{dx_0}{dt} \\ y &= A y_0 + B \frac{dy_0}{dt} \end{aligned} \right.$$

where A and B for their first few terms are

(20)

$$A = 1 - \frac{1}{2}\frac{t^2}{r_0^{\,3}} + \frac{1}{2}\frac{t^3}{r_0^{\,4}}\frac{dr_0}{dt} + \frac{t^4}{24}\left[\frac{1}{r_0^{\,6}} - \frac{12}{r_0^{\,5}}\left(\frac{dr_0}{dt}\right)^2 + \frac{3}{r_0^{\,4}}\frac{d^2r_0}{dt^2}\right] + \cdots$$

$$B = t - \frac{1}{6}\frac{t^3}{r_0^{\,3}} + \frac{1}{4}\frac{t^4}{r_0^{\,4}}\frac{dr_0}{dt} + \frac{3}{10}\frac{t^4}{r_0^{\,5}}\left(\frac{dr_0}{dt}\right)^2 + \cdots$$

In these series we have taken $t_0 = 0$. They may be written as series in $t - t_c$ by use of the Weierstrassian theory of the continuation of power-series.

Since a power-series serves in every way to define the behavior of the function from which it is derived as long as we remain within its circle of convergence, we can deal with the series (19) as with quantities which obey all the laws of ordinary algebra (association, commutation, &c.), such as ordinary polynomials or rational quantities, and the resulting series will be convergent. (See CHRYSTAL's *Algebra*, Vol. II, pp. 139–143).

15. *Ratios of the Triangles.* We denote the triangles between the positions of two radii vectores of the comet's orbit by the expression $[r_i, r_j]$ where i and j denote the orders of any two of the three observations; also, in general, we denote the three coordinates of the first, second, and third observations by the subscripts 1, 2, 3 respectively. Now the ratios of the triangles $[r_i, r_j]$ are equal to the ratios of their projections on any plane, which may be expressed thus:

$$\left. \begin{aligned} \frac{[r_2, r_3]}{[r_1, r_2]} &= \frac{x_2y_3 - y_2x_3}{x_2y_1 - y_2x_1} \\ \frac{[r_1, r_2]}{[r_1, r_3]} &= \frac{x_2y_1 - y_2x_1}{x_1y_3 - y_1x_3} \end{aligned} \right\} \qquad (21)$$

Let now $x_2, y_2, z_2, \frac{dx_2}{dt}, \frac{dy_2}{dt}, \frac{dz_2}{dt}$ be taken as the zero-values of the coordinates and velocities in the expansions (19) and (20). Then we get

$$\left. \begin{aligned} x_1 &= A_1x_2 + B_1\frac{dx_2}{dt} \\ y_1 &= A_1y_2 + B_1\frac{dy_2}{dt} \\ x_3 &= A_3x_2 + B_3\frac{dx_2}{dt} \\ y_3 &= A_3y_2 + B_3\frac{dy_2}{dt} \end{aligned} \right\} \qquad (22)$$

where A_1, B_1, A_3, B_3 are defined by

(23)

$$A_1 = 1 - \frac{1}{2}\frac{(t_1-t_2)^2}{r_2^{\,3}} + \ldots, \quad B_1 = (t_1-t_2) - \frac{1}{6}\frac{(t_1-t_2)^3}{r_2^{\,3}} + \ldots$$

$$A_3 = 1 - \frac{1}{2}\frac{(t_3-t_2)}{r_2^{\,3}} + \ldots, \quad B_3 = (t_3-t_2) - \frac{1}{6}\frac{(t_1-t_2)^3}{r_2^{\,3}} + \ldots$$

Now, the series (22) are all convergent within the same circle. It follows that, since $x_2, y_2, \frac{dx_2}{dt}, \frac{dy_2}{dt}$ are not in general equal to zero, the series (23) are also convergent in this same circle (see CHRYSTAL's *Algebra*, Vol. II, p. 178, 5). It follows also, since for such series the law of distribution holds (*Ibid*, pp. 142–143), that the products A_1B_3 and A_3B_1 are also convergent series. Hence, from the law of addition, we have that $A_1B_3 - A_3B_1$ is also convergent.* (*Ibid*, p. 141).

* Prof. HARZER, in Kiel Publications, Vol. XI, Sect. 2, has shown from a direct consideration of the series by processes of successive simplifications and diminishing of the realm of convergency, that the time-intervals may be taken small enough so that the series converge. The same result can be inferred from JORDAN's *Cours d'Analyse*, Vol. III, p. 99. These results are of little practical value because the radius found differs so widely from the true value. By the method of this paper we have the ***upper limit of the time-intervals for which the series are convergent and that for any case that may arise whatever.***

We are now at liberty to substitute the values of x_1, y_1, x_3, y_3, as given by (22) in the ratios on the right of (21). We get after the substitution indicated, and by cancelling the factor $x_2\frac{dy_2}{dt} - y_2\frac{dx_2}{dt}$, from the two members of the ratio,*

$$(24)\quad \begin{cases} \dfrac{[r_2\,,\,r_3]}{[r_1\,,\,r_2]} = -\dfrac{B_3}{B_1} = \dfrac{(t_3-t_2)}{(t_2-t_1)}\left[1-\frac{1}{6}\,\dfrac{(t_3-t_2)^2-(t_1-t_2)^2}{r_2^{\,3}}+\frac{1}{4}\,\dfrac{(t_3-t_2)^3+(t_2-t_1)^3}{r_2^{\,4}}\,\dfrac{dr_2}{dt}+\ldots\right] \\[2ex] \dfrac{[r_1\,,\,r_2]}{[r_1\,,\,r_3]} = \dfrac{-B_1}{A_1B_3-A_3B_1} = \dfrac{(t_2-t_1)}{(t_3-t_1)}\left[1+\frac{1}{6}\,\dfrac{(t_3-t_1)^2-(t_2-t_1)^2}{r_2^{\,3}}-\frac{1}{4}\,\dfrac{(t_3-t_2)[(t_3-t_2)(t_3-t_1)-(t_2-t_1)^2]}{r_2^{\,4}}\,\dfrac{dr_2}{dt}+\ldots\right] \end{cases}$$

where A_1, B_1, A_3, B_3, have the meaning given by (23).

Now, B_1 and B_3 are series with arguments t_1-t_2 and t_3-t_2 respectively. They hold: *i.e.*, are convergent, as long as t_1-t_2 and t_3-t_2 obey the relations

$$(25)\quad \begin{cases} |\,t_1-t_2\,| < \dfrac{p^{\frac{3}{2}}}{3}\sqrt{\dfrac{9}{p^3}(t_2-\Pi)^2+1} \\[2ex] |\,t_3-t_2\,| < \dfrac{p^{\frac{3}{2}}}{3}\sqrt{\dfrac{9}{p^3}(t_3-\Pi)^2+1} \end{cases}$$

Also the series $A_1B_3-B_1A_3$, which has two arguments, viz.: t_1-t_2 and t_3-t_2, is convergent so long as (25) hold.

16. *The Zeroes of B_1 and $A_1B_3-B_1A_3$.* If B_1 should vanish, or if $A_1B_3-B_1A_3$ should vanish, then the fractions on the right of (24) evidently would no longer be legitimate. As to this question we state two theorems which are easily proved; but the proofs of which are too long to present here. They are as follows:

Theorem I. The expressions $x_1y_3-y_1x_3$ and $x_1y_2-y_1x_2$ can vanish only for real values of v_3-v_1.

Theorem II. In all cases where the times of the observations are distinct, and where the differences of the longitudes of the comet in its orbit are not equal to an odd multiple of π, the expression $(r_i\,,\,r_j)$ cannot vanish, and the expressions $\dfrac{B_3}{B_1}$, $\dfrac{B_1}{A_1B_3-B_1A_3}$ are legitimate fractions which may be expressed as series, each of which is convergent for all

$$|\,t_3-t_2\,| < \frac{p^{\frac{3}{2}}}{3}\sqrt{1+\tau_3^{\,2}} \quad \text{and} \quad |\,t_1-t_2\,| < \frac{p^{\frac{3}{2}}}{3}\sqrt{1+\tau_1^{\,2}}$$

The first terms of these series are written out in the right members of (24).

Computation by Use of the Series. The fractions on the right of (24) have both numerators and denominators in the form of series. Their radius of convergency is

$$R_i = \frac{p^{\frac{3}{2}}}{3}\sqrt{1+\tau_2^{\,2}} \quad : \quad \text{where } \tau_2 = \frac{3}{p^{\frac{3}{2}}}\,(t_2-\Pi)$$

If we make $k = \frac{1}{60}$, the last line of the following table will give the corresponding maximum intervals of time in days for different values of p for which the series are convergent when t_2 is taken equal to Π:

TABLE I.

p	4.0	3.0	2.5	2.0	1.5	1.25	1.0	.8	.6	.4	.25	.2	.1	.08	.05	.02
q	2.0	1.5	1.25	1.0	.75	.63	.5	.4	.3	.2	.125	.1	.05	.04	.025	.01
da	160.0	103.4	79.8	56.0	36.2	27.4	20.0	14.1	9.0	5.0	2.5	1.8	.6	.44	.22	.06

It is evident that for any particular value of p the time-intervals should be well within the limit of values for which the series are convergent. This is especially true if we would have the most rapid convergence — a thing most desirable from the standpoint of the computer. In fact, as is well known, it is imperative to have this convergence so rapid that at most but one or two terms will give sufficiently approximate values of the ratios. The reason for this is at once evident when we consider that the series are transcendental in character. Thus the quantities $\frac{dr_2}{dt}$ and r_2 which enter into the terms higher than the first are essentially unknown from the start, and cannot even be guessed at with any degree of certainty until an approximate value of p has been obtained. It cannot be too strongly insisted upon, therefore, that, in order to get the closest determination of the ratios of the triangles, the greatest care must be taken to secure a set of time-intervals which, by their coordination with the parameter of the orbit in hand, will make the series rapidly convergent. It is true that this is more or less a question of trial to start with; yet when a value of p has been once computed by means of any set of time-intervals, it will be seen at once whether the value so obtained is one for which the series are sufficiently convergent for the time-intervals employed. If this is not the case, then new time-intervals should be taken and the computation made over again.

Beloit College, Beloit, Wis.

* Prof. HARZER, in *l.c.*, Sect. 1, has carried the expansion to the 10th degree in the time-intervals.

OBSERVATIONS OF HELIOMETER COMPARISON-STARS,

MADE WITH THE 6-INCH TRANSIT CIRCLE OF THE U. S. NAVAL OBSERVATORY,

BY PROFESSOR M. UPDEGRAFF AND COMPUTER J. C. HAMMOND.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

The following are the results of observations of the stars comprising the last five lists of heliometer comparison-stars proposed by Sir DAVID GILL, H.M. Astronomer, Cape of Good Hope, in his circular of April 2, 1901. They are a continuation of the lists published in *A.J.*, No. 528.

The same notation has been employed as in the publication of the previous lists. The instrument was reversed during each series except that for *Uranus*. The mean epochs of the observations are in the order of the lists, 1902.57, 1902.62, 1902.62, 1902.65 and 1902.80 respectively.

On the nights that Mr. HAMMOND observed, the microscopes on Circle A were read, and all the recording was done by Mr. H. R. MORGAN.

The lists here given and those in *A.J.* No. 528 are not corrected for magnitude equation.

FUNDAMENTAL STARS, 1902.0.

Star	Mag.	R.A.	Decl.	Star	Mag.	R.A.	Decl.
		h m s	° ′ ″			h m s	° ′ ″
σ Scorpii	3.1	16 15 13.810	−25 21 28.05	*π Capricorni*	5.2	20 21 42.760	−18 31 59.01
ρ Ophiuchi	4.7	16 19 42.392	−23 13 14.79	*υ Capricorni*	5.4	20 34 28.324	−18 29 1.46
τ Scorpii	2.9	16 29 46.810	−28 0 46.36	*θ Capricorni*	4.2	21 0 26.360	−17 37 20.97
μ Sagittarii	4.0	18 7 54.139	−21 5 4.77	*ι Capricorni*	4.3	21 16 47.479	−17 15 7.34
δ Sagittarii	2.8	18 14 43.213	−29 52 11.76	*γ Capricorni*	3.8	21 34 39.757	−17 6 18.18
λ Sagittarii	2.9	18 21 55.370	−25 28 33.98	*κ Capricorni*	4.8	21 37 11.221	−19 18 47.07
30 *Sagittarii*	6.1	18 44 57.006	−22 16 27.65	*δ Capricorni*	3.0	21 41 37.976	−16 34 19.65
π Sagittarii	3.0	19 3 56.174	−21 10 46.44	*λ Aquarii*	3.9	22 47 30.144	− 8 6 4.16
ψ Sagittarii	4.9	19 9 31.920	−25 25 32.83	*β Piscium*	4.6	22 58 53.397	+ 3 17 32.54
d Sagittarii	5.0	19 11 54.089	−19 7 38.97	*φ Aquarii*	4.4	23 9 14.839	− 6 34 38.61
h_2 Sagittarii	4.6	19 30 44.654	−25 6 0.31	*λ Piscium*	4.6	23 37 2.753	+ 1 14 26.40
f Sagittarii	5.1	19 40 38.766	−19 59 48.60	25 *Piscium*	6.3	23 48 3.590	+ 1 32 44.72
62 *Sagittarii*	4.6	19 56 38.001	−27 58 56.88	27 *Piscium*	5.0	23 53 39.361	− 4 5 58.66
4 *Capricorni*	6.0	20 12 15.977	−22 6 46.01	30 *Piscium*	4.7	23 56 56.055	− 6 33 31.31
β Capricorni	3.2	20 15 30.391	−15 5 27.66				

STARS FOR *Uranus*, 1901, 1902, 1903 AND 1904.

OBSERVERS, UPDEGRAFF AND HAMMOND.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
a	5.7	16 36 7.95	−19 44 13.6	5	*y*	6.5	17 29 24.73	−21 58 41.4	4
b	8.0	16 37 12.49	−21 9 23.3	3	*z*	8.0	17 29 33.76	−24 33 40.4	4
c	6.5	16 39 14.92	−23 0 6.4	5	*α*	7.9	17 31 49.57	−23 19 43.2	4
d	8.7	16 43 14.15	−20 16 59.6	2	*β*	6.3	17 32 51.45	−21 51 17.4	4
e	7.0	16 43 44.24	−21 40 49.8	4	*γ*	7.8	17 34 51.15	−23 47 1.0	5
f	8.5	16 44 4.58	−23 16 41.3	2	*δ*	5.0	17 37 33.42	−21 38 9.0	4
g	7.3	16 48 47.11	−21 43 10.3	5	*π*	8.8	17 38 53.04	−24 38 27.2	4
h	5.6	16 50 53.31	−22 59 41.7	5	*ε*	8.3	17 39 8.51	−22 50 46.5	4
k	7.0	16 54 39.20	−21 18 46.2	5	*ρ*	7.4	17 43 57.85	−24 10 30.7	4
l	7.5	16 57 26.42	−23 0 39.7	5	*μ*	8.7	17 44 45.31	−21 54 6.4	4
m	6.6	17 0 20.64	−21 25 44.8	5	*ψ*	7.0	17 45 10.89	−22 53 27.3	4
n	8.7	17 2 38.13	−23 5 53.0	4	*σ*	6.2	17 48 52.08	−24 52 3.8	4
o	7.7	17 6 28.34	−22 48 21.2	4	*χ*	7.0	17 50 27.35	−21 56 22.8	4
p	6.8	17 6 47.60	−21 29 14.9	4	*ν*	8.0	17 50 31.80	−23 22 27.9	4
q	7.0	17 12 7.94	−23 57 54.1	4	*ξ*	4.6	17 53 48.52	−23 48 26.3	4
r	8.8	17 12 59.96	−22 36 12.5	4	*η*	8.1	17 53 57.50	−25 4 46.0	3
s	4.5	17 15 7.77	−21 0 28.9	5	*ζ*	6.0	17 55 58.15	−22 46 40.7	5
t	8.0	17 17 17.27	−22 54 54.6	4	*τ*	6.5	17 59 9.89	−24 24 14.1	5
u	6.5	17 18 50.29	−21 21 0.8	5	*υ*	6.4	18 1 18.68	−21 27 15.1	4
v	4.5	17 20 23.07	−24 5 8.2	4	*θ*	8.3	18 1 48.32	−23 6 58.8	3
w	8.5	17 22 46.69	−22 29 59.6	5	*φ*	5.3	18 5 44.51	−23 43 16.4	5
x	4.9	17 25 26.13	−23 53 13.9	5					

Probable error of a single observation in α, 0s.016; in δ, 0″.30 for Circle A, 0″.33 for Circle B.

STARS FOR *Saturn* AND *Jupiter*, 1901.

OBSERVERS, UPDEGRAFF AND HAMMOND.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
a	5.9	18 27 54.27	−24 6 20.0	7	*m*	8.0	18 45 51.64	−24 46 10.5	5
b	8.0	18 29 26.55	−22 10 3.9	6	*n*	5.0	18 48 15.18	−22 51 56.4	7
c	6.3	18 32 2.36	−21 28 44.3	6	*q*	3.5	18 51 53.03	−21 14 8.6	6
d	5.8	18 32 33.12	−23 35 19.5	6	*r*	6.5	18 55 43.30	−22 50 1.0	6
e	6.2	18 35 53.01	−23 55 29.7	7	*s*	3.9	18 58 48.66	−21 53 7.0	6
f	7.3	18 37 25.72	−22 30 23.0	6	*t*	7.1	19 2 49.22	−23 20 40.0	6
g	5.7	18 38 48.14	−25 6 34.1	6	*u*	8.6	19 3 37.85	−22 32 1.9	5
h	5.6	18 40 26.05	−22 29 42.0	6	*v*	3.1	19 3 56.17	−21 10 46.5	6
k	8.4	18 41 58.91	−23 21 49.2	6	*w*	7.3	19 8 16.76	−22 13 38.0	6
l	6.1	18 44 57.05	−22 16 28.0	6					

NOTE: *p* too faint to observe.

Probable error of a single observation in α, $0^s.016$; in δ, $0''.23$ for Circle A, $0''.27$ for Circle B.

STARS FOR *Saturn*, 1902.

OBSERVER, UPDEGRAFF.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
a	5.1	19 40 38.77	−19 59 48.4	4	*d*	8.2	19 46 8.67	−19 56 43.9	7
b	7.2	19 40 42.13	−21 45 39.9	6	*e*	6.7	19 53 45.76	−22 28 37.2	7
c	8.2	19 44 25.61	−23 1 35.7	7	*f*	7.0	19 54 48.28	−20 7 29.8	6

Probable error of a single observation in α, $0^s.024$; in δ, $0''.38$ for Circle A, $0''.35$ for Circle B.

STARS FOR *Jupiter*, 1902, AND *Saturn*, 1903.

OBSERVERS, UPDEGRAFF AND HAMMOND.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
α	5.6	20 24 16.88	−18 54 27.6	6	*n*	7.0	20 46 39.90	−20 0 39.5	6
z	8.3	20 27 44.93	−21 13 49.3	7	*m*	6.9	20 47 57.17	−19 29 0.9	7
y	7.8	20 28 49.62	−19 43 57.0	6	*l*	6.0	20 49 15.67	−18 17 40.3	6
x	8.5	20 29 49.78	−18 7 26.5	7	*a*	6.2	20 54 2.10	−19 24 55.0	7
w	7.2	20 30 46.26	−20 55 26.7	6	*b*	6.5	20 55 20.88	−17 54 47.2	6
v	8.7	20 33 55.27	−20 0 58.2	7	*c*	8.0	20 58 29.93	−18 29 56.9	7
u	5.3	20 34 28.30	−18 29 1.4	6	*d*	4.3	21 0 26.36	−17 37 20.4	6
t	8.8	20 36 25.33	−21 37 26.3	7	*e*	6.9	21 1 56.72	−19 28 49.0	7
s	7.3	20 38 18.65	−19 41 44.1	6	*f*	7.7	21 4 41.60	−16 5 59.2	4
r	8.0	20 41 2.64	−17 31 8.0	6	*g*	8.3	21 4 53.67	−17 21 21.5	4
q	8.0	20 41 48.18	−18 58 45.2	7	*h*	8.0	21 5 1.20	−18 43 44.9	4
o	8.0	20 43 35.29	−20 59 16.2	6	*k*	6.4	21 9 37.70	−17 45 2.3	6
p	6.7	20 43 47.08	−18 23 51.2	7					

Probable error of a single observation in α, $0^s.020$; in δ, $0''.39$ for Circle A, $0''.32$ for Circle B.

STARS FOR *Jupiter*, 1903.

OBSERVER, HAMMOND.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
a	4.2	23 9 14.86	−6 34 38.8	12	*g*	7.0	23 21 30.36	−7 8 45.6	5
b	5.7	23 14 19.16	−5 39 35.5	6	*h*	8.2	23 21 35.73	−5 46 18.0	6
c	6.5	23 15 11.04	−4 27 9.4	4	*i*	6.3	23 24 28.13	−5 4 0.2	6
d	6.5	23 15 37.79	−6 26 34.9	4	*k*	6.8	23 25 57.96	−6 49 40.4	6
e	8.0	23 16 10.58	−7 33 36.1	4	*l*	7.2	23 28 25.83	−4 56 32.1	6
f	8.9	23 19 44.81	−5 35 3.8	7					

Probable error of a single observation in α, $0^s.014$; in δ, $0''.25$ for Circle A, $0''.18$ for Circle B.

OBSERVATIONS OF COMET *b* 1902 (*PERRINE*),*

MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY,

BY J. P. McCALLIE.

1902 Charl. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Sept. 10 12 7 1	1	5,10	−1 8.71	+4 30.1	3 3 53.42	+39 18 55.1	*n*0.867	0.221	+4.43	+ 2.1
27 14 39 24	2	8,6	−1 34.68	+2 36.8	0 46 57.21	+55 15 52.0	0.836	*n*0.215	+6.01	+18.5
28 16 13 0	3	10,8	+1 19.01	+4 43.9	0 22 42.45	+56 11 0.5	1.052	9.736	+5.92	+21.8
29 15 41 59	4	10,8	−1 33.84	−3 38.0	23 57 21.79	+56 46 42.9	1.058	9.643	+5.80	+24.3
Oct. 1 14 39 10	5	10,8	+1 22.01	+4 18.4	23 0 14.05	+56 52 11.8	0.987	*n*9.904	+5.11	+30.2
7 13 2 31	6	8,8	−1 18.53	−3 59.1	20 6 0.70	+44 16 13.7	0.986	0.587	+2.55	+34.6
8 12 52 36	7	10,8	+1 43.29	+2 38.4	19 45 45.69	+40 48 14.5	0.961	0.640	+2.33	+33.2
14 10 10 18	8	*d*14,12	−0 0.38	+4 22.8	18 29 54.36	+21 12 0.2	0.862	0.653	+2.02	+23.9
17 8 50 30	9	8,8	+1 10.29	−1 13.0	18 8 54.27	+13 45 8.2	0.815	0.660	+1.99	+20.0
21 7 54 15	10	10,8	+0 37.83	+6 0.8	17 49 7.82	+ 6 13 32.1	0.785	0.698	+2.03	+16.1
22 7 55 48	11	*d* 8,8	−0 28.84	−1 40.1	17 45 3.1	+ 4 39.9	0.793	0.709	+2.04	+15.4
24 7 31 3	12	8,8	−0 54.20	−6 50.7	17 38 1.89	+ 1 56 31.8	0.782	0.722	+2.05	+14.0
28 7 6 33	13	*d*14,12	−0 16.88	−2 15.6	17 26 5.95	− 2 29 45.1	0.786	0.743	+2.04	+11.4
29 6 30 39	14	6,5	−2 55.86	+3 35.5	17 23 30.1	− 3 24.8	0.751	0.750	+2.05	+11.1
31 6 11 3	15	4,4	−3 1.40	−8 6.0	17 18 26.64	− 5 7 54.0	0.742	0.759	+2.04	+10.4
Nov. 1 6 9 42	16	8,8	+1 13.08	−7 2.3	17 15 59.89	− 5 55 27.5	0.750	0.762	+2.02	+ 9.6

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	3 4 57.70	+39 14 22.9	Lund, A.G. 1634	9	18 7 41.99	+13 46 1.2	Leipzig I, A.G. 6476
2	0 48 25.88	+55 12 56.7	Camb., (U.S.) A.G. 400	10	17 48 27.98	+ 6 7 15.6	Leipzig II, A.G. 8141
3	0 21 17.52	+56 5 54.8	Hels.-Gotha, A.G. 322	11	17 45 29.9	+ 4 41.3	DM. +4°3523
4	23 58 49.83	+56 49 56.6	Hels.-Gotha, A.G.14633	12	17 38 54.02	+ 2 3 7.7	Albany, A.G. 5893
5	22 58 46.93	+56 47 23.2	Hels.-Gotha, A.G.13683	13	17 26 20.78	− 2 27 41.2	Paris III, 22273
6	20 7 16.68	+44 19 38.0	Bonn, A.G. 13855	14	17 26 23.9	− 3 28.6	DM. −3°4120
7	19 44 0.07	+40 45 3.0	Bonn, A.G. 13394	15	17 21 25.97	− 4 59 59.0	Paris III, 22116
8	18 29 52.74	+21 7 13.8	Berlin B, A.G. 6540	16	17 14 44.76	− 5 48 35.4	Paris III, 21919

NOTES. Those comparisons marked *d* were taken directly by micrometrical measurements. — Corrections for refraction have been made Oct. 7. Seeing 2. Images fuzzy. — Oct. 17. Seeing 1. Comet very faint. Hazy and full moon. — Oct. 21. Tail extends across 5-inch finder. — Oct. 28. Seeing 3. Slightly tremulous.

*From Supplement to Nos. 531-532.

OBSERVATIONS OF COMET *b* 1902 (*PERRINE*),*

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY.

[Communicated by Captain COLBY M. CHESTER, U.S.N., Superintendent.]

These observations give a correction of $+5^s$ in α and $-1'.2$ in δ to the ephemeris of ELIS STRÖMGREN in A.N. 3821.

1903 Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 5 11 0 8	1	23 , 5	+1 37.18	−5 24.2	8 3 49.73	−33 23 32.7	*n*7.890	0.922	+2.26	−17.7
" 5 11 16 47	1	20 , 4	+1 32.90	−4 32.7	8 3 45.45	−33 22 41.2	8.477	0.922	+2.26	−17.7

Mean Place of Comparison-Star for the beginning of the year.

*	α	δ	Authority
	h m s	° ′ ″	
1	8 2 10.29	−33 17 50.8	C.G.C. 10728

$\Delta\alpha$ determined by transits. First observation by FREDERICK, second by DINWIDDIE.

*From Supplement to Nos. 531-532.

THE MISSING *DURCHMUSTERUNG* STAR +44°3585,

By ZACCHEUS DANIEL.

On 1900 September 14, while examining the region near *α Cygni* with the 10-inch telescope I found the star DM. +44°3585, 9ᴹ.4, missing from the place given on the chart. The region was carefully examined on 1900 Oct. 18, Nov. 14, Dec. 19; 1901 July 20; 1902 Aug. 8, Sept. 10; and 1903 Feb. 5, but no star brighter than the eleventh or twelfth magnitude was ever seen within 10′ of the place. In addition to the dates given above, I have looked for the star with the same result, probably at least fifty times without recording the observation.

The *Bonner Sternverzeichniss* contains the following position of the missing star:

$$\alpha = 20^h\,42^m\,55^s.3\quad;\quad \delta = +44^\circ\,29'.3\ (1855).$$

This is $1^m\,52^s$ following and 9′ north of 7456 *RR Cygni*. There are two stars of about the twelfth magnitude near this position and one of them may be the star in question, but that can be determined only by exact measurements, which I have not the facilities for making at present. However, no variation in the brightness of either star has been noticed.

Bucknell University, Lewisburg, Penn., 1903 *February* 7.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Captain C. M. Chester, U.S.N., Superintendent.]

1901 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				Eurynome.						
			m s	′ ″	h m s	° ′ ″			s	″
Aug. 20 10 44 52	1	10 , 5	+0 16.46	+ 6 33.5	22 9 14.30	−3 59 17.8	n9.240	0.773	+4.10	+25.9
22 11 46 50	2	11 , 6t	+4 19.23	−10 56.7	22 7 26.71	−4 11 26.1	n8.541	0.777	+4.12	+26.0
Sept. 3 10 45 5	3	12 , 12	−0 43.27	+ 6 55.6	21 57 2.60	−5 30 3.5	n8.636	0.787	+4.17	+26.5
4 10 28 39	4	12 , 12	+0 6.12	− 1 41.5	21 56 13.53	−5 36 53.9	n8.824	0.788	+4.17	+26.6
7 9 20 7	5	12 , 12	−0 19.37	− 0 45.4	21 53 51.72	−5 57 20.0	n9.235	0.787	+4.17	+26.7
9 10 21 38	6	28 , 7t	+1 20.26	+ 6 7.5	21 52 18.33	−6 11 22.9	n8.527	0.793	+4.17	+26.6
				Parthenope.						
Oct. 16 9 43 25	7	34 , 7t	−1 30.11	− 3 31.3	2 41 21.77	+7 27 27.8	n9.545	0.688	+4.51	+18.1

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	22 8 53.74	−4 6 17.2	U.S.N. Obs'y Tran. Cir. Pos.	5	21 54 6.92	−5 57 1.3	⅓(Mun. I 29974 + Rümk. 9700 + W.B. XXI, 1209)
2	22 3 3.36	−4 0 55.4	U.S.N. Obs'y Tran. Cir. Pos.				
3	21 57 41.70	−5 37 25.6	⅓(Mu. I 30092 + Mu. II 12203)	6	21 50 53.90	−6 17 57.0	½(Mun. I 29848 + Schj. 8922)
4	21 56 3.24	−5 35 39.0	U.S.N. Obs'y Tran. Cir. Pos.	7	2 42 47.37	+7 30 41.0	Leipzig II, A.G. 1033

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METHOD OF FORMING THE RIGHT-ASCENSIONS OF THE CATALOGUE OF 627 PRINCIPAL STANDARD STARS (*A.J.* 532–533),

By LEWIS BOSS.

In developing a new system of right-ascensions of the principal stars we must examine the testimony of observation from three practically distinct points of view. First, we must ascertain the relation of the stars to the moving equinox; secondly, we must find the errors of observation depending for their argument on the right-ascension of the star observed which, combined with the correction for equinox, we designate as $\Delta\alpha_a$; and thirdly, we must evaluate the errors of observation having declination as the argument, $\Delta\alpha_\delta$, which are due to the failure of the several transits to describe an accurate hour-circle on the sky.

In a subsequent chapter will be exhibited the observed values of $\Delta\alpha_m$, the personal equation dependent on magnitude of the star observed, for each catalogue of observation. From the deductions drawn therefrom it will appear that, in a further approximation toward a better system, upon the basis of the catalogue under consideration, it will be feasible and necessary to take into account this fourth factor in the problem; but in establishing the present system this element was not considered.

The rigorous treatment of our problem implies the solution of equations containing a multitude of unknown quantities; but practically it cannot be handled in this form. The solution must be reached through a series of approximations taking into consideration the principal elements separately and in succession. In the present attempt the neglect of terms in $\Delta\alpha_m$ has doubtless influenced the results for $\Delta\alpha_a$ and $\Delta\alpha_\delta$ in a sensible degree, yet this effect must always have been minute. With the present catalogue as a basis the convergence toward the true result ought to be both rapid and satisfactory.

Corrections Having the Form, $\Delta\alpha_a$.

It will be more convenient to consider first the motion of the equinox, together with periodic terms having the argument right-ascension. The combination of these terms we shall call $\Delta\alpha_a$.

For this purpose I have taken as my starting point the right-ascensions of 97 stars contained in the list of "Time Stars" for which Newcomb gives five-year ephemerides in his work on *Standard Clock and Zodiacal Stars* (pp.300–314). In general the choice was confined to stars situated between $+30°$ and $-20°$ of declination; but *α Scorpii*, *α Lyrae*, and *α Pisc. Austr.*, were included. Of the stars between the chosen limits of declination, *α Can. Maj.* and *α Can. min.* were omitted on account of periodicity in proper motion; 11 *Orionis*, *ν Orionis*, *b Ophiuchi*, 1 *Aquilae*, *τ Aquilae*, 1 *Pegasi*, *π Aquarii*, and *θ Piscium* were omitted as insufficiently observed for the purpose required. The choice of this preliminary system, N_1, presents manifest advantages for testing questions relating to $\Delta\alpha_a$, since both points involved have been investigated by Professor Newcomb, and with a degree of success which leaves little opportunity for improvement, even with the aid of the additional accumulation of observations which has appeared since these investigations were made.

Position and Motion of the Equinox.

Let us first examine the question as to what correction may be required for the preliminary right-ascensions, N_1, on account of error in the adopted equinox. This equinox was established in Newcomb's work on the *Equatorial Fundamental Stars*, and includes the testimony of observations down to about 1870.

Subsequently, in the course of his planetary investigations, Professor Newcomb had occasion to investigate anew the position and motion of the equinox. From observations of the sun he found as the correction to the right-ascensions of N_1 (*Astronomical Constants*, pp. 88 and 96),

$$+0''.07 \;\; -0''.34\, T$$

wherein T is the interval in centuries from 1850. He also derives similar corrections through the observations of *Mercury*, *Venus*, and *Mars*, and combining these with the result from solar observations, in a manner not readily presented in brief, obtains as the correction for N_1,

$$-0''.48 \;\; +0''.30\, T$$

Later, (*A.J.*, XV, p. 188) Professor NEWCOMB gives as revised values of the secular terms, $-0''.37\,T$ and $\mp 0''.40\,T$ as the results, respectively, for observations of the sun alone and of these in combination with those of the planets.

The question whether the result from observations of the planets should have any weight is one which cannot be decided off-hand; but it may fairly be doubted whether the relative weight for the planetary observations should be so great as that which Professor NEWCOMB has assigned. For the further purposes of this investigation I prefer to employ a result from solar observations alone.

In looking over Professor NEWCOMB's tables from which his $\alpha+dl''$ and dl'' are computed (pp. 22 and 30, *Astronomical Constants*) it will be seen that considerable weight has been attributed to the older observations to which I have attached no weight in forming the present system of right-ascensions. In discussions of this kind for the determination of fundamental quantities it seems to me that there are few meridian observations of a date earlier than 1820 which deserve any consideration whatever. In the discussion of secular terms the apparent advantage of employing these older observations is very tempting; but it can be shown that these supposed advantages are really illusory, since the unknown systematic errors of these older observations are liable to be so great in comparison with those of the more precise modern observations as to overbalance the advantage due to the greater interval. A strong illustration of this fact will be found in the comparison of the declinations of AUWERS's reduction of BRADLEY's observations with those of the present Standard Catalogue.

In the present instance, however, Professor NEWCOMB's employment of the older observations has not materially influenced the result. I have taken from his tables the observed values of $c\,(=\alpha+dl'')$ and dl'' which result from observations of mean date, 1820 and later, except that in the case of the Paris results I have not employed the observations made previous to the mean for 1837. Adopting NEWCOMB's weights, I have solved the equations for c and dl'' with the following result for the correction of the right-ascension of N_1:

$$+0''.04 \quad -0''.28\,T$$

in which T is the interval in centuries from 1850. This is almost precisely the same as that already cited which NEWCOMB obtains from all the observations of the sun.

The latest observations included in Professor NEWCOMB's results are those of Greenwich for 1892. In *A.J.*, XXI, 141-2, Professor NEWCOMB discusses the Washington observations of the sun, 1894–1899, and finds for the correction of the right-ascension of N_1:

$$+0^s.009$$

He also shows that a combination of the equinox determitions at Greenwich, 1893–1899, results in a similar correction to N_1 of

$$+0^s.005$$

From NEWCOMB's discussion of the solar observations to 1892 already cited (see *Astronomical Constants*) the correction of N_1 for 1896 would be

$$-0^s.015$$

As previously shown this is confirmed by the discussion which excludes observations earlier than 1820. But, since the Washington observations would give a correction of $-0^s.053$, in the mean to Professor NEWCOMB's ephemeris right-ascensions of the sun, he finally expresses the opinion that, at the date in question, the system, N_1, appears to require a correction of $+0^s.02$ or $+0^s.03$.

Without prolonging the examination further I think it may safely be stated that, so far as the existing determinations of the position and motion of the equinox from observations of the sun throw light on the question, it is uncertain whether the position of the equinox upon which N_1 is founded requires a positive or a negative correction at any epoch. Because I am actually in doubt upon this point I have not ventured upon any correction whatever. I have adopted the equinox of N_1, preferring to await a new influx of determinations before attempting to correct it. It is earnestly to be hoped that these will soon be offered not from Greenwich only, but also from many other observatories situated in latitudes more favorable for the purpose.

Examination of N_1 as to the Periodic Part of $\Delta\alpha_\delta$.

The system, N_1, may be regarded as having been established after an exhaustive and trustworthy discussion of systematic errors depending on the right-ascension. This may be regarded as the special feature of NEWCOMB's work on the equatorial fundamental stars. He called the attention of observers to the special means whereby such errors could be eliminated from their observations; and his precepts have been followed with effect at the Greenwich Observatory and elsewhere. At the same time very little has been contributed during the last thirty years which can be utilized as an independent test of N_1 as to periodic errors in $\Delta\alpha_\delta$. I think the following short table comprises all of these contributions. The numbers are in the sense of corrections to N_1, and result from final comparison with my catalogue of 627 standard stars, which virtually represents it. The comparison includes all stars between $+30°$ and $-22°$ of declination.

Observatory	Corrections to N_1	
Pulkowa, 1865	$-0^s.0022 \sin\alpha$	$+0^s.0033 \cos\alpha$
Greenwich, 1882	$+0.0038$	-0.0026
Pulkowa, 1884	-0.0028	$+0.0056$
Greenwich, 1894	-0.0001	-0.0001

As will be seen the corrections are practically insignificant. If we combine the three later determinations, giving Greenwich 1882 half weight, because during that period the process of eliminating an initial periodic error in the right-ascensions was in progress, we have as the correction to N_1:

$$1888, \qquad -0^{s}.0004 \sin\alpha \; +0^{s}.0017 \cos\alpha$$

A very slight alteration in the method of comparison, either as to the limits of declination employed, or as to the representative of N_1, might alter these numbers either way by more than their entire amount.

It might also be remarked that in Volume II (p. (vii)) of the Strassburg Annals there is a quasi determination of the periodic correction, $\Delta\alpha_\alpha$, required for the *Fundamental-Katalog* of Dr. Auwers. From this I find,

$$+0^{s}.0135 \sin\alpha \; +0^{s}.0076 \cos\alpha$$

as the corresponding correction of N_1. But under the circumstances set forth in the discussion cited this cannot be regarded as entitled to very great weight.

For the foregoing reasons I have considered the Catalogue of 627 Standard Stars to require no present revision as to errors of the form $\Delta\alpha_\alpha$.

It seems rather surprising that so little attention by recent observers has been given to the elimination of this form of error in their observations. Whenever, for any reason, observations of the stars are made in the daytime, it is comparatively easy to arrange the night observations in such a way as to render this elimination possible. There has been no period in the history of astronomy when such strenuous attention has been given, as now, to the perfection of clocks and of their installation. Elaborate meridian marks have been established. Thus the means exist for the most rigorous determination of the periodic part of the error, $\Delta\alpha_\alpha$, in the adopted right-ascensions of the observing lists; and yet they are seldom employed for this purpose. In order to appreciate the situation, one has only to cite the observations of Bessel, Struve, and other observers near their time, who used ordinary clocks exposed to the extreme variations of temperature in the observing rooms, and who were yet able to produce systems of fundamental right-ascension almost wholly free from error of this form.

Table I exhibits the final results for determination of $\Delta\alpha_\alpha$, derived from a comparison of each catalogue of observation with the right-ascensions of the Catalogue of 627 Standard Stars between the limits of $+30°$ and $-22°$ of declination, — the constant part, at the same time, being determined from the limits, $+37°.5$ to $-22°$.

TABLE I.

List of Final Determinations of $\Delta\alpha_\alpha$ for the Principal Catalogues.

Greenw. (A-B),	1755	$-0^{s}.079$	$-0^{s}.010 \sin\alpha$	$+0^{s}.004 \cos\alpha$
Königsb. (North),	1820	−0.034	0.000	0.000
Königsb. (Zod.),	1823	+0.025	+0.014	−0.009
Dorpat,	1824	−0.020	+0.011	+0.004
Cape,	1830	+0.013	+0.032	+0.014
Abo,	1829	+0.015	+0.003	−0.001
Greenwich,	1830	−0.062	−0.009	+0.001
St. Helena,	1832	−0.046	0.000	−0.019
Cape,	1833	+0.021	−0.003	−0.014
Cambridge,	1831	−0.024	−0.004	+0.005
Madras (D),	1835	−0.052	+0.017	−0.010
Cape,	1837	−0.004	−0.011	+0.002
Greenwich,	1838	+0.082	−0.028	+0.012
Greenwich,	1844	+0.023	−0.018	+0.017
Radcliffe,	1845	−0.023	−0.024	+0.046
Paris,	1845	+0.025	−0.008	+0.002
Pulkowa,	1845	+0.023	0.000	−0.004
Santiago,	1851	+0.004	−0.008	+0.008
Greenwich,	1851	0.000	−0.015	+0.011
Washington,	1856	+0.015	−0.024	+0.017
Greenwich,	1857	+0.019	−0.009	+0.016
Radcliffe,	1857	+0.032	−0.017	+0.002
Cape,	1859	+0.024	−0.012	+0.010
Paris,	1860	+0.038	−0.018	+0.012
Melbourne,	1862	+0.047	−0.030	+0.015
Greenwich,	1864	+0.032	−0.009	+0.009
Cape,	1865	−0.012	0.000	−0.001
Brussels,	1865	$+0^{s}.050$	$-0^{s}.037 \sin\alpha$	$+0^{s}.018 \cos\alpha$
Harvard,	1865	−0.028	−0.017	−0.004
Pulkowa,	1865	−0.005	+0.002	−0.003
Melbourne,	1867	+0.035	−0.033	+0.015
Washington,	1871	−0.001	−0.003	−0.002
Greenwich,	1872	+0.029	−0.007	+0.010
Madras,	1875	+0.047	+0.004	−0.005
Harvard,	1875	−0.006	+0.001	−0.007
Pulkowa,	1876	+0.002	+0.005	−0.005
Paris,	1876	+0.040	−0.024	+0.018
Cape,	1876	+0.037	−0.006	+0.009
Cordoba,	1877	0.000	−0.032	+0.022
Melbourne,	1877	+0.034	−0.010	+0.021
Greenwich,	1882	+0.035	−0.004	+0.003
Cape,	1883	+0.019	−0.011	+0.004
Pulkowa,	1884	+0.020	+0.003	−0.006
Radcliffe,	1885	+0.017	−0.002	+0.008
Strassburg,	1886	+0.016	−0.002	−0.001
Cape,	1889	+0.022	−0.005	+0.002
Madison,	1890	+0.008	−0.002	0.000
Berlin,	1890	+0.020	−0.004	+0.001
Lisbon,	1890	+0.016	−0.001	−0.002
Greenwich,	1894	+0.045	0.000	0.000
Mt. Hamilton,	1895	+0.028	−0.004	−0.004
Berlin,	1895	+0.020	−0.002	+0.002
Albany,	1898	0.000	0.000	0.000

In all cases I have taken the formula, $a \sin\alpha + b \cos\alpha$, as the expression to be adopted for the periodic part of $\Delta\alpha_\alpha$. This expression is naturally suggested by the association of the observations with the annual cycle of the seasons, and with the diurnal range of temperature. It is doubtless true that in particular instances the real corrections may deviate quite sensibly from this form, but the result of my experience is that this formula usually represents the observed residuals for $\Delta\alpha_\alpha$ in a satisfactory manner. But, even were this formula more deficient than it actually is in the representation of the errors of observation, a strong reason for its adoption would still exist in the requirements of investigations based upon a study of observed right-ascensions. The formulas for determination of precession and solar motion, for instance, contain terms of the form, $a \sin\alpha + b \cos\alpha$; so that in preparing observations for such an investigation it would be more desirable to remove all vestiges of an error of this form than it would be to have even a more exact removal of the errors of observation on the whole associated with a less perfect removal of the periodic term. In fact, curve-drawing in the representation of residual phenomena should be looked upon as an unsatisfactory process to be avoided whenever practicable, and when circumstances warrant the use of a formula.

The original values of $\Delta\alpha_\alpha$, which I obtained from comparison with Newcomb's *Standard and Zodiacal Stars* in the manner already described, do not differ very materially from those of Table I in the case of any important catalogue. Following are the values of $\Delta\alpha_\alpha$, employed throughout the computations for the catalogue, for which the differences from the corresponding values from Table I are much larger than usual.

Dorpat,	1824	$-0^s.020$	$+0^s.010 \sin\alpha$	$+0^s.008 \cos\alpha$
Cape,	1837	-0.004	-0.015	-0.002
Greenwich,	1838	$+0.076$	-0.033	$+0.007$
Greenwich,	1857	$+0.019$	-0.013	$+0.017$
Melbourne,	1862	$+0.047$	-0.024	$+0.014$
Harvard,	1865	-0.024	-0.010	-0.008
Melbourne,	1867	$+0.035$	-0.034	$+0.010$
Cordoba,	1877	$+0.002$	-0.028	$+0.018$
Radcliffe,	1885	$+0.017$	-0.008	$+0.008$

Investigation of Errors having the Form, $\Delta\alpha_\delta$.

Having ascertained that the system, N_1, in the light of all the observations, does not require a decided correction that is discoverable either as to the adopted equinox or as to the periodic part of $\Delta\alpha_\alpha$, we proceed next to construct a system which shall be as far as possible, in like manner and degree, free from errors depending on the declination. In the attempt to do this it will be advisable to recur to the observations themselves, and to build up from them an independent system. If now we could find a list of as many as 200 stars, well distributed in declination, of which the right-ascension of each star had been determined in each valuable authority with approximately the same weight for each star, our task would be comparatively simple. We could first settle upon the standard weight in the fundamental sense to which each catalogue of observation is entitled, and by use of these weights, after correction of each catalogue for $\Delta\alpha_\alpha$, compute the right-ascension of each star from a combination of all the authorities entitled to weight in the fundamental sense. We should at once have our fundamental catalogue, from comparison with which the systematic errors of each of the principal catalogues of observation could be determined with practical finality. A slight subsequent improvement of the right-ascensions of each individual star might then be obtained through the employment of the systematic corrections already ascertained, in a new solution for each star in which the weights would be assigned according to the value of each catalogue in the purely differential sense; so that many valuable series of observations would then be introduced which make no pretensions to value as independent determinations.

In practice the case is far different. I have been unable to find more than 46 stars for which the ideal conditions are even approximately fulfilled; and this number is too few for anything better than a rough preliminary determination of the systematic corrections, $\Delta\alpha_\delta$. Perhaps the necessary method of procedure will be more easily understood through a brief exposition of the successive steps actually adopted in the present investigation.

In arriving at my published right-ascensions three successive approximations were employed. It will not be necessary to exhibit these in detail, since the only object in doing this would be to afford the means of judging what degree of confidence attaches to the final result, — an object which is sought otherwise through a special discussion further on. But some account of the preliminary steps may have an interest.

First, 45 stars (and *Polaris*) were selected (see Table II) on the ground that they are the stars of which the positions are most effectively determined in the greatest number of important catalogues. All the catalogues were then corrected for the values of $\Delta\alpha_\alpha$ determined in the manner already described. The positions may now be supposed to be affected only by systematic errors of the form, $\Delta\alpha_\delta$.

The next process is to obtain a series of values of α and μ for these 45 stars which shall be as nearly as possible homogeneous. At this stage it is not important that the systematic errors should be zero; though it is very desirable that the casual errors of computed mean α and μ should be as small as possible, and that, whatever the systematic errors of these computed values of right-ascension may be, they should be approximately the same within any one comparatively narrow zone of declination. Therefore in selecting the catalogues to be employed in this preliminary

step, precision in the differential sense is of more importance than fundamental character. Hence the average weight assigned to each catalogue for service in this preliminary operation was virtually based upon its precision in the differential sense. This first essay is not for the purpose of computing a system of right-ascensions free from systematic error, but to provide a system which can be corrected easily and with accuracy through the means which actually exist.

The 45 values of α and μ obtained in this way must next be freed from systematic error of the form, $\Delta\alpha_s$. Each catalogue which is entitled to any weight for the determination of $\Delta\alpha_s$ furnishes a correction of each of the 45 computed right-ascensions that are common to it. Within any comparatively narrow zone we may suppose that these corrections will be approximately the same, except for the casual errors of observation; so that, if we combine four stars in a single zone, and our catalogue of observation contains only two of these stars, the mean of the two corrections will be systematically the same as would have been the mean of all four had the catalogue contained all of them, — the only difference being that the casual error may be larger, and therefore less likely to be compensated by those of other catalogues in the same zone. This process is entirely analogous to that which is followed in the reduction of broken transits.

Thus for each of the nine zones (see Table II) into which the 45 right-ascensions were divided, a "zone-correction" was formed from the corrections of the preliminary right-ascension given by each catalogue supposed to have weight in the determination of $\Delta\alpha_s$. As a matter of fact there were few catalogues which did not contain nearly all the stars in each of the zones. The deficiencies were greatest north of +30° of declination.

Taking now one of the zones as a representative of the mean of the stars in that zone we find that we have a series of corrections to that representative given by each of the selected catalogues, arranged in chronological order. The list of selected catalogues, together with the weights assigned to each, is exhibited in the first three columns of Table III; but Lisbon 1890 must be excepted, since it was not published until after the final computations for right-ascension had been completed. The mean date of each catalogue as given in the second column was estimated for the purposes of this computation, and the adopted weights appear in the third column. In assigning these weights various restrictions will suggest themselves to the investigator. In the first place the casual errors of observation may be so great, or the number of observations may be relatively so small, as to vitiate the value that a catalogue might otherwise have for this purpose. Cape 1830 (Fallows) falls in this category. Cambridge 1831 (Airy) is very near the boundary of rejected catalogues for this reason.

The contributions of an observatory during any given period must be judged as a whole, — obviously so, when the same transit is concerned in the production of distinct series of right-ascensions in comparatively close succession. Thus, each of the contributions from the Greenwich Transit Circle has received a weight less than that to which it would have been entitled had it stood as the sole representative of the observations of that instrument. On the other hand, when the extreme interval in a long series of productions, like those from the Greenwich Transit Circle, is so great, the total weight assigned may be much greater than that which could possibly be due to a single representative.

Another question is presented in the criterion of independence relative to terms in $\Delta\alpha_s$ that should be exacted. The character of the meridian, or hour-circle, described by the transit, as fixed by the definitive reduction of the observations, depends essentially upon three elements:

1. *Upon the character of the determinations of collimation.* Frequent reversal of the transit eliminates this question in a material degree. The Lisbon observations, where the transit was reversed in the observation of each star, illustrates this in a remarkable manner. For the non-reversible transit a systematic error in this respect amounting to Δc introduces a systematic error in the final result equal to

$$\Delta c\left(\tan\frac{P}{2}-\tan\frac{P'}{2}\right)$$

in which P is the polar distance of the star observed, and P' is the effective mean polar distance of the clock stars.

2. *Upon the character of the pivots.* For the most part we must rely upon our experience and judgement as to the quality of work produced by the respective makers of instruments. In a few instances the pivots have been investigated in a manner which inspires confidence in the results. On the other hand, my impression is that the pivots now produced by the best makers are so perfect that the errors originating through irregularities in them are not very important in relation to those from other sources. The error in right-ascension introduced by a relative ellipticity of pivots has the form, $e\sin(2\delta+\psi)\sec\delta$, in which ψ is an auxiliary angle to be determined. This error is most to be feared in non-reversible transits; but even in the case of a reversible transit, where either of the axes of the relative ellipse happens to be nearly parallel to the line of collimation, this form of error is not eliminated. In many instances where the systematic correction is large and reaches a maximum at about 45° of declination, while it is quite small near the pole, I suspect that the instrumental error may be due to relative ellipticity of pivots. The right-ascensions of Dorpat 24, Greenwich 30, Harvard 65, Brussels 65 and Albany 98 may be open to suspicion of this kind.

3. *Upon the determination of the polar deviation, n.* To secure independence in this respect the value of n should be ascertained through successive transits of close circumpolar stars. If this criterion were to be insisted on it would considerably restrict the choice of catalogues to be employed in determination of $\Delta\alpha_\delta$. This restricted choice I have indicated through the weights assigned in the fourth column of Table III. But we may be permitted to look upon the right-ascensions of the close circumpolar stars, especially of such stars as α and δ *Ursae minoris*, as something approximating the nature of astronomical constants. Admission into the problem of determination of $\Delta\alpha_\delta$ of certain catalogues of right-ascensions which rest on predicted right-ascensions of the close circumpolar stars would mean on the one hand that we are virtually assigning a larger weight than nominally appears to those catalogues upon which the predicted positions depend; and on the other hand that we regard the errors, $\Delta\alpha_\delta$, to be feared in the region intermediate between the pole and the equator as likely to be notably larger than those which we find in investigations of the polar zone. This is really the case, and seems to justify the decision to assign some weight in the determination of $\Delta\alpha_\delta$ to certain series of right-ascension in the reduction of which predicted places of the close circumpolars have been employed. But these catalogues were not used in determination of $\Delta\alpha_\delta$ for the polar zone. (See Table III.)

Added to these three conspicuous elements to be considered in determining $\Delta\alpha_\delta$ are others of a nature less certain and more obscure. These in some instances may have exerted a sensible influence upon the errors of the catalogues. There are systematic errors arising from faulty illumination of the transit threads. There is also the error at the zenith which may exist relative to the apparent direction of transit. This, in the case of Romberg at Pulkowa, may have been quite sensible, and there are slight traces of it in other instances. In general, however, the evidence is that this form of error is little to be feared. Associated with this are errors in the line of collimation which may arise from looseness, or weakness, in the structure of the transit, — especially in the fastening of the objective and ocular. I suspect that the right-ascensions of the Washington Transit Circle (1865–1890) may have suffered somewhat from this cause. The respective zenith-points of the principal observatories in the northern hemisphere range over more than 20° of declination; and, therefore, the results for $\Delta\alpha_\delta$ from the mass of observations afford an efficient test for any one of them in respect to these two sources of errors. Personal equations dependent on declination of the star observed may also exist, but their effects would probably be merged to a considerable extent in the determination of the polar deviation of the transit.

Resuming now the zone-corrections for the amendment of the preliminary right-ascensions of 45 stars we may form equations in each zone from the corrections given by the respective catalogues arranged in chronological order, and weighted in the manner already described. Let $\Delta\alpha_0$ be the common correction to the preliminary right-ascensions for 1875 of a given zone, and $\Delta\mu_0$, the common correction for the proper motions. We then form equations of condition in the usual manner, and obtain from the resulting normal equations adopted values of these quantities. Then, correcting each preliminary right-ascension in a given zone by the values of $\Delta\alpha_0$ and $\Delta\mu_0$ for that zone we arrive at right-ascensions for each star which, provisionally, may be regarded as free from systematic error of the form, $\Delta\alpha_\delta$, so far as this result can be obtained from the testimony under consideration.

As already pointed out, a slight inconsistency attaches to these solutions in the fact that, for a zone here and there and for a few catalogues, the normal weight was reduced. It was necessary to take this course in some instances on account of the casual errors which might arise from the unusual weakness of the catalogues in question for certain of the zones, — especially for those north of +30°. Practically, however, the results of this inconsistency are not important; and in many instances they are remedied in a later approximation.

Furthermore, it should be explained that in the case of Romberg's Pulkowa catalogue for 1875, of Battermann's Berlin catalogue for 1895, and of Greenwich 1890, the catalogue right-ascensions were modified in this part of the work. Romberg's catalogue right-ascensions were first corrected in a way to remove the systematic corrections which he applied (*Int.*, p. 8) in order to reduce them to systematic conformity with Pulkowa, 1865. In this way we may suppose that we have obtained the right-ascensions as they would have been given by the meridian circle itself; and I have assumed that these would be valuable as an independent factor in the determination of $\Delta\alpha_\delta$. A similar course was followed with the Berlin catalogue for 1895 by the use of systematic corrections printed in the introduction (p. 14) which exhibit the deviation of the instrumental meridian from that of the *A.G.C.* In the case of Greenwich 1890 I have applied the correction for systematic error of collimation in the form recommended in the introduction (p. 5) of the catalogue.

But in the tables of systematic correction, $\Delta\alpha_\delta$, hereafter to be given, the numbers are applicable to the catalogue positions.

The results obtained for the 45 stars through the combination of operations already described are exhibited in Table II. The spaces indicate the division into zones. The corrections, $\Delta\alpha$ and $\Delta\mu$, when added to the right-ascensions of the 45 stars, produce those of the final cata-

logue after all the approximations. It will be seen in a general way that this first approximation represents in the systematic sense the final result with a very considerable degree of accuracy. In no case is the correction for an individual star above $0^s.01 \sec\delta$, and usually it is much less. Systematically the results of this first approximation are practically the same as those of the final catalogue. This, of course, could not have been foreseen at the time; but if the case had been otherwise the same final result would have been reached.

TABLE II.

Right-Ascensions, B′, of 46 Primary Standard Stars Determined in the First Approximation, Defined through the Corrections to them Given by the Right-Ascensions, B, of the Final Catalogue for the Epoch 1875.

		B — B′				B — B′	
	δ	$\Delta\alpha$	$\Delta\mu$		δ	$\Delta\alpha$	$\Delta\mu$
		.001	.0001			.001	.0001
	°	s	s		°	s	s
α Urs. min.	+89	−25	−13	*α Androm.*	+28	+2	−1
				β Tauri	28	0	0
ζ Urs. min.	+78	+ 2	− 1	*β Geminor.*	28	−3	0
γ Cephei	76	+14	+ 9	*α Cor. Bor.*	27	−1	−1
β Urs. min.	75	+ 6	+ 6	*α Arietis*	23	+1	+1
β Cephei	70	+12	+ 6				
				α Bootis	+20	−3	−1
α Urs. Maj.	+62	− 5	− 4	*α Tauri*	16	+1	0
α Cephei.	62	−21	− 8	*β Leonis*	15	−2	−1
α Cassiop.	56	− 2	− 2	*α Herculis*	15	0	−1
γ Urs. Maj.	54	+ 7	− 3	*α Pegasi*	15	+1	+1
				γ Pegasi	14	−2	0
θ Urs. Maj.	+52	− 2	− 1	*α Leonis*	13	−2	0
β Draconis	52	+12	+ 5	*α Ophiuchi*	13	−2	−2
γ Draconis	52	− 6	− 4				
η Urs. Maj.	50	+ 2	− 2	*γ Aquilae*	+10	0	0
				α Aquilae	9	−1	0
				α Orionis	7	+4	0
α Persei	+49	− 1	+ 1	*α Serpentis*	7	0	0
ι Urs. Maj.	49	− 7	− 1	*β Aquilae*	6	−2	0
α Aurigae	46	− 7	− 2	*α Ceti*	+4	−4	−1
α Cygni	45	+ 3	+ 3	*α Aquarii*	−1	−2	0
				β Orionis	−8	+1	+1
α Can. Ven.	+39	+ 4	+ 2	*α Hydrae*	8	0	0
α Lyrae	39	− 4	− 2	*α Virginis*	10	0	0
β Androm.	35	− 1	+ 1	*α Capricorni*	13	−1	+1
β Lyrae	33	+ 1	− 1	*α Librae*	15	−1	−1

With this preliminary Standard Catalogue of 45 Stars between +79° and −15° of declination the first preliminary corrections of the principal catalogues were determined, embracing those enumerated in Table III, together with others valuable in the differential point of view. In drawing the curves of correction minor peculiarities were not regarded at this stage; but the most painstaking care was exercised to secure practical equality of positive and negative deviations from the general trend of the curves of correction on the part of individual residuals. This is really one of the most critical operations in the entire process, and one upon which the actual gain of precision in successive approximations very much depends; since a fairly good approximation to a perfect standard catalogue already obtained may subsequently be lost through an unlucky combination of errors in the adopted curves of correction.

The corrections were adopted at this stage for stars between +80° and −20° of declination. These preliminary systematic corrections were now employed in determining standard right-ascensions for 113 additional stars for which the weight of determination in view of the collected material seemed to be the greatest. The adopted weights in the star-solutions were now based upon the supposed value of the various catalogues in the differential sense. We now have a standard catalogue of 158 stars to which may be added 70 others within the limits, −20° to −40°, which were taken from the best determined stars of my paper on 179 *Southern Standard Stars* (*A.J.* 450), modified by systematic corrections exhibited in my paper published in the *Astronomical Journal*, No. 499, since I considered those to have reached a grade of approximation in the progress toward a true normal system comparable with that now attained for the 158 northern standard stars. Preliminary to plotting the revised zone-corrections for curve-drawing all of them were multiplied by $\cos\delta$, a process which is absolutely necessary to the attainment of real precision in the higher declinations. In drawing these curves more attention was given to subsidiary inflections in the trend of corrections. In regard to minor deviations of the curves from a bold general sweep they are abundantly justified by experience. There is logical reason for such deviations in the probable minute deformations of the pivots of transit instruments combined with other sources of error. In many instances where, on account of the comparative poverty of material in this approximation, I have resisted the inclination to follow an indicated deflection from a free sweeping curve, on a later approximation, with much more material, I have been compelled to recognize it. In general, throughout, I have not pushed the maxima and minima of these curves to the extremes indicated by the observations; but at such points I have usually left an interval from the zone-correction (mean for 5°) equal to its probable error, and in some cases to as much as three or four times the probable error. The weights of the group-correction being known it is possible with care to arrange the residual errors left by the curve so that to some extent they shall follow the law of distribution indicated by the theory of probable error.

TABLE III.

Observed Corrections, $-\Delta\alpha_s \cos\delta$, to the System of Right-Ascensions of 627 Standard Stars, Given by Catalogues of Observation.

Northern Hemisphere.

Decl. of Zone				+80°			+60°			+45°			+30°			+15°			0°		
Catal. and Date	1800+	Weights		**	p	C	**	p	C	**	p	C	**	p	C	**	p	C	**	p	C
Dorpat	15	2	2	23	2	-.003	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
Konigsb.	23	3	3	8	3	+.002	12	3	+.004	18	3	+.016	5	3	-.010	10	3	.000	5	3	+.008
Dorpat	24	3	3	19	3	-.006	29	3	-.018	34	3	-.022	30	3	-.015	21	3	-.004	18	3	+.005
Abo	29	3	–	12	3	-.016	20	3	-.018	32	3	-.011	29	3	+.001	26	3	+.005	33	3	+.001
Greenw.	30	1	1	30	1	-.007	37	1	-.035	49	1	-.018	50	1	-.014	72	1	-.012	67	1	+.007
Cape	33	2	2	. .	. .	. .	. .	. .	. .	4	0	-.066	11	1	-.037	16	2	-.001	18	2	+.016
Camb.	31	1	1	6	0.5	-.011	14	0.5	-.002	8	0.5	-.002	20	1	.000	47	1	-.009	44	1	+.004
Cape	37	2	1	. .	. .	. .	. .	. .	. .	3	0	-.034	16	1	-.016	43	2	+.015	34	2	+.007
Greenw.	38	1	1	18	1	+.031	19	1	+.032	16	1	+.004	26	1	+.007	48	1	+.009	51	1	-.007
Greenw.	44	2	2	22	2	+.015	26	2	+.028	46	2	+.007	32	2	+.011	58	2	-.002	53	2	-.005
Radcl.	45	1	–	38	1	+.009	38	1	-.015	50	1	-.016	19	1	-.023	50	1	-.018	50	1	+.024
Paris	45	3	–	35	0	-.009	32	3	-.014	47	3	-.003	56	3	+.006	71	3	-.005	75	3	.000
Pulkowa	45	8	8	29	8	-.005	36	8	-.001	51	8	+.007	55	8	+.005	61	8	+.001	50	8	-.005
Greenw.	51	1	–	24	1	+.021	19	1	+.014	35	1	-.002	52	1	+.011	70	1	+.001	66	1	-.004
Wash.	56	2	–	33	0	-.001	30	2	+.016	41	2	-.002	47	2	-.008	73	2	-.001	71	2	+.002
Greenw.	57	2	2	26	2	+.007	21	2	+.025	33	2	+.026	45	2	+.022	68	2	+.007	67	2	-.012
Radcl.	57	1	–	28	1	+.009	26	1	+.029	38	1	-.007	49	1	+.004	69	1	-.010	68	1	-.011
Cape	59	1	1	. .	. .	. .	. .	. .	. .	5	0	-.033	25	1	-.007	41	1	+.007	43	1	-.002
Paris	60	3	–	16	0	-.001	21	3	-.007	36	3	-.011	56	3	-.006	72	3	-.003	71	3	+.001
Melb.	62	1	1	. .	. .	. .	. .	. .	. .	3	0	-.006	22	1	+.019	38	1	+.002	29	1	.000
Greenw.	64	2	2	33	2	-.002	20	2	-.003	34	2	+.011	38	2	+.018	67	2	+.001	63	2	-.010
Bruss.	65	2	–	36	0	+.009	34	2	+.050	47	2	+.012	63	2	+.007	70	2	+.009	75	2	-.001
Harv.	65	1	–	31	0	+.005	34	1	-.023	24	1	-.016	33	1	-.015	41	1	-.006	38	1	+.008
Pulkowa	65	10	10	30	10	+.008	37	10	+.009	50	10	+.015	56	10	+.010	59	10	-.003	48	10	-.008
Melb.	67	2	2	. .	. .	. .	. .	. .	. .	3	0	-.024	18	1	-.006	30	2	-.007	28	2	+.007
Wash.	71	4	4	37	4	+.003	31	4	+.010	54	4	+.015	58	4	-.009	74	4	-.008	64	4	+.009
Greenw.	72	2	1	37	2	-.012	36	2	-.002	46	2	.000	60	2	+.013	73	2	+.003	66	2	-.009
Harv.	75	2	–	37	0	+.003	60	2	-.004	50	2	-.002	56	2	-.005	64	2	-.001	49	2	.000
Pulkowa	76	4	–	27	0	+.032	36	4	+.023	53	4	-.004	62	4	-.008	71	4	+.002	64	4	+.003
Paris	76	2	–	. .	. .	. .	8	0	+.024	35	2	-.015	56	2	-.007	64	2	-.005	68	2	+.003
Cape	76	1	–	. .	. .	. .	. .	. .	. .	. .	. .	. .	24	1	-.003	54	1	+.006	64	1	-.002
Cordoba	77	3	2	. .	. .	. .	. .	. .	. .	. .	. .	. .	14	2	+.006	32	3	-.003	49	3	+.003
Melb.	77	1	–	. .	. .	. .	. .	. .	. .	9	0	-.006	27	1	-.005	49	1	-.004	56	1	+.014
Greenw.	82	2	2	37	2	-.010	35	2	-.005	47	2	+.004	62	2	+.008	71	2	.000	65	2	-.005
Cape	83	2	2	. .	. .	. .	. .	. .	. .	27	0	+.013	51	2	+.011	65	2	.000	64	2	+.005
Pulkowa	84	10	10	27	10	+.002	35	10	+.002	50	10	+.001	55	10	+.002	58	10	+.005	46	10	-.003
Strassb.	86	8	8	12	8	-.001	19	8	-.016	36	8	-.009	22	8	+.003	50	8	+.006	58	8	-.008
Cape	89	2	2	. .	. .	. .	. .	. .	. .	18	0	-.024	57	2	-.006	74	2	-.001	68	2	+.006
Madison	90	3	–	17	0	-.006	34	3	-.014	46	3	-.004	57	3	-.008	62	3	-.003	45	3	+.004
Lisbon	90	2	–	. .	. .	. .	9	2	+.001	44	2	+.007	48	2	+.003	57	2	-.003	43	2	-.005
Greenw.	94	3	3	37	3	.000	34	3	-.001	53	3	+.002	63	3	+.003	75	3	.000	76	3	-.001
Mt. Hamil.	95	3	–	33	0	-.015	. .	. .	. .	. .	. .	. .	15	3	.000	28	3	-.007	50	3	+.004
Berlin	95	2	–	. .	. .	. .	33	2	-.002	41	2	+.014	49	2	.000	58	2	+.004	38	2	+.002
Albany	98	3	1	11	3	-.011	3	1	-.013	14	3	-.027	17	3	-.031	29	3	-.004	21	3	+.006

TABLE III.

OBSERVED CORRECTIONS, $-\Delta\alpha_{\iota}\cos\delta$, TO THE SYSTEM OF RIGHT-ASCENSIONS OF 627 STANDARD STARS, GIVEN BY CATALOGUES OF OBSERVATION.

SOUTHERN HEMISPHERE.

Decl. of Zone		−15°			−30°			−45°				−60°				−80°			
Catal. and Date	1800+	**	p	C	**	p	C	**	p	C′	C	**	p	C′	C	**	p	C′	C
				s			s			s	s			s	s			s	s
Konigsb.	23	7	3	+.011															
Dorpat	24	15	2	+.013															
Abo	29	33	2	−.011															
Greenw.	30	93	1	+.023	30	0	+.028												
Cape	33	24	2	+.011	16	2	−.006	24	2	+.005	+.006	25	2	+.005	+.005	16	2	.000	−.001
Camb.	31	61	1	.000	8	0	+.011												
Cape	37	55	2	−.007	63	2	−.004	63	2	−.001	−.009	56	2	−.003	−.013	33	2	.000	−.012
Greenw.	38	57	1	−.015	21	0.5	+.005												
Greenw.	44	57	2	−.002	20	1	+.011												
Radcl.	45	61	1	+.014	14	0	+.010												
Paris	45	92	3	.000	37	2	+.002												
Pulkowa	45	64	6	−.001															
Greenw.	51	88	1	−.006	24	0.5	−.005												
Wash.	56	88	2	+.001	61	2	−.009	7	0	−.065	−.065								
Greenw.	57	67	2	−.012	34	1	−.004												
Radcl.	57	76	1	+.017	29	0	+.035												
Cape	59	40	1	.000	57	1	−.012	57	1	−.025	−.030	60	1	−.027	−.031	38	1	.000	−.002
Paris	60	85	3	+.007	30	2	+.021												
Melb.	62	37	1	−.009	33	1	+.005	16	1	+.015	+.012	25	1	+.022	+.018	33	1	−.004	−.009
Greenw.	64	80	2	−.006	21	1	+.001												
Bruss.	65	90	2	−.015	37	1	−.014												
Harv.	65	39	1	+.011	27	1	+.036												
Pulkowa	65	39	6	+.002															
Melb.	67	24	2	+.009	35	2	+.006	65	2	−.006	+.002	62	2	+.010	+.018	42	2	+.003	+.012
Wash.	71	75	4	+.008	54	4	+.009	13	1	+.003	+.003								
Greenw.	72	81	2	.000	28	1	−.002												
Harv.	75	66	2	+.004	31	2	−.009												
Pulkowa	76	51	4	.000															
Paris	76	84	2	+.007	21	1	+.002												
Cape	76	78	1	−.013	65	1	−.037	69	1	−.012	−.026	65	1	+.023	+.016	43	1	.000	+.003
Cordoba	77	96	3	−.011	66	3	−.005	71	3	+.023	+.018	65	3	+.017	+.014	43	3	+.002	+.002
Melb.	77	79	1	−.012	39	1	−.039	4	0	...	−.062	14	1	+.003	−.003	29	1	+.005	+.009
Greenw.	82	85	2	−.001	36	1	−.002												
Cape	83	89	2	−.014	53	2	−.030	54	2	−.004	−.018	43	2	+.001	−.008	39	2	+.002	+.001
Pulkowa	84	37	6	−.003															
Strassb.	86	84	8	−.002	32	5	+.006												
Cape	89	81	2	+.001	65	2	−.024	78	2	−.017	−.027	60	2	−.011	−.020	37	2	.000	−.007
Madison	90	15	3	+.011															
Lisbon	90	53	2	+.007	20	2	+.006												
Greenw.	94	97	3	−.005	37	2	−.001												
Mt. Hamil.	95	28	3	+.010	30	3	+.007	5	1	−.003	−.003								
Berlin	95	21	2	−.006															
Albany	98	21	3	+.014	66	3	+.030	13	0	+.019	+.019								

Provided with this second set of systematic corrections, $\Delta\alpha_s$, the catalogue of standard stars was now expanded to include 401 stars distributed over the entire sky; and the process already described was repeated, except that new positions were not computed for the 158 northern standard stars, nor for the 70 included in the zone $-20°$ to $-40°$. The curves of correction ascertained in this approximation were the ones finally adopted in computing the right-ascensions which appear in the catalogue. They do not differ in any very material way from the curves of correction derived from the final catalogue-places themselves, which are the corrections to be published at the end of this series of papers.

In relation to the two zones embraced within a radius of 8° from either pole it is proper to say that an amount of labor was expended upon this part of the work which was possibly not fully justified by its importance. The final positions of Newcomb, or of Auwers, were as far as possible utilized as the basis of position to be corrected. An ephemeris of each star at five-year intervals and extending over the entire range of observation was computed by means of the trigonometrical formulas, assumed proper-motions having been taken strictly into account. Annual and secular variations were then computed for each epoch, and also the third derivative, where necessary, by means of differences in the secular variations. Then the work of expansion was tested by mechanical integration with the result that in no case was more than a trifling modification found necessary. The observations were then compared with the ephemeris, and the usual mode of procedure, ending with the formation of zone-equations, was followed, the residuals as in other instances relative to high declinations having been first multiplied by cos δ. In case of the northern zone the first approximation was reached by means of a special treatment of the four stars, *α Urs. min.*, 51 H *Cephei*, δ and λ *Urs. min.* In the case of *α Urs. min.*, which played the principal role, I employed all the strictly independent determinations known to me, whether contained in the principal catalogues or not. In deciding upon the curves of correction for the polar zones their trend in the adjoining zones was taken into account.

In extending the work from $-40°$ to the southern pole a process was followed which is set forth in connection with the test-computation (Table III), except that the zones treated were 5° instead of 15° in width.

Final Test of the Right-Ascensions.

In order to test the entire work and to perfect the curves of correction, I have carefully compared the Catalogue of 627 Standard Stars (including also about 100 standard right-ascensions which remain unpublished) with each of the catalogues of observation employed in its construction, and with some others for which it is of special interest to know the systematic corrections. An abridged outline of the results for all the catalogues to which weight was assigned in the systematic sense is exhibited in Table III. This part of the work, as well as all other essential operations connected with the formation of the catalogue, was performed in duplicate.

Throughout the work the values of $\Delta\alpha_s$ for the principal catalogues were retained as they were first computed in the manner already described; but in this final revision of the systematic corrections a new determination of each individual value of $\Delta\alpha_s$ was obtained by means of comparison with the Catalogue of 627 Standard Stars.

The observed values of $\Delta\alpha_s$ were computed for zones 5° wide in such a manner that the mean of each zone should fall very nearly at some multiple of 5°. Throughout this and all similar computations in this work the differential weights were rigorously employed, and the resulting weights were always attached to the combinations that were made in means. For the purposes of the present illustration, and in order to bring the entire matter into a form convenient for general inspection, the results for these zones of 5° were condensed into zones 15° in width, except for the polar zones of which the radii are 22°.5.

The table exhibits in the first column the name of the observatory. In the second column is found the estimated mean epoch of observation; in the third, the weights which have been assigned to the respective catalogues in the fundamental sense in this investigation as to $\Delta\alpha_s$; in the fourth, the weights which might have been assigned under more strict rules of selection commented upon in the foregoing. The line of numbers for each catalogue in this latter class to which has been assigned a weight of two or more is printed in full-faced type. For the respective zones indicated in the headings of the succeeding columns are given the values of $\Delta\alpha_s \cos\delta$ derived from comparison of the observed right-ascensions of the various catalogues with the computed right-ascensions of the standard catalogue in the sense, observed *minus* computed. The number of stars on which each comparison rests, and the specially assigned fundamental weights (in general accordance with those of the third column) are also given. As for the weights in the differential sense, these are omitted for economy of space. The corresponding probable error of the values of $\Delta\alpha_s \cos\delta$, to which any weight is assigned, in no case exceeds $\pm 0^s.01$, and is sometimes nominally very slightly greater than $\pm 0^s.001$.

For the generality of catalogues of a date later than 1840 the nominal differential probable error in the principal zones will average about $\pm 0^s.0025$, so that much the larger part of the discrepancies is due to systematic differences.

The residuals for Pulkowa 1875, Greenwich 1894, and Berlin 1895 have been modified to represent the respec-

tive instrumental meridians in the manner previously explained.

Thus the quantities C or C' (in the sense Obs.—Comp.) are corrections applicable to the published right-ascensions of this standard catalogue given by the respective catalogues of observation. They are discordant; but by means of the weights attached, or by means of those in the fourth column, or through any other system of weights, we may determine in each of the zones any general correction of the standard catalogue which may still be required. This determination I have effected for each system of weights; and the results are exhibited in Table IV. It will be noticed that in the zones for $-45°$, $-60°$, and $-80°$ there are two sets of values of $\Delta\alpha_s \cos\delta$, C' and C. The first, C', is that which is employed with the attached weights, and C is employed with the weights of the fourth column. The latter, C, is the original value of $\Delta\alpha_s \cos\delta$ resulting directly from the comparison. C' was formed in the following manner. After solution of the equations formed from the values of C (with weights attached) in zones $-15°$ and $-30°$, the values of the observed quantities in those columns were corrected for the concluded values of $\Delta\alpha_0$ and $\Delta\mu_0$ obtained from the solutions. These were trifling in amount as will be seen from the results in Table IV. Then for the Southern catalogues their errors of the form $\Delta\alpha_s$ at the declination, $-22°$, are supposed to be known from the means of the corrected values taken from zones $-15°$ and $-30°$. The close circumpolar zone was supposed to give another value of these at $-87°$ in which some confidence may be felt, since the discordances in the various independent determinations are exceedingly small as will be seen from the following list of them.

Systematic Polar Deviations $\Delta\alpha_s \cos\delta$ (O−C) at −87°.

		O−C			O−C
Cape	33	−0.001	Cape	76	+0.006
Cape	37	−0.013	Cordoba	77	+0.001
Cape	59	−0.001	Melbourne	77	+0.007
Melbourne	62	−0.006	Cape	85	+0.001
Melbourne	67	+0.009	Cape	95	−0.006

The further computations of the far southern zones are conducted under the supposition that the corrections due to the catalogues of observation, supposed to be known at $-22°$ and $-87°$ from superior weight of material, vary uniformly between these points, and that the residuals, C', after removing these are observed corrections to the system of the standard catalogue.

The principle adopted in this course is empirical, and is based upon experience with the values of $\Delta\alpha_s$ for the catalogues of the northern hemisphere. If the correction, C, is comparatively large at a distance of 20° or 30° from the zero usually occurring near +10° of declination, and especially if in that region it is found to be increasing toward the pole it is not likely to change sign at a declination smaller than +75°, or +80°. Under such conditions the use of a preliminary interpolated correction is much more likely to decrease the peculiar error of any one catalogue of observation than it is to increase it. On the other hand if the original value of the correction at −22° is small the others are not materially affected by this process.

Putting the matter in another way, our procedure gives us in the values of C' approximately those which would result from the respective series of observation if they should be recomputed with assumed right-ascensions of the close polar stars taken from the present catalogue, and of the clock-stars also taken from the same source, but confined between the limits of −7°.5 and −37°.5 of declination.

TABLE IV.

Solution of Zone-Equations to Determine Observed Systematic Corrections of the Right-Ascensions of the Standard Stars.

	Solution A				Solution B			
Zone	$\Delta\alpha_s \cos\delta$ 1875	$100\,\Delta\mu \cos\delta$	$\Delta\alpha_s$ 1900	$100\,\Delta\mu$	$\Delta\alpha_s \cos\delta$ 1875	$100\,\Delta\mu \cos\delta$	$\Delta\alpha_s$ 1900	$100\,\Delta\mu$
+80	+0.0004	+0.001	+0.0035	+0.006	+0.0010	0.000	+0.0058	0.000
60	+0.0011	+0.001	+0.0026	+0.002	−0.0001	−0.007	−0.0038	−0.014
45	+0.0004	−0.004	−0.0008	−0.006	+0.0033	−0.008	+0.0020	−0.011
30	0.0000	+0.002	+0.0005	+0.002	+0.0032	+0.011	+0.0037	+0.012
+15	0.0000	+0.002	+0.0004	+0.002	+0.0010	+0.006	+0.0025	+0.006
0	−0.0007	−0.005	−0.0019	−0.005	−0.0026	−0.009	−0.0049	−0.009
−15	+0.0006	−0.003	−0.0001	−0.003	+0.0013	−0.014	−0.0022	−0.014
30	+0.0004	+0.008	+0.0029	+0.009	−0.0010	−0.002	−0.0016	−0.002
45	−0.0005	−0.007	−0.0047	−0.010	−0.0069	−0.028	−0.0195	−0.043
60	+0.0050	+0.005	+0.0126	+0.010	−0.0018	−0.013	−0.0102	−0.026
−80	+0.0012	+0.003	+0.0109	+0.017	−0.0003	+0.004	+0.0040	+0.023

This would be advantageous only under the supposition that the system can be established from existing observations with very much greater weight for zones, $-15°$, and $-30°$ than for zones further south. As a matter of fact the computed weight for zone, $-15°$, is six times that for $-45°$. Under all the circumstances, therefore, I think the result from solutions employing the values of C' for the southernmost zones is greatly to be preferred.

The results for all the solutions are presented in Table IV, which exhibits under the designation, "Solution A," those in which the attached weights (corresponding to those in the third column) were employed; and also under the designation, "Solution B," the results which are obtained when the weights are taken from the fourth column of Table III under a more rigid criterion as to the independence of catalogues.

Upon careful consideration of this question with the present opportunity for reconsideration after the computations had been laid aside for many months, it seems to me that the results from Solution A are to be preferred not only as to the far southern stars, but throughout. From the north pole down to $-15°$ the differences in the results from the two solutions are not, however, of serious importance.

Probable Error of the System as to $\Delta\alpha_\delta$.

Regarding the discrepancies, C and C' in Table III, as due entirely to systematic differences the nominal probable errors of the system as to $\Delta\alpha_\delta$ can be computed for the several zones. These have been derived for Solution A. They are shown in the following table in equatorial seconds in order to correspond to Table III, and to facilitate the comparison between different zones.

		(Probable Errors) × (cos δ) for			
Zone	Mean Ep.	α, ep.	100μ	α, 1900	α, 1755
+80	1863	$\pm^s.0014$	$\pm^s.005$	$\pm^s.0024$	$\pm^s.006$
60	1865	.0021	.009	.0038	.010
45	1866	.0016	.007	.0028	.008
30	1867	.0012	.005	.0021	.006
+15	1867	.0006	.003	.0010	.003
0	1867	.0007	.003	.0013	.004
−15	1867	.0009	.004	.0016	.005
30	1872	.0020	.011	.0036	–
45	1867	.0033	.016	.0061	–
−60	1866	.0035	.017	.0071	–

The probable errors under "α, ep." are the minima for right-ascension and correspond to the epochs in the preceding column. Those in the last column correspond equally for epochs not far from 1975. Zone $-80°$ is omitted, since from the nature of the computation the computed probable errors for that zone would be much too small. The probable errors for zones near the equator are necessarily small, since they are very near the artificial zeros of $\Delta\alpha_\delta$.

These probable errors might be regarded as fair indications of the uncertainty of the system as to $\Delta\alpha_\delta$ could we be assured that the weights employed in Solution A (Table III) are sufficiently homogeneous. We have the means for a rough test of this point in the residuals for the zones $+30°$ $+45°$, and $+60°$. From these I find as the probable error of the unit of weight, $\pm 0^s.013$ computed from 67 residuals having weights 1 and 2; while from 45 residuals having weight greater than 2 I find the probable error of the unit to be $\pm 0^s.016$. This agreement is, perhaps, as good as could have been anticipated; and, at any rate, it is good enough to inspire some confidence in the reality of the probable errors of the table within a reasonable limit of uncertainty.

The entire probable error of the system would depend upon a proper combination of these probable errors for $\Delta\alpha_\delta$ with the probable errors appertaining to the position and motion of the adopted equinox. It would be very difficult to estimate what common error may possibly subsist in all the observations of the sun for right-ascension; but in reference to the centennial motion of the equinox I think, perhaps, that $\pm 0^s.02$ may not be regarded as an underestimate of its probable error.

In basing opinions upon probable errors like these it should always be borne in mind that the unfavorable chance may be taken. Mathematically there is one chance in five that the true error will turn out to be twice, and one chance in 23 that will turn out to be thrice, the correctly computed probable error. Therefore, aside from an error which may be common to all the right-ascensions, and due to error in adopted equinox, and assuming that our computations of probable error as to $\Delta\alpha_\delta$ are fairly well founded, we may consider that there is small chance that the systematic error as to $\Delta\alpha_\delta$ for 1900 in our right-ascensions is numerically larger than $0^s.01 \sec\delta$ for any point in the northern hemisphere, or larger than $0^s.02 \sec\delta$ for any point in the southern hemisphere; and that generally it will be much less.

Systematic Corrections of the Form, $\Delta'\alpha_\alpha \tan\delta$.

Hitherto in our discussion the correction $\Delta\alpha_\alpha$, principally due to periodic error in the adopted places of clock-stars, has been regarded as equally appertaining to stars at all distances from the equator, as if it were the sole correction of the kind which is needed. But it is evident that we may have an additional periodic error of this kind due to imperfection in the determination of polar deviation of the transit. In general this error could be represented by the expression, $\Delta'\alpha_\alpha \tan\delta$. For some of the catalogues included in the list of Table III such a term appears to be sensible; and the same may be true of others. For example: we have made a thorough comparison of PIAZZI's Catalogue with the present standard. $\Delta\alpha_\alpha$ was determined precisely

in the same manner as for other catalogues, after the application of a preliminary correction for $\varDelta\alpha_\delta$. We have

$$\varDelta\alpha_a = +0^s.118 + 0^s.002 \sin\alpha - 0^s.018 \cos\alpha$$

The individual residuals of the comparison in question, at all declinations, were then freed from the effect of both $\varDelta\alpha_a$ and $\varDelta\alpha_\delta$. The error dependent on tan δ was then supposed to have the periodic form which has been assigned to $\varDelta\alpha_a$ throughout this work. The solution of the equations formed in the three zones resulted as follows:

Zone	Weight	$\varDelta'\alpha_a$	
-42° to -22°	0.3	$-0^s.114 \sin\alpha$	$+0^s.046 \cos\alpha$
$+30$ $+59$	1.0	-0.068	$+0.043$
$+60$ to $+89$	1.0	-0.098	$+0.054$
Adopted:		$-0.087 \sin\alpha$	$+0.048 \cos\alpha$

The last expression, $+0^s.099 \sin(\alpha+151^\circ) \tan\delta$, has been adopted as a supplementary correction of Piazzi's right-ascensions. The use of it greatly improves the accordance of Piazzi with other authorities.

Very likely a periodic term of the form adopted may really vary quite materially from the true form of the correction in some hours of right-ascension; but the danger of attempting to draw a curve under the circumstances seems to me to hold out greater possibilities of error than those which attach to the adoption of the formula.

The method of computation adopted in the foregoing illustration concerning Piazzi's right-ascensions will serve to illustrate the procedure with other catalogues. It seemed to be desirable to have some evidence whether such terms have existence in any case. For this purpose the residuals for all catalogues, corrected for $\varDelta\alpha_a$ and $\varDelta\alpha_\delta$, were divided into zones, as follows:

$+79^\circ$ to $+70^\circ$	-22° to -50°
$+69$ to $+60$	-51 to -65
$+59$ to $+30$	-65 to -79

The residuals in each zone were collected in weighted means covering in each instance not more than two hours of right-ascension. Then in each zone the means were multiplied by mean cotan δ for that zone; afterward the results for all zones were combined in one general mean. When it was clearly seen that the mean residuals in the several zones indicated no decided trace of a periodic law, either in common or separately, no further discussion of them was attempted. The following list embraces all catalogues for which it was considered to be advisable to evaluate a formula of correction. Those marked with an asterisk are the only ones which appear to be very sensible in relation to the probable error of determination.

I am scarcely prepared to recommend any of these for adoption except, perhaps, those for Radcliffe 57, Melb. 62, and Brussels 65. Dr. Auwers finds a very decided periodic variation for Melb. 62 of the form, $\varDelta'\alpha_a \tan\delta$, applicable south of declination -50°. This limitation seems to be fully warranted.

Corrections of the Form, $\varDelta'\alpha_a \tan\delta$.

Dorpat,	24*	$(-0^s.014 \sin\alpha$	$-0^s.002 \cos\alpha) \tan\delta$
Greenwich,	30	$+0.005$	$+0.007$
Cambridge,	30	-0.010	$+0.010$
Madras,	35	-0.016	$+0.005$
Greenwich,	44	-0.004	-0.011
Washington,	56	$+0.009$	-0.011
Radcliffe,	57*	-0.007	$+0.024$
Paris,	60	-0.007	$+0.005$
Melbourne,	62*	$+0.022$	$+0.023$
Greenwich,	64	$+0.006$	$+0.004$
Brussels,	65*	$+0.035$	-0.003
Greenwich,	72	$+0.007$	-0.006
Pulkowa,	75	-0.015	$+0.005$
Harvard,	75	-0.003	$+0.005$
Strassburg,	85*	-0.008	-0.004

The systematic corrections to the standard catalogue of right-ascensions indicated in Solutions A (Table IV), are so small that they can safely be neglected at present. For their improvement it is desirable that the several observatories of the southern hemisphere in possession of suitable instruments should devote at least one or two years to the observation of the principal and secondary standard stars. So far as possible all other stars down to the seventh magnitude, which are situated between -30° and the southern pole, should also receive two or three observations each on the part of two or three different observatories. For clock-stars one should employ the principal standard stars situated between $+30^\circ$ and -22° of declination, and should determine the polar deviations of the transits, where practicable, from successive transits of close polar stars.

Several observatories of the United States and of Southern Europe should likewise make special observations in the southern sky upon stars down to the seventh magnitude, at least, working from -10° or -15° of south declination to a zenith-distance of 76° or 77°. It would facilitate future investigations of systematic error if the observed right-ascensions of the clock-stars were also to be included in the respective catalogues for those nights when eight or more have been observed.

Especially, the relation of instrumental meridian to the meridian of the standard catalogue from which the clock-stars are taken should be determined through preliminary computations; so that, either by correcting the standard positions, or by correcting the instrumental results, systematically consistent clock-corrections may be determined from stars in widely separated parallels of declination. The Pulkowa Catalogues for 1855 and 1875 furnish good examples of this method, which is indispensable to the attainment of the best results. The neglect of this precaution has proved a serious blemish in the reduction of several valuable series of observations.

THE BENJAMIN APTHORP GOULD FUND.

Applications for grants of money in aid of astronomical investigation may be made by letter to any of the Directors undersigned stating the amount desired, the nature of the proposed investigation, and the manner in which the money is to be expended. The following information is given for the guidance of applicants.

The BENJAMIN APTHORP GOULD FUND was established in 1897 by Miss ALICE BACHE GOULD, to advance the science of astronomy, and to honor the memory of her father by ensuring that his power to accomplish scientific work shall not end with his death. The principal is $20,000, vested in the National Academy of Sciences as Trustee. The income is to be administered by the undersigned and their successors to assist the prosecution of researches in astronomy.

In recognition of the fact that during Dr. GOULD'S lifetime his patriotic feeling and ambition to promote the progress of his chosen science were closely associated, it is preferred that the Fund should be used primarily for the benefit of investigators in his own country or of his own nationality. But it is further recognized that sometimes the best possible service to American science is the maintenance of close communion between the scientific men of Europe and of America, and that therefore, even while acting in the spirit of the above restriction, it may occasionally be best to apply the money to the aid of a foreign investigator working abroad.

In all cases work in the astronomy of precision will be preferred to work in astrophysics, both because of Dr. GOULD'S especial predilection and because of the present existence of generous endowments for astrophysics.

Finally, the BENJAMIN APTHORP GOULD FUND is intended for the advancement and not for the diffusion of scientific knowledge, and is to be used to defray the actual expenses of investigation, rather than for the personal support of the investigator during the time of his researches, without absolutely excluding the latter use under exceptional circumstances.

In addition to the above call for applications the Directors, desiring to stimulate the participation of American astronomers in the attempt to bring up the arrears of cometary research, renew the offer to them of the sum of $500 for computation of the "definitive" orbits of comets (see list in *A.J.* 493, p. 104); this sum to be distributed at the average rate of $100 for each computation, — the amount to vary according to the relative difficulty of the computation, and to be determined by the Directors of the GOULD FUND.

LEWIS BOSS, SETH C. CHANDLER, ASAPH HALL.

1903 *March.*

OBSERVATIONS OF THE DECLINATION OF *VESTA*,

MADE WITH THE 5-INCH VERTICAL CIRCLE, AT THE U.S. NAVAL OBSERVATORY,

BY GEORGE A. HILL.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent.]

Date	Wash. M.T.	Obs'd Decl.	O—C
	h m s	° ′ ″	″
1902 July 2	13 4 45	−22 14 17.7	+2.5
5	12 50 14	22 34 37.1	+2.0
8	12 35 36	22 54 56.6	+3.5
11	12 20 52	23 15 8.8	+2.8
13	12 11 2	23 28 24.3	+4.0
14	12 6 6	23 35 0.8	+1.5
16	11 56 15	23 47 55.9	+3.7
22	11 26 50	−24 25 0.5	+1.9
1902 Aug. 3	10 29 27	−25 27 57.5	+0.7
7	10 11 6	25 45 4.7	−1.8
22	8 56 38	−26 31 16.8	−1.0

These places are corrected for parallax, and the comparison in the last column has been made with the ephemeris of *Vesta*, as published in the *British Nautical Almanac for* 1902.

OBSERVATIONS OF THE RIGHT-ASCENSION OF *VESTA*,

MADE WITH THE 5.3-INCH TRANSIT INSTRUMENT AT THE U.S. NAVAL OBSERVATORY,

BY EVERETT I. YOWELL.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent.]

The clock correction was obtained from a group of four clock stars, level and collimation by reversal over the mercury, both before and after the series; azimuth from λ *Ursae minoris* or 51 H *Cephei* s.p. The residuals O—C have been obtained by direct comparison with the ephemeris in the *British Nautical Almanac for* 1902.

Date	Clamp	Observed R.A.	O—C
		h m s	s
1902 July 2	E	19 46 6.15	+4.31
11	E	37 35.04	+4.38
16	W	32 36.96	+4.35
22	E	26 45.81	+4.40
31	W	19 18 49.99	+4.27
1902 Aug. 7	E	19 13 53.57	+4.12
18	W	9 6.41	+3.81
21	E	8 29.91	+3.69
29	W	8 22.68	+3.54
Sept. 4	E	9 42.42	+3.37
13	W	13 52.02	+3.15
16	E	15 47.36	+3.07
27	W	24 52.46	+2.76
29	E	26 50.64	+2.78
Oct. 3	W	31 2.76	+2.69
9	E	37 57.87	+2.53
12	E	53 41.00	+2.36
25	E	19 59 23.62	+2.26

OBSERVATIONS OF COMET *d* 1902 (*GIACOBINI*),

MADE WITH 26-INCH REFRACTOR OF THE LEANDER MCCORMICK OBSERVATORY, UNIVERSITY OF VIRGINIA,

BY J. P. McCALLIE.

1902 Charl. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Dec. 5 13h 10m 2s	1	8 , 6	+2m 32.80s	−4′ 4.5″	7h 16m 49.74s	−1° 32′ 54.3″	n9.149	9.746	+4.33s	−12.7″
" 7 13 29 52	2	8 , 7	+1 13.69	−0 24.8	7 16 10.34	−1 14 48.4	n8.810	9.744	+4.36	−13.1

Mean Places of Comparison-Stars for 1902.0.

*	α	δ	Authority
1	7h 14m 12.61s	−1° 28′ 37.2″	Paris III, 9000
2	7 14 52.29	−1 14 10.5	I München, 2474

Approximate corrections for refraction have been applied.

In my observations of Comet *b* 1902, in *A.J.* 533, p. 57, the values of log pΔ in R.A. were given in arc instead of in time, so that log 15 should be subtracted from them throughout.

Charlottesville, Va.

OBSERVATIONS OF MINOR PLANETS,

MADE AT THE VASSAR COLLEGE OBSERVATORY,

BY MARY W. WHITNEY AND CAROLINE E. FURNESS.

1901–2 Greenw. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(354) *Eleanora.*					
Nov. 8 16h 17m 3s	1	12 , 6	+0m 45.26s	+ 2′ 6.6″	5h 41m 16.90s	− 1° 37′ 7.9″	n9.511	0.776	+4.28s	+ 1.1″[1]
20 16 8 30	2	10 , 4	−1 11.50	− 1 8.6	5 34 57.27	− 2 22 39.3	n9.414	0.783	+4.54	0.0[1]
21 15 29 24	2	8 , 4	−1 51.05	− 3 45.8	5 34 17.73	− 2 25 16.7	n9.492	0.782	+4.55	− 0.2[1]
Dec. 5 16 2 52	3	4 , 4*	−0 16.94	+ 1 39.6	5 23 4.39	− 2 40 39.9	n9.173	0.788	+4.79	− 1.3[1]
					(82) *Alkinene.*					
Dec. 18 15 25 56	4	12 , 6	−0 55.68	− 4 10.0	7 5 49.65	+27 19 8.3	n9.519	0.468	+5.81	−16.7[1]
Jan. 11 15 29 33	5	10 , 8*	−0 22.20	− 6 1.9	6 42 37.19	+28 6 18.5	n8.998	0.324	+2.50	− 9.5[2]
13 14 36 16	6	10 , 8	−0 41.05	−10 0.3	6 40 43.72	+28 7 52.3	n9.280	0.358	+2.50	− 9.3[2]
					(356) *Liguria.*					
Jan. 25 14 28 13	7	8 , 8*	−0 37.85	− 4 10.7	7 9 39.58	+35 24 7.3	n9.263	0.066	+2.74	− 9.7[2]
28 14 0 58	8	6 , 4	+1 44.96	+ 2 58.2	7 7 13.59	+35 11 49.5	n9.325	0.105	+2.71	− 9.1[1]
30 13 31 48	9	8 , 8*	−0 13.60	+ 5 34.6	7 5 45.52	+35 2 54.9	n9.404	0.158	+2.69	− 9.0[1]
					(17) *Thetis.*					
Jan. 27 14 15 23	10	8 , 8*	+0 28.12	+ 0 24.5	7 55 36.04	+20 5 17.5	n9.402	0.554	+2.44	−13.4[2]
30 14 52 52	11	8 , 8*	+0 3.27	− 5 33.7	7 52 37.19	+20 19 25.7	n9.195	0.522	+2.44	−13.1[1]
Feb. 3 14 4 25	13	7 , 9*	−0 27.86	+ 4 12.2	7 48 52.25	+20 37 22.4	n9.326	0.531	+2.45	−13.2[2]
4 13 49 55	14	7 , 4	+2 44.89	+ 1 16.9	7 47 58.46	+20 41 37.5	n9.361	0.536	+2.46	−13.1[1]
5 12 30 28	14	12 , 6	+1 54.46	+ 5 19.8	7 47 8.04	+20 45 40.4	n9.544	0.591	+2.47	−13.1[1]
					(347) *Pariana.*					
Feb. 4 15 0 48	15	8 , 8*	+0 22.69	− 5 33.6	9 10 57.15	+32 9 38.6	n9.442	0.294	+2.56	−15.7[2]
5 14 47 35	15	8 , 8*	−0 35.45	+ 2 45.2	9 9 59.02	+32 17 57.5	n9.467	0.306	+2.57	−15.6[2]
10 15 37 8	17	7 , 4	+1 9.90	− 5 29.5	9 5 5.04	+32 56 5.6	n9.150	0.161	+2.62	−14.8[1]
11 12 46 56	17	8 , 8*	+0 19.71	+ 0 27.6	9 4 14.85	+33 2 2.8	n9.645	0.469	+2.62	−14.7[1]
14 14 21 19	18	10 , 8	+1 23.76	+ 3 49.1	9 1 21.69	+33 21 5.3	n9.418	0.238	+2.64	−14.2[2]
15 12 41 50	18	10 , 8	+0 33.23	+ 9 4.8	9 0 31.16	+33 26 21.0	n9.628	0.534	+2.64	−14.2[1]
					(68) *Leto.*					
Mar. 10 16 5 44	19	8 , 9*	+0 16.79	− 9 30.3	11 12 13.39	+15 48 51.5	n9.001	0.588	+2.46	−17.0[2]
14 16 9 32	20	4	. . . .	+ 1 12.6	. . . .	+16 3 50.6	. .	0.580	. .	−16.8[2]
14 16 17 10	20	8 *	−0 29.38	. . . .	11 8 49.16	. . . .	n8.601	. .	+2.47	. . [2]
18 15 9 20	21	10 , 8	+0 57.89	− 7 1.7	11 5 33.27	+16 16 39.1	n9.134	0.585	+2.50	−16.7[2]

1902 Greenw. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				(103) *Hera.*						
			m s	′ ″	h m s	° ′ ″			s	″
Mar. 18 16 27 33 h m s	22	8, 8*	+0 11.05	− 2 53.3	12 28 50.98	+ 3 24 0.3	n9.145	0.738	+2.52	−15.9[1]
25 16 1 20	23	10, 5*	+0 0.61	− 3 34.1	12 23 19.64	+ 4 10 30.8	n9.105	0.730	+2.55	−16.0[1]
26 15 44 23	23	12, 6	−0 47.00	+ 2 54.3	12 22 32.03	+ 4 16 59.2	n9.175	0.730	+2.55	−16.0[1]
				(393) *Lampetia.*						
Apr. 24 15 5 40	24	5, 6*	−0 4.50	+ 0 41.5	11 39 27.18	− 7 32 8.4	8.882	0.823	+2.52	−18.7[1]
May 8 15 51 1	25	5, 5†	+0 37.64	− 5 14.2	11 35 26.39	− 5 33 52.4	9.416	0.802	+2.37	−18.2[1]
9 14 27 42	25	8, 8	+0 31.34	+ 1 44.0	11 35 20.08	− 5 26 54.2	9.091	0.808	+2.36	−18.2[1]

* Δα measured directly.

† Observation made with square bar micrometer.

[1] Mary W. Whitney, Observer.

[2] Caroline E. Furness, Observer.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	5 40 27.36	− 1 39 15.6	Nicolajew A.G. 1464	14	7 45 11.11	+20 40 33.7	Berlin B, A.G. 3144
2	5 36 4.23	− 2 21 30.7	Schjellerup 1879	15	9 10 31.90	+32 15 27.9	Leiden A.G. 3809
3	5 23 16.54	− 2 42 18.2	Strassburg A.G. Zones	16	9 2 5.65	+32 56 7.3	Leiden A.G. 3763
4	7 6 39.52	+27 23 35.0	Camb. Eng. A.G. 3815	17	9 3 52.52	+33 1 49.9	Micr. Comp. with *16
5	6 42 56.89	+28 12 29.9	Camb. Eng. A.G. 3509	18	8 59 55.29	+33 17 30.4	Leiden A.G. 3750
6	6 41 22.27	+28 18 1.9	Camb. Eng. A.G. 3492	19	11 11 54.14	+15 58 38.8	Bonn VI
7	7 10 14.69	+35 28 27.7	Lund. A.G. 3767	20	11 9 16.07	+16 2 54.8	Berlin A, A.G. 4390
8	7 5 25.92	+35 9 0.4	Lund. A.G. 3736	21	11 4 32.88	+16 23 57.5	Berlin A, A.G. 4369
9	7 5 56.43	+34 57 29.3	Lund. A.G. 3740	22	12 28 37.41	+ 3 27 9.5	Albany A, G. 4512
10	7 55 5.48	+20 5 6.4	Berlin B, A.G. 3212	23	12 23 16.48	+ 4 14 20.9	Albany A, G. 4493
11	7 52 31.48	+20 25 12.5	Berlin B, A.G. 3194	24	11 39 29.16	− 7 32 31.2	Munich I 7151
12	7 48 26.38	+20 26 10.7	½[Bonn VI + Yar. 3280]	25	11 34 46.38	− 5 28 20.0	Munich I 7045
13	7 49 17.66	+20 33 23.4	Micr. Comp. with *12				

SEARCHING EPHEMERIS FOR APPEARANCE IN 1903 OF COMET 1896 V,

[Abridged from M. Ebell's communication in *A.N.* 3848.]

Berlin M.T.	Assumed Per. Pass. June 6.5			Assumed Per. Pass. June 22.5			Assumed Per. Pass. July 8.5		
	α	δ	Br.	α	δ	Br.	α	δ	Br.
1903	h m	° ′		h m	° ′		h m	° ′	
Apr. 3.5	21 24.5	− 5 50	0.86	20 55.0	− 7 14	0.83	20 25.3	−8 32	0.79
11.5				21 17.6	− 5 20	0.95			
19.5				21 40.5	− 3 17	1.09			
27.5				22 3.7	− 1 7	1.23			
May 5.5	23 0.3	+ 2 44	1.29	22 27.1	+ 1 8	1.39	21 51.7	−0 32	1.50
13.5				22 50.8	+ 3 24	1.55			
21.5				23 14.7	+ 5 41	1.72			
29.5				23 38.7	+ 7 54	1.88			
June 6.5	0 37.6	+11 5	1.77	0 2.8	+10 1	2.03	23 23.2	+8 43	2.46

Unit of brightness assumed as for 1897 January 4, when last seen. At discovery in 1896 it was 11^{M}–12^{M} (Br. = 2.93).

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NEW FORMULAS FOR FINDING THE MEAN ERROR OF AN OBSERVATION AND SOME LIKELY ERRORS OF THE MOST PROBABLE VALUES OF THE UNKNOWN QUANTITIES IN INDIRECT OBSERVATIONS,

By J. MIDZUHARA.

When a limited number of observation-equations containing independent unknown quantities is given, it is usual, in finding the mean error of the single observation, to adopt the well known formula of GAUSS, which is said to be best approximative; but whether it is truly approximative or not may be questioned, for, it was not the result of the comparison with the true one. I have lately spent much time on the discussion of this point, and finally conceived a formula sensibly different from that of GAUSS; and as the consequence of this discussion I have also found formulas giving some likely errors (I have adopted this name to distinguish from the word "probable error" in the usual meaning) of the most probable values of the unknown quantities. The details of the results of this discussion are as follows.

1. *Notation.* It is to be noticed that in the present paper I have adopted, for the most part, the notation described in the Spherical and Practical Astronomy, Vol. II, by CHAUVENET, except some few parts which will be especially explained as occasion requires.

2. *On a Substitution of Mean Value for the True One.* The square of the true error of $[an]$ is $[au]^2$, and the square of its mean error $[aa]\epsilon^2$; the system of such relations will be expressed, for shortness, by saying that "the mean value of

$\{[au]^2 \pm [bu]^2 \pm \ldots\}$ is equal to $[aa]\epsilon^2 \pm [bb]\epsilon^2 \pm \ldots$"

The expression which we have now to determine is

$$\text{(A)} \qquad [au]\Delta x + [bu]\Delta y + [cu]\Delta z + \ldots$$

and, remembering that (for example)

$$\begin{aligned}\text{mean value of } [au][bu] &= \text{mean value of } \frac{[(a+b)u]^2 - [au]^2 - [bu]^2}{2} \\ &= [ab]\epsilon^2\end{aligned}$$

it may be easily seen that the mean value of every term of the expression (A) is ϵ^2, which has been substituted for the true by GAUSS; but since the ratios of the different functions of the errors to the corresponding mean values are not necessarily constant (see Art. 3 and last part of Art. 5), it is possible that, even if any function of the errors be identical with that mean value, yet another function of the errors can not be considered so; that is to say, there must be, sometimes, a better value than the mean to be substituted for the true one. On this account I shall now discuss by what expressions the functions

$$[au]^2,\ [bu]^2,\ [cu]^2 \ldots, \qquad [au][bu],\ [au][cu],\ [bu][cu],\ \ldots,$$

of which the expression (A) is composed, must be substituted. It is sufficient to consider the case of three unknown quantities.

3. *Discussion of the Functions* $[au]^2$, $[bu]^2$, $[cu]^2$ and $[au][bu]$, $[au][cu]$, $[bu][cu]$.

Let us suppose that

$$[aa] = [bb] = [cc]$$

and

$$[au]^2 \geqq [bu]^2 \geqq [cu]^2$$

then we may put

$$[au]^2 = (A+B)[aa]\epsilon^2 = (A'+B')[aa]\epsilon^2 \qquad (1)$$

$$[bu]^2 = (A-B)[aa]\epsilon^2 = (A''+B'')[aa]\epsilon^2 \qquad (2)$$

$$[cu]^2 = (A'-B')[aa]\epsilon^2 = (A''-B'')[aa]\epsilon^2 \qquad (3)$$

and therefore

$$[au][bu] = \pm\sqrt{A^2-B^2}\,[aa]\epsilon^2 \qquad (4)$$

$$[au][cu] = \pm\sqrt{A'^2-B'^2}\,[aa]\epsilon^2 \qquad (5)$$

$$[bu][cu] = \pm\sqrt{A''^2-B''^2}\,[aa]\epsilon^2 \qquad (6)$$

where $A, A', \ldots B, B', \ldots$ are indeterminate positive numbers. Now, if we put

(7) $[au]\,[bu] = A\,.\,f[ab]\,\epsilon^2$*

(8) $[au]\,[cu] = A'.\,f[ac]\,\epsilon^2$

(9) $[bu]\,[cu] = A''.\,f[bc]\,\epsilon^2$

we get

(10)

$$[cu]^2 = \frac{A\left\{1+\sqrt{1-\left(\frac{f[ab]}{[aa]}\right)^2}\right\}\left\{1-\sqrt{1-\left(\frac{f[ac]}{[aa]}\right)^2}\right\}[aa]\,\epsilon^2}{1+\sqrt{1-\left(\frac{f[ac]}{[aa]}\right)^2}}$$

(11)

$$= \frac{A\left\{1-\sqrt{1-\left(\frac{f[bc]}{[aa]}\right)^2}\right\}\left\{1-\sqrt{1-\left(\frac{f[ab]}{[aa]}\right)^2}\right\}[aa]\,\epsilon^2}{1+\sqrt{1-\left(\frac{f[bc]}{[aa]}\right)^2}}$$

which, if we have

(12) $[ab] = [ac] = [bc],$

give $f[ab] = f[ac] = f[bc] = \pm[aa];$

and since, for instance, we may change the values of $[ab]$ and $[ac]$, without changing the value of $[bc]$, in the equations (12), and moreover the latter may be any value between zero and $[aa]$, for all possible values of $[ab]$, $[ac]$ and $[bc]$ we must always have

$$f[bc] = \pm[aa],$$

and, by the same reasoning,

$$f[ab] = f[ac] = \pm[aa].$$

Therefore, from (1), (2), (7) and (10), we must also have

(13) $$\left\{\begin{aligned}&[au]^2 = [bu]^2 = [cu]^2 = A\,[aa]\,\epsilon^2\\ &\quad = A \times \text{mean value of } [au]^2 = \text{\&c.}\end{aligned}\right.$$

and

(14)

$$\left\{\begin{aligned}&[au][bu] = \pm[au][cu] = \pm[bu][cu] = \pm A[aa]\,\epsilon^2\\ &\quad = \pm\frac{A\,[aa]}{[ab]} \times \text{mean value of } [au][bu] = \text{\&c.}\end{aligned}\right.$$

* It is easily seen that we can not adopt the following assumption:

(7)′ $[au]\,[bu] = A\,[ab]\,\epsilon^2$

(8)′ $[au]\,[cu] = A'[ac]\,\epsilon^2$

(9)′ $[bu]\,[cu] = A''[bc]\,\epsilon^2$

as that of GAUSS. For let us consider the following particular case:

a	b	c	
+1	+1	+1	$[ab] = \frac{1}{2}[aa]$
+1	+1	+1	$[ac] = \frac{1}{2}[aa]$
+1	+1	−1	$[bc] = 0$
−1	+1	−1	

then since $[bc] = 0$

from (9)′ we have $[cu] = 0$

therefore " (8)′ " " $A' = 0$

" " (3) " " $B' = 0$

" " (1) " " $[au] = 0$

" " (7)′ " " $A = 0$

" " (1) " " $B = 0$

" " (2) " " $[bu] = 0$

Thus, in order that the hypotheses (7)′, (8)′, (9)′ shall be true, we must have $[au] = [bu] = [cu] = 0$ which can not be accepted.

which show that the ratios of $[au]^2$, &c., to their mean values are not the same as those of $[au]\,[bu]$, &c., to the corresponding mean values. The ambiguous signs of the expression of $[au]\,[bu]$ (for instance) may be determined by the following consideration. Let us transform the expression

$$\frac{[bu]}{[au]}$$

by substituting mean values for true ones, as follows:

$$\frac{[bu]}{[au]} = \frac{[bu]\,[au]}{[au]^2} = \frac{[ab]\,\epsilon^2}{[aa]\,\epsilon^2} = \frac{[ab]}{[aa]} \qquad (15)$$

$$\frac{[bu]}{[au]} = \frac{[bu]^2}{[au]\,[bu]} = \frac{[bb]\,\epsilon^2}{[ab]\,\epsilon^2} = \frac{[aa]}{[ab]} \qquad (16)$$

then comparing both the results it is evident that the result (15) generally gives a less value, and the result (16) a larger one, showing again that the ratios of the different functions of the errors to the corresponding mean values are not necessarily constant; but each of them determines the sign of

$$\frac{[bu]}{[au]}$$

to be identical with that of $[ab]$, and the geometrical mean of them is equal to unity; therefore we have probably

$$\frac{[bu]}{[au]} = \frac{[ab]}{[ab]_0} \qquad (17)$$

or

$$[au]\,[bu] = \frac{A\,[ab]\,[aa]\,\epsilon^2}{[ab]_0} \qquad (18)$$

where $[ab]_0$ denotes the numerical value of $[ab]$. Now, since the above results come from the supposition that

$$[aa] = [bb] = [cc]$$

to apply them for the practical purpose we must first transform the observation-equations so that the transformed equations have the relation

$$[aa] = [bb] = [cc]$$

as may be easily effected in the following manner.

4. *Transformation of the Observation-Equations.* If the series of the coefficients $a, b, c, \ldots\ldots$ of the unknown quantities $x, y, z, \ldots\ldots$ in the observation-equations be multiplied by constant quantities $L_1, L_2, L_3, \ldots\ldots$ such that

$$[aa]\,L_1^2 = [bb]\,L_2^2 = [cc]\,L_3^2 = \ldots\ldots$$

and, at the same time, the unknown quantities be divided by the same constants $L_1, L_2, L_3, \ldots\ldots$ respectively, then it is evident that the new values of $[vv]$, $[uu]$, and of $[au]\Delta x$, $[bu]\Delta y$, $[cu]\Delta z$, $\ldots\ldots$, are the same as those

of the original equations, and the new values of $[aa]$, $[bb]$, $[cc]$, ... now become $[aa]L_1^2$, $[bb]L_2^2$, $[cc]L_3^2$, ..., all of which are equal. In the following article it is to be understood that we have always treated of such transformed equations, though their notations are not distinguished between the new and the old equations.

Now, before we consider the expression

$$[au]\Delta x + [bu]\Delta y + [cu]\Delta z + \ldots.$$

in the general case, it is more convenient to first treat the case of two unknown quantities; for it is very simple and the results of its discussion may be extended to the general case almost by similar considerations.

5. *Determination of the Mean Error of an Observation and Some Likely Errors of the Most Probable Values of the Unknown Quantities when the Observation-Equations contain Two Unknown Quantities x and y.* By solving

$$(19)\qquad \begin{aligned}[aa]\Delta x + [ab]\Delta y &= [au]\\ [ab]\Delta x + [bb]\Delta y &= [bu]\end{aligned}$$

we have

$$(20)\qquad \Delta x = \frac{[au][bb]-[bu][ab]}{[aa][bb]-[ab]^2}$$

and therefore

$$(20')\qquad [au]\Delta x = \frac{[au]^2\left\{[aa]-\frac{[bu]}{[au]}[ab]\right\}}{[aa]^2-[ab]^2}$$

which, being transformed by the equations (17) and (13), becomes

$$(21)\qquad [au]\Delta x = \frac{[au]^2\{[aa]-[ab]_0\}}{[aa]^2-[ab]^2}$$

$$(22)\qquad = \frac{A\epsilon^2}{1+\frac{[ab]_0}{[aa]}}$$

By the same reasoning we have the same expression of $[bu]\Delta y$, and therefore we have

$$(23)\qquad \epsilon^2 = \frac{[vv]}{m-\frac{2A}{1+\frac{[ab]_0}{[aa]}}}$$

Now from this result it is to be observed that if we put $A=1$ and $[ab]_0 = 0$ then the equation (23) is identical with that of GAUSS; and as the value of $[ab]_0$ increases, the denominator of ϵ^2 gradually increases; when the value of $[ab]_0$ becomes a maximum (for instance the case in which $a'=b'$, $a''=b''$, $a'''=b'''$, &c.), it becomes the same form as that in the case of the single unknown quantity, which coincides with the practical condition since in that case $x+y$ may be considered as the single unknown quantity.

I shall now consider some likely values of Δx and Δy. Let us first consider the rigorous expressions of Δx, Δy, $\Delta x + \Delta y$, as follows:

$$\Delta x = \frac{[au][bb]-[bu][ab]}{[aa]^2-[ab]^2} \qquad (24)$$

$$\Delta y = \frac{[bu][aa]-[au][ab]}{[aa]^2-[ab]^2} \qquad (24)'$$

$$\Delta x + \Delta y = \frac{[au]+[bu]}{[aa]+[ab]} \qquad (24)''$$

Now, since $[aa]=[bb]$, it is quite probable that the first terms of the expressions of Δx and Δy are larger than the second terms of them respectively, and therefore the signs of Δx and Δy must be the same as those of $[au]$ and $[bu]$ respectively; but when the value of $[ab]$ is positive and not small, it being quite probable that $[au]$ and $[bu]$ have the same signs, Δx and Δy must then have the same signs, so that they vary with $\Delta x + \Delta y$; therefore, since it is evident from the above equation (24)″ that $\Delta x + \Delta y$ does not increase infinitely as the value of $[ab]$ increases, the values of Δx and Δy must also be so; this evidently proves that

$$\text{mean error of } x = \frac{\sqrt{[aa]}\,\epsilon}{\sqrt{[aa]^2-[ab]^2}}$$

$$\text{mean error of } y = \frac{\sqrt{[bb]}\,\epsilon}{\sqrt{[aa]^2-[ab]^2}}$$

are not approximate values of Δx and Δy respectively, for they gradually increase as the value of $[ab]$ increases, and finally become infinity when $[ab]=[aa]$. Indeed, the most likely values of Δx and Δy will be obtained in the following manner; viz., from the equation [21] we have immediately

$$\Delta x = \frac{[au]\{[aa]-[ab]_0\}}{[aa]^2-[ab]^2} \qquad (25)$$

$$= \frac{[au]}{[aa]+[ab]_0} \qquad (26)$$

where $[au] = \sqrt{A[aa]}\,\epsilon$. By the same reasoning

$$\Delta y = \frac{[bu]}{[aa]+[ab]_0} \qquad (27)$$

Therefore, adding (26) to (27) we have

$$\Delta x + \Delta y = \frac{[au]+[bu]}{[aa]+[ab]_0} \qquad (28)$$

which is identical with the rigorous equation (24)″. It is also to be noticed that by comparing the value of Δx in the equation (26) with the mean value of it we have

$$(\Delta x)^2 = \frac{([aa]-[ab]_0)\,A}{[aa]+[ab]_0} \times \text{mean value of } (\Delta x)^2$$

which shows that the ratio of $(\Delta x)^2$ to its mean value is a function of $[ab]_0$.

I have, now, to consider the general case.

6. *Determination of the Mean Error of an Observation and Likely Errors of the most Probable Values of the Unknown Quantities when the Observation-Equations contain any number of the Unknown Quantities* $x, y, z, \ldots$. Let

D = the determinant formed from all of the coefficients of the unknown quantities in the normal equations.

$D_a, D_b, D_c, \ldots$ = the minors corresponding to the constituents $[aa], [bb], [cc], \ldots$ respectively.

$D_{ab}, D_{ac}, \ldots$ = the minors corresponding to the constituents $[ab], [ac], \ldots$

then since

$$(33)\qquad D\Delta x = [au]D_a + [bu]D_{ab} + [cu]D_{ac} + \ldots$$

we have

$$(34)\qquad [au]\Delta x = \frac{[au]^2}{D}\left\{D_a + \frac{[bu]}{[au]}D_{ab} + \frac{[cu]}{[au]}D_{ac} + \ldots\right\}$$

but since, by (17)

$$\frac{[bu]}{[au]} = \frac{[ab]}{[ab]_0}$$

and similarly

$$\frac{[cu]}{[au]} = \frac{[ac]}{[ac]_0}$$

we have

$$(35)\qquad [au]\Delta x = \frac{[au]^2}{D}\left\{D_a + \frac{[ab]D_{ab}}{[ab]_0} + \frac{[ac]D_{ac}}{[ac]_0} + \ldots\right\}$$

By the same reasoning

$$(36)\qquad [bu]\Delta y = \frac{[bu]^2}{D}\left\{\frac{[ab]D_{ab}}{[ab]_0} + D_b + \frac{[bc]D_{bc}}{[bc]_0} + \ldots\right\}$$

. . . .

Therefore, if we put

$$(37)\qquad \Sigma D_a = D_a + D_b + D_c + \ldots$$

$$(38)\qquad \sum\frac{[ab]D_{ab}}{[ab]_0} = \frac{[ab]D_{ab}}{[ab]_0} + \frac{[ac]D_{ac}}{[ac]_0} + \ldots + \frac{[bc]D_{bc}}{[bc]_0} + \ldots$$

we have

$$(39)\qquad [au]\Delta x + [bu]\Delta y + [cu]\Delta z + \ldots = \frac{[au]^2}{D}\left\{\sum D_a + 2\sum\frac{[ab]D_{ab}}{[ab]_0}\right\}$$

or

$$(40)\qquad \epsilon^2 = \frac{[vv]}{m - \dfrac{A[aa]}{D}\left\{\sum D_a + 2\sum\dfrac{[ab]D_{ab}}{[ab]_0}\right\}}$$

This equation gives the best value of the mean error of the single observation. But for the practical purpose we must find some approximate formula.

Now, since the values of $[ab]_0, [ac]_0, \ldots$ are usually small in comparison with $[aa], [bb], \ldots$ we may approximately substitute the mean of $[ab]_0, [ac]_0, [ad]_0, \ldots [bc]_0, [bd]_0, \ldots$ for each of them involved in the equations (35), (36), ...; then since

$$D = [aa]D_a + [ab]D_{ab} + [ac]D_{ac} + \ldots \qquad (41)$$

comparing this with (35) we have, nearly,

$$[au]\Delta x = \frac{[au]^2}{D}\left\{D_a + \frac{D - [aa]D_a}{\lambda^2}\right\}$$

$$= \frac{[au]^2\{D - (\alpha^2 - \lambda^2)D_a\}}{D\lambda^2} \qquad (42)$$

where

$\alpha^2 = [aa] = [bb] = \ldots$

λ^2 = the mean of $[ab]_0, [ac]_0, \ldots [bc]_0, [bd]_0, \ldots$

It is to be noticed that though the formula (42) was deduced from the supposition that the values of $[ab]_0$, $[ac]_0$, &c., are small, yet it is well satisfied to the particular case in which the values of $[ab], [ac]$, &c., are equal to each other, even if they are large.

Now since, accurately to the second order of $\frac{\lambda^2}{\alpha^2}$ in the expression of D, we may put

$$D = \begin{vmatrix} \alpha^2 & \lambda^2 & \lambda^2 & \ldots \\ \lambda^2 & \alpha^2 & \lambda^2 & \ldots \\ \lambda^2 & \lambda^2 & \alpha^2 & \ldots \\ \vdots & \vdots & \vdots & \end{vmatrix} \qquad (43)$$

by the application of the following theorem:

$$D = \phi(\lambda^2) - \lambda^2\frac{\partial\phi}{\partial(\lambda^2)}$$

where

$$\phi(\lambda^2) = (\alpha^2 - \lambda^2)^\mu$$

we have

$$D = (\alpha^2 - \lambda^2)^{\mu-1}\{\alpha^2 + (\mu-1)\lambda^2\}$$

$$D_a = D_b = D_c = \ldots = (\alpha^2 - \lambda^2)^{\mu-2}\{\alpha^2 + (\mu-2)\lambda^2\}$$

Substituting these, (42) becomes:

$$[au]\Delta x = \qquad (44)$$

$$\frac{[au]^2\{(\alpha^2-\lambda^2)^{\mu-1}[\alpha^2+(\mu-1)\lambda^2] - (\alpha^2-\lambda^2)^{\mu-1}[\alpha^2+(\mu-2)\lambda^2]\}}{\lambda^2(\alpha^2-\lambda^2)^{\mu-1}\{\alpha^2+(\mu-1)\lambda^2\}}$$

$$= \frac{A\epsilon^2}{1+(\mu-1)\dfrac{\lambda^2}{\alpha^2}} \qquad (45)$$

and therefore

$$\epsilon^2 = \frac{[vv]}{m - \dfrac{A\mu}{1+(\mu-1)\dfrac{\lambda^2}{\alpha^2}}} \qquad (46)$$

which shows that the value of ϵ generally increases with

that of μ, except that, when $\lambda^2 = \alpha^2$, they become independent of each other.

Now, the best values of Δx, Δy, ... must be found from (35), (36), ...; but their practical values may be easily found from the formula (44), and similar ones, as follows:

(47)

$$\Delta x = \frac{[au]}{\alpha^2 + (\mu - 1)\lambda^2}$$

(48)

$$(\Delta x)^2 = \frac{\left(1 - \frac{\lambda^2}{\alpha^2}\right) A}{\left\{1 + (\mu-1)\frac{\lambda^2}{\alpha^2}\right\}\left\{1 + (\mu-2)\frac{\lambda^2}{\alpha^2}\right\}} \times \left\{ \begin{matrix} \text{mean value} \\ \text{of } (\Delta x)^2 \end{matrix} \right.$$

. . . .

therefore

(49)
$$\Delta x\, \Delta y = \frac{[au]\,[bu]}{\{\alpha^2 + (\mu-1)\lambda^2\}^2}$$

. . . .

I shall now give a few theorems which may be easily proved from the results of the above discussion.

A. All of the expressions $[au]\,\Delta x$, $[bu]\,\Delta y$, $[cu]\,\Delta z$, ... have positive signs.

This may be evidently seen from the form of the equation (45), which is the approximate value of the second member of (34).

B. $\Delta x \Delta y$, $\Delta x \Delta z$, ... have the same signs with $[au]\,[bu]$, $[au]\,[cu]$, ... respectively.

This immediately follows from the theorem *A.* It is also evident from (49).

C. $\Delta x \Delta y$, $\Delta x \Delta z$, ... have the same signs with $[ab]$, $[ac]$, ... respectively.

This immediately follows from the theorem *B* and the following hypothesis:

"$[au]\,[bu]$, $[au]\,[cu]$, ... have the same signs with $[ab]$, $[ac]$, ... respectively."

which is quite probable when the numerical values of $[ab]$, $[ac]$, ... are large.

D. All the terms of the equation $[au] = [aa]\,\Delta x + [ab]\,\Delta y + [ac]\,\Delta z + \ldots.$ have the same signs.

This may be easily proved by multiplying the equation by Δx, and comparing every term of it with the theorem *C.* As a corollary of the present theorem we may say that:

If, whilst the values of $[aa]$ and $[au]$, $[bu]$, ... are put in unchanged, the numerical values of $[ab]$, $[ac]$, ... be increased then those of Δx, Δy, Δz, ... are decreased.

E. If we have $a' = \pm b' = \pm c' = \ldots.$, $a'' = \pm b'' = \pm c'' = \ldots.$, $a''' = \pm b''' = \pm c''' = \ldots.$, &c., then the value of $[au]\,\Delta x + [bu]\,\Delta y + [cu]\,\Delta z + \ldots.$ becomes the same as that in the case of one unknown quantity.

This is evident from the equation (45).

F. The equations

$$[au] = [aa]\,\Delta x + [ab]\,\Delta y + [ac]\,\Delta z + \ldots.$$
$$[bu] = [ab]\,\Delta x + [bb]\,\Delta y + [bc]\,\Delta z + \ldots.$$

. . . .

are just satisfied by my values of Δx, Δy, Δz, ..., but not by the mean values of them.

It is also important to compare the following results:

$$\text{mean value of } (\Delta x)^2 = \frac{D_a}{D}\,\epsilon^2 \qquad (a)$$

$$\Delta x = \frac{[au]}{\lambda^2}\left\{1 - (\alpha^2 - \lambda^2)\frac{D_a}{D}\right\} \text{(from (42))} \qquad (b)$$

For clearness of thought, let us take the following example; viz., let there be given observation-equations involving any number of unknown quantities in which the approximate value w_0 of the unknown quantity w is known, and they be solved by supposing that

1. $w = w_0$
2. $w = w_0 + \Delta w$

then from (*a*) we can not say that, even if the value of ϵ^2 in the second solution be less than that in the first, the mean value of $(\Delta x)^2$ in the second solution is less than that in the first (see my theorem in the *A.J.*, No. 521); but from (*b*) we may say that, if the values of λ^2s and of ϵs in both solutions be equal, the value of Δx in the second solution is generally less than that in the first.

We must now determine the value of *A.* But this being, indeed, a very vague problem, so that we can not now find any reliable value of it, we are compelled still to adopt "$A = 1$," which agrees with the theory of the mean errors.

Finally, it is to be remarked that, since we have hitherto supposed that the observation-equations have been transformed to have the relation $[aa] = [bb] = [cc = \ldots.$, for instance, to apply the formula (46) for the original equations we must understand that

$$\frac{\lambda^2}{\alpha^2} = \text{the mean of } \frac{[ab]_0}{\sqrt{\{[aa]\,[bb]\}}}\,,\ \frac{[ac]_0}{\sqrt{\{[aacccc]\}}}\,,\ \ldots.$$

Tokyo Astronomical Observatory, 1902 October.

OBSERVATIONS OF COMET 1900 II,

MADE AT THE U.S. NAVAL OBSERVATORY,

BY GEORGE K. LAWTON.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1900 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
d h m s			m s	′ ″	h m s	° ′ ″			s	″
July *27 14 44 31.7	1	25, 5	−0 56.50	+ 0 1.2	2 47 48.01	+24 13 38.1	n9.617	0.535	+3.02	+ 7.4
27 15 13 46.3	1	25, 5	−0 55.60	+ 3 36.7	2 47 48.91	+24 17 13.6	9.574	0.497	+3.02	+ 7.4
30 15 4 25.0	2	25, 5	−2 0.74	+ 3 6.8	2 51 29.75	+33 29 54.7	n9.614	0.317	+3.28	+ 4.4
31 15 24 41.9	3	25, 5	+3 11.26	+ 5 54.8	2 52 56.13	+36 38 38.1	n9.588	0.147	+3.40	+ 3.7
Aug. 1 15 35 30.5	4	20, 4	+2 3.32	+ 4 30.8	2 54 27.45	+39 44 50.4	n9.578	9.921	+3.50	+ 2.6
3 15 48 16.8	5	19, 4	−3 56.47	− 4 16.7	2 57 54.13	+45 49 30.7	n9.577	n9.310	+3.73	+ 0.2
5 15 41 29.6	6	25, 5	−3 43.28	− 1 59.4	3 1 58.07	+51 36 47.6	n9.635	n9.982	+4.03	− 1.5
6 14 23 22.3	7	25, 5	−1 25.56	− 7 55.7	3 4 10.88	+54 14 41.5	n9.808	7.598	+4.21	− 2.2
7 15 18 21.2	8	25, 5	+2 49.70	+ 0 36.3	3 6 51.43	+57 2 26.1	n9.742	n0.138	+4.42	− 2.7
10 14 54 4.7	9	20, 4	+3 4.90	−12 1.0	3 16 30.58	+64 27 47.0	9.887	n0.283	+5.12	− 4.9
17 11 0 34.0	10	20, 4	−3 45.09	+ 3 12.1	4 1 58.81	+77 52 44.2	n0.313	0.527	+7.77	−11.0
17 12 23 52.0	10	19, 4	−3 0.81	+ 9 25.3	4 2 43.09	+77 58 57.4	n0.344	9.993	+7.77	−11.0
28 12 34 40.5	11	17, 17	−0 50.9	− 2 6.2	11 14 12.5	+84 43 23.3	9.401	0.865	−9.9	−14.4
30 13 17 17.7	12	14, 8	+3 50.4	− 6 28.2	12 11 16.3	+83 13 52.2	9.484	0.871	−9.0	−10.5
Sept. 1 11 16 28.4	13	4, 4	−0 37.7	− 4 9.7	12 42 55.1	+82 5 24.0	0.336	0.771	−8.1	− 7.6
2 11 52 51.8	14	19, 4	−3 17.4	− 2 42.8	12 55 14.6	+81 21 59.4	0.228	0.810	−7.4	− 6.4
*23 11 41 59.6	15	24, 9	+0 43.18	+ 0 36.7	14 16 42.74	+70 49 37.4	9.914	0.848	−2.73	− 3.1
Oct. 15 11 43 42.7	16	18, 4	−4 59.27	− 2 49.5	14 50 57.93	+66 16 56.4	9.639	0.904	−2.35	− 4.2
17 12 12 10.0	17	15, 6	+5 20.44	+ 2 26.8	14 53 54.22	+66 5 45.8	9.442	0.919	−2.28	− 6.0
19 12 7 25.0	18	16, 7	−1 33.30	+ 4 37.7	14 56 48.17	+65 56 50.6	9.438	0.920	−2.31	− 5.2
20 11 43 7.4	18	†11, 16	−0 7.60	+ 0 55.8	14 58 13.85	+65 53 8.4	9.574	0.911	−2.33	− 5.5

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	2 48 41.49	+24 13 29.5	Berlin B, A.G. 852	10	4 5 36.13	+77 49 43.1	Kasan, A.G. 655
2	2 53 27.21	+33 26 43.5	Leiden, A.G. 1122	11	11 15 13.3	+84 45 43.9	Carrington 1679
3	2 49 41.47	+36 32 39.6	Lund, A.G. 1498	12	12 7 34.9	+83 20 30.9	Carrington 1815
4	2 52 20.63	+39 40 17.0	Lund, A.G. 1517	13	12 43 40.9	+82 9 41.3	Carrington 1897
5	3 1 46.87	+45 53 47.2	Bonn, A.G. 2630	14	12 58 39.4	+81 24 48.6	Carrington 1936
6	3 5 37.32	+51 38 48.5	Camb., U.S., A.G. 1413	15	14 16 2.29	+70 49 3.8	Dorpat, A.G. Zones
7	3 5 32.23	+54 22 39.4	Camb., U.S., A.G. 1411	16	14 55 59.55	+66 19 50.1	Newcomb 951
8	3 3 57.31	+57 1 52.5	Hels.&Gotha, A.G. 2813	17	14 48 36.06	+66 3 25.0	Christiania, A.G. 2215
9	3 13 20.56	+64 39 52.9	Hels.&Gotha, A.G. 2920	18	14 58 23.78	+65 52 18.1	Christiania, A.G. 2235

* The observations from July 27 to Sept. 2 were made with the 12-inch equatorial; those from Sept. 23 to Oct. 20 on the 26-inch equatorial. † Micrometer measurement. NOTE: — Oct. 20, "Exceedingly faint for all measures."

2387 —*GEMINORUM.*

A cable dispatch of March 27, from Dr. KREUTZ, announced the discovery by Prof. TURNER, at Oxford University Observatory, of a new star in the position,

$$\alpha = 6^h\ 37^m\ 48^s \quad , \quad \delta = +30°\ 3'$$

but which might possibly be a variable. Its magnitude on March 16 was 8.0. The cipher words of the dispatch, *trouble valerian*, are construed as meaning that it was not visible on the plates of February 16.

On March 28, telegrams were received from Harvard College Observatory, communicating a dispatch from Prof. HALE, at Yerkes Observatory, giving the position,

$$1903\ \text{Mar.}\ 27.750, \quad \alpha = 6^h\ 37^m\ 49^s \quad , \quad \delta = +30°\ 2'\ 38''$$

and the magnitude as 8.5; and stating that the spectrum contains bright lines or bands; also a dispatch from Capt. CHESTER, Superintendent of Naval Observatory, stating that the star was photographed there on March 27, and that the magnitude was 8.5, color red.

By Prof. BARNARD's note printed on another page of this number, the accurate position, observed at the Yerkes Observatory, is

$$\alpha = 6^h\ 37^m\ 48^s.96 \quad , \quad \delta = +30°\ 2'\ 38''.3 \quad (1900.0)$$

OBSERVATION OF THE POSITION OF TURNER'S "*NOVA*," (2387 — *GEMINORUM*),

By E. E. BARNARD.

The position of this star was observed with the micrometer of the 40-inch on March 27. No comparison-star was available for direct measurement. The comparison-star used (A.G. Camb., Eng., 3482), was too distant in declination for differences of right-ascension. A star of 12^M was therefore used as an intermediate.

$$\Delta\alpha \;\; \textit{Nova} \text{ and } 12^M \text{ star}, \;\; 5' \; 5''.1 = 23^s.49 \;\; (4)$$
$$\Delta\delta \;\; \text{"} \qquad \text{"} \qquad 2 \;\; 16.0 \;\; (3)$$

The 12^M star was s.f.

The small star was then referred to A.G. 3482.

$$\Delta\alpha \;\; 3482 \text{ and } 12^M \text{ star} \;\; 1^m \, 58^s.36 \;\; (12)$$
$$\Delta\delta \;\; \text{"} \qquad \text{"} \qquad 3 \;\; 41.6 \;\; (2)$$

The 12^M star n.p.

The *Nova* was also referred direct to 3482 in declination.

$$\Delta\delta \;\; 5' \, 57''.0 \;\; (2) \;\; \textit{Nova} \text{ north.}$$

These observations give

$$\textit{Nova} - 3482, \quad \Delta\alpha = -2^m \, 21^s.85$$
$$\textit{Nova} - 3482, \quad \Delta\delta = +5' \, 57''.3$$

Yerkes Observatory, Williams Bay, Wis., 1903 *March* 28.

The place of the comparison-star (A.G., Camb., Eng. 3482) is

$$1903.0 \quad \alpha = 6^h \, 40^m \, 22^s.31 \quad \delta = +29^\circ \, 56' \, 30.7$$

This gives the place of the *Nova*,

1903.0	$6^h \, 38^m \, 0^s.46$	$+30^\circ \, 2' \, 28''.0$
1900.0	6 37 48.96	+30 2 38.3

At $11^h \, 0^m$, a direct estimate made the *Nova* $8^M.2$. It was estimated to be $\frac{2}{10}$ of a magnitude less than the comparison-star, which is given as $8^M.2$ in DM. This would make the *Nova* $8^M.4$ on the DM. scale. It was of a strong red color.

Careful tests were made for focus with a power of 700 diameters. The results are

Focus for *Nova*,	2.12 inches	(4 obs.)
Focus for 8^M star,	2.10 "	(6 obs.)
Focus, *Nova* — Star =	+0.02 inch.	

The difference is too small to mean anything, though it is in the right direction for a *Nova*.

NOTES ON VARIABLE STARS, — No. 37,

By HENRY M. PARKHURST.

Approximate Maxima. When from scarcity of decisive observations it becomes necessary to substitute the maximum from the elements for the observed maximum, as indicated by E in the column of weights, there may yet be an indication from the light-curve shown by the observations, of an approximate maximum, which may be given by its Julian date in the column headed "Mag.," and thus either confirm the elements or suggest their approximate correction.

Results of Observations.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	W.	Mag.	Factors			Remarks
				1902-3								
6888	*RW Sagittarii*	–	–	–	–	–	–	–	–	–	–	No period ascertained
6892	*RX Sagittarii*	Max.	6018	Sept. 15	6	–	E	6045 :	–	–	–	320ᵈ *A.J.* 456
6900	*W Aquilae*	Max.	6041	Oct. 18	7	–	E	–	–	–	–	*A.J.* 393
7118	*X Aquilae*	Min.	6066	Nov. 12	10	–	E	–	–	–	–	*A.J.* 347
7155	*RR Aquilae*	Max.	6096	Dec. 12	7	–	5	–	–	–	–	Approximate period 389ᵈ
7162	*RS Aquilae*	–	–	–	–	–	–	–	–	–	–	Period about 400ᵈ probably
7242	*S Aquilae*	Max.	6049	Oct. 26	92	–	E	–	–	–	–	Correct'n apparently increas'g
7244	*RW Aquilae*	Max.	6114	Dec. 30	194	– 1	6	–	–	–	–	Period 7.87. *A.J.* 490
7261	*R Delphini*	Max.	6113	Dec. 29	48	–66	6	7.9	–	–	–	Shortening of period confirmed
(7416)	– 19°5892	–	–	–	–	–	–	–	–	–	–	Third catalogue, supplement
7458	*V Delphini*	Max.	6042	Oct. 19	8	–	E	6107 :	–	–	–	Probably much later
(7484)	– *Cygni*	–	–	–	–	–	–	–	–	–	–	Third catalogue, supplement
(8104)	– *Aquarii*	–	–	–	–	–	–	–	–	–	–	" " "
(8106)	– *Pegasi*	–	–	–	–	–	–	–	–	–	–	" " "
7896	*V Pegasi*	Max.	6118	Jan. 3	9	–	E	6089 :	–	–	–	Probably much earlier
8230	*S Aquarii*	Max.	6058	Nov. 4	56	–	E	6058 :	–	–	–	
8290	*R Pegasi*	Max.	6106	Dec. 22	50	– 2	6	10.1	–	–	–	Correction probably too small
8373	*S Pegasi*	Max.	6161	Feb. 15	48	–19	5	–	–	–	–	Low in west
8512	*R Aquarii*	Max.	6127	Jan. 12	86	–16	9	6.14	1.22	0.91	16ᵈ	
8562	*Z Aquarii*	Max.	4984	Nov. 25	2	–	2	6095 :	–	–	–	1899
8622	*W Ceti*	Max.	6122	Jan. 7	7	+72	7	–	–	–	–	Sky very uncertain

Individual Observations.

Including Observations by Arthur C. Perry.

6888 *RW Sagittarii.*
(Cont. from 456. Comp. Stars 421)

Julian	Calendar	Mag.
	1899	
4843.6	July 7	10.11
4867.6	31	9.4P
	1900	
5228.6	July 27	10.0
5246.6	Aug. 14	9.6
5257.6	25	9.7
5280.5	Sept. 17	9.7
5312.5	Oct. 19	9.4
5318.5	25	9.2
	1901	
5609.6	Aug. 12	10.0
5673.5	Oct. 15	9.2
	Range of 7 dates	
	1902	
6082.5	Nov. 28	9.7
6085.5	Dec. 1	9.6

6892 *RX Sagittarii.*
(Cont. from 456. Comp. Stars 421)

Julian	Calendar	Mag.
	1899	
4843.6	July 7	13]
4928.6	Sept. 30	12]P
	1900	
5228.6	July 27	to
5318.5	Oct. 25	13]
	6 dates	
	1901	
5609	Aug. 12	to
5664	Oct. 6	13]
	6 dates	
	1902	
6082.5	Nov. 28	10.0
6085.5	Dec. 1	9.8

6900 *W Aquilae.*
(Cont. from 456. Comp. Stars 339)

Julian	Calendar	Mag.
	1901	
5609.6	Aug. 12	10.9
5631.6	Sept. 3	11.0
5634.6	6	11.2
5643.6	15	11.4
5665.6	Oct. 7	13]
	1902	
6089	Dec. 5	9.8:
6102	18	9]

7118 *X Aquilae.*
(Cont. from 490. Comp Stars 339)

Julian	Calendar	Mag.
	1902	
6082	Nov. 28	12.1]
6102	Dec. 18	11.8]

7155 *RR Aquilae.*
(Cont. from 490. Comp. Stars 393)

Julian	Calendar	Mag.
	1901	
5609.5	Aug. 12	12]
5632.5	Sept. 4	12]
5643.5	15	12]
5664.5	Oct. 6	9.1
5665.5	7	9.00_2
5668.5	10	8.77_2
5671.5	13	8.85_2
5677.5	19	8.91_2
	1902	
6082.5	Nov. 28	8.0
6085.5	Dec. 1	8.2
6089.5	5	8.2
6092.5	8	8.2
6101.5	17	8.1
6102.5	18	8.0
6107.5	23	8.1
6111.5	27	8.2
6114.5	30	8.2

7162 *RS Aquilae,*
(Cont. from 490. Comp. Stars 464)

Julian	Calendar	Mag.
	1901	
5607.6	Aug. 10	to
5664.5	Oct. 6	13]
	4 dates	
	1902	
6082	Nov. 28	to
6102	Dec. 18	13]
	3 dates	

7242 *S Aquilae.*
(Cont. from 425. Comp. Stars 464)

Julian	Calendar	Mag.
	1902	
6085	Dec. 1	9.3
6102	18	9.4

7244 *RW Aquilae.*
(Cont. from 490. Comp. Stars 464)

Julian	Calendar	Mag.
	1902	
6085.5	Dec. 1	8.7
6090.5	6	8.6
6091.5	7	8.8
6092.5	8	8.6
6093.5	9	8.7
6101.5	17	8.7
6102.5	18	8.9
6103.5	19	8.8
6107.5	23	8.9
6108.5	24	9.1
6111.5	27	8.7
6112.5	28	8.6
6114.5	30	8.6
6115.5	31	8.8

7261 *R Delphini.*
(Continued from 498.)

Julian	Calendar	Mag.
	1901	
5585	July 19	9.7
5604	Aug. 7	11.2
	1902	
6085	Dec. 1	8.9
6089	5	8.6
6092	8	8.8
6101	17	8.4
6102	18	8.7
6107	23	8.8
6111	27	7.9
6115	31	8.1
	1903	
6121	Jan. 6	8.7

(S.DM)
(7416) —19°5892.

Julian	Calendar	Mag.
	1896	
3777	Aug. 5	8.9
3809	Sept. 6	9.0
3860	Oct. 27	8.7
3897	Dec. 3	8.7
	1897	
4111	July 5	8.8
4114	8	8.9
4144	Aug. 7	8.63_2
4166	29	9.08_2
4215	Oct. 17	8.97_2
4256	Nov. 27	8.82_2
	1898	
4587	Oct. 24	8.92

7458 *V Delphini.*
(Cont. from 482. Comp. Stars 311)

Julian	Calendar	Mag.
	1902	
6085	Dec. 1	10.7]
6102	18	10.7]
6107	23	10.0:
6111	27	11.5

(7484) — *Cygni.*

Julian	Calendar	Mag.
	1896	
3776	Aug. 4	to
	1897	
3948	Jan. 23	12.5 ±.3
	10 dates	
	1897	
4078	June 2	to
4283	Dec. 24	12.5 ±.3
	5 dates	

(7484) — *Cygni.*—Cont.

Julian	Calendar	Mag.
	1898	
4579	Oct. 16	to
4639	Dec. 15	12.5 ±.3
	3 dates	

7896 *V Pegasi.*
(Continued from 482.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	9.8
6101	17	9.8]
6111	27	11.3]

(8104) — *Aquarii.*
(Continued from 372.)

Julian	Calendar	Mag.
	1896	
3813	Sept. 10	12.2
3870	Nov. 6	12.5
	1897	
4138	Aug. 1	12.2
4257	Nov. 27	12.3
	1898	
4545	Sept. 12	12.0

(8106) — *Pegasi.*
(Continued from 372.)

Same dates as (8104)
No perceptible change

8230 *S Aquarii.*
(Continued from 487.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	9.2
6111	27	10.5
	1903	
6123	Jan. 8	10.7]

8290 *R Pegasi.*
(Continued from 487.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	10.1
6103	19	10.4
6107	23	9.9
6114	30	10.6

8373 *S Pegasi.*
(Cont. from 487. Comp. Stars 464)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	11.5
6103	19	9.9
6107	23	9.8
6114	30	9.7
	1903	
6123	Jan. 8	9.5
6128	13	9.3

8373 *S Pegasi.*—Cont.

Julian	Calendar	Mag.
	1903	
6132	Jan. 17	9.3
6136	21	9.3
6143	28	8.9
6145	30	8.7
6151	Feb. 5	8.5
6158	12	7.93_2
6160	14	7.76_2
6168	22	8.4

8512 *R Aquarii.*
(Continued from 498.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	7.9
6092	8	7.6
6101	17	7.4
6102	18	8.1
6107	23	7.7
6114	30	6.8
	1903	
6123	Jan. 8	6.2
6127	12	6.0
6128	13	5.8
6132	17	6.78
6136	21	6.76_2
6141	26	6.8

8562 *Z Aquarii.*

Julian	Calendar	Mag.
	1899	
4984.6	Nov. 25	8.1P
	1902	
6101	Dec. 17	8.5
6102	18	8.8
6107	23	8.4
6114	30	9.0
	1903	
6123	Jan. 8	9.7
6127	12	9.8
6128	13	9.6
6136	21	10]

8622 *W Ceti.*
(Continued from 498.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	8.3
6101	17	9.0
6102	18	7.8
6107	23	7.8
6114	30	7.8
	1903	
6123	Jan. 8	7.8
6128	13	7.4
6132	17	7.91
6136	21	8.49
6141	26	8.7

CONTENTS.

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DETERMINATION OF ABSOLUTE MAGNITUDE-EQUATION FOR THE CATALOGUE OF 627 STANDARD STARS (*A. J.* 531–2),

By LEWIS BOSS.

In the computation for the right-ascensions of the Catalogue of 627 Standard Stars the numerical data for each star occupy a single sheet. Upon each is entered at the outset, in the chronological order of catalogues, the comparison, O—C, of each catalogue of observation with a previously assumed right-ascension and annual motion. This is the permanent basis of the subsequent computations. In succeeding columns are arranged the residuals corrected for systematic errors of the form, $\Delta\alpha_a$ and $\Delta\alpha_\delta$, in the successive approximations. The last column on each star-sheet contains the final residuals, C—O, resulting from the definitive computations for the Catalogue of Standard Stars. These residuals are freed from the effect of $\Delta\alpha_a$ and $\Delta\alpha_\delta$ according to the values of these quantities adopted in the final computation, — values which are, in all cases, close approximations to those which have been finally computed subsequently to the formation of the definitive right-ascensions. It is evident, therefore, that these residuals are well suited for the accurate discussion of any type of systematic correction which it is desired to investigate independently of $\Delta\alpha_a$ and $\Delta\alpha_\delta$.

Throughout the computations for the catalogue it had seemed to me altogether probable that the subject of magnitude-equation is not yet amenable to precise formulation and definite numerical treatment. But when the operations for deducing the finally adopted values of $\Delta\alpha_a$ and $\Delta\alpha_\delta$ had been completed, it occurred to me that certain points of interest relative to the magnitude-effect ought not to be neglected. Abundant well tested material for such discussions was already available in a perfectly convenient form; and no relatively great loss of labor would result in case the investigation should prove to be barren of definite conclusions.

Once entered upon, the course of the investigation followed almost without volition on the part of the computer, and without reference to his preconceived notions. Therefore it is quite natural that the results should be presented in the order in which they were obtained. This course is especially appropriate since each conclusion stands as it was first produced and without subsequent revision.

Description of Table I.

The entire investigation derives its material of observation from Table I; therefore, for a clear comprehension of what follows, it will be necessary to understand the construction of this table. All the "star-sheets" were arranged in the order of magnitude in three zones, as follows:

Zone	Limits
I	$+60^\circ$ to $+30^\circ$
II	$+29$ to -22
III	-23 to -50

The adopted magnitudes are those of the Harvard Photometry.

Then in each zone the separate residuals, C—O, for each catalogue are collected in groups falling as nearly as may be upon an even magnitude. Zone II (the equatorial zone) is readily recognized in the tables through the group of eight stars of mean, $0^{\mathrm{M}}.9$. It is in the second column for observatories having northern latitude and in the first column for those having southern latitude. Each of the mean residuals is a statement of the mean observed correction given by the Standard Catalogue to the catalogue of observation in question, as a result of the mean by weights of all the residuals contained in the given zone for the particular group of magnitudes to which it refers.

Since very closely approximate values of $\Delta\alpha_a$ and $\Delta\alpha_\delta$ were employed in the derivation of the final residuals upon the star-sheets, and since the means of Table I involve all hours of right-ascension and a wide range of declination, it is evident that any outstanding errors which could be attributed to defective knowledge of $\Delta\alpha_a$ and $\Delta\alpha_\delta$ are practically of no importance at all, except possibly in a few instances where the number of stars involved in the mean is very small. We may, therefore, assume that Table I exhibits the effect of differences of magnitude-equation combined with casual errors of observation. For each zone

is given in the first column the number of stars participating in the respective means, and in the second column the combined weight upon a unit of which the probable error is $\pm 0^s.01$. Where the probable error is supposed to be greater than $\pm 0^s.015$ the weight is suppressed.

To prepare the mean residuals for examination it was decided to take as the zero of reference, $3^M.5$, and to establish this through the arithmetical mean of the groups for $3^M.0$ and $4^M.0$. This procedure is illustrated by the following example in which the residuals for Zone II in the case of Cordoba 1875 are treated.

Mag.	**	Wt.	$\Delta'\alpha_m$	$\Delta\alpha_m$
$0^M.9$	8	7	$+0^s.006$	$+0^s.008$
2.0	13	9	+0.003	+0.005
3.0	41	30	−0.001	+0.001
4.0	66	30	−0.003	−0.001
5.0	50	17	−0.004	−0.002

The numbers in the column, $\Delta'\alpha_m$, are the mean residuals as they were directly computed from the separate residuals collected from the star-sheets. The mean for the groups, $3^M.0$ and $4^M.0$, is $-0^s.002$. Adding to the numbers in column, $\Delta'\alpha_m$, $+0^s.002$, so as to make the sum of the residuals for $3^M.0$ and $4^M.0$ zero, we derive the adopted residuals in the last column, $\Delta\alpha_m$, which are thus supposed to be referred to $3^M.5$ as the zero-origin. In precisely this manner, without exception, all of the residuals for each of the zones were referred to the zero-origin, $3^M.5$; and where this was not possible for a certain zone on account of the scanty weight for one or the other of the residuals for $3^M.0$ or $4^M.0$, that zone has been omitted from the computations.

All the work up to this point was done in a purely mechanical way upon a uniformly prescribed plan, carried out with undeviating rigor, regardless of what might seem to be discordances or anomalies.

The last column in each of the sub-tables which make up Table I is the mean by weights of the individual zones; and the weight belonging to each residual is the sum of the corresponding weights in the separate zones. It is scarcely necessary to remark that much the larger part of the weight in the final residuals is due to the equatorial zone.

TABLE I.

Final Results for Magnitude-Equations, with Observed Values of $\Delta\alpha_m$.

Greenwich 1755 (A.–B.) $\Delta'\alpha_m = -^s.004$ $\Delta\alpha_m = -^s.011$

M	**	p	$\Delta\alpha_m$ (s)	M	**	p	$\Delta\alpha_m$ (s)	M	$\Delta\alpha_m$ (s)
				0.9	8	2	−.001	0.9	−.001
1.8	14	1	+.037	2.0	14	2	+.002	1.9	+.014
3.1	26	1	.000	3.0	51	5	−.002	3.0	−.002
3.9	38	1	.000	4.0	124	10	+.002	4.0	+.002
4.8	5	–	−.070	5.0	87	6	−.009	5.0	−.011

Königsberg 1825 $\Delta'\alpha_m = -^s.004$ $\Delta\alpha_m = -^s.012$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
				0.9	8	5	+.004	0.9	+.004
1.6	9	3	+.027	2.1	7	4	+.029	1.9	+.028
3.1	7	2	−.005	2.9	8	5	+.005	2.9	+.001
3.9	6	1	+.005	4.3	3	1	−.007	4.1	.000
4.7	3	1	+.001					4.7	+.001

Dorpat 1824 $\Delta'\alpha_m = -^s.004$ $\Delta\alpha_m = -^s.011$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
				0.9	8	7	+.012	0.9	+.012
1.9	15	2	−.019	2.1	8	7	+.004	2.0	−.002
3.0	17	1	−.010	2.9	16	8	+.001	2.9	.000
3.9	23	1	+.010	3.9	25	5	−.001	3.9	+.001
4.7	4	–	+.004	5.0	10	1	−.015	5.0	−.012

St. Helena 1830 $\Delta'\alpha_m = {}^s.000$ $\Delta\alpha_m = -^s.008$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
0.9	8	2	−.008					0.9	−.008
2.1	10	2	.000	2.1	18	1	−.006	2.1	−.002
3.0	32	5	.000	3.0	39	1	+.001	3.0	.000
4.0	47	5	.000	4.0	50	1	−.001	4.0	.000
5.0	35	3	−.005	4.9	30	1	−.021	5.0	−.008

Åbo. 1830 $\Delta'\alpha_m = -^s.004$ $\Delta\alpha_m = -^s.012$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
				0.9	8	9	+.010	0.9	+.010
1.9	12	7	+.002	2.1	8	9	+.004	2.0	+.003
3.0	13	6	+.006	3.0	22	13	+.004	3.0	+.005
4.0	16	6	−.006	4.0	44	14	−.004	4.0	−.005
4.9	4	1	−.006	5.1	26	7	−.006	5.1	−.006

Greenwich 1830 $\Delta'\alpha_m = -^s.003$ $\Delta\alpha_m = -^s.010$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
				0.9	8	2	+.002	0.9	+.002
1.9	15	1	−.002	2.0	14	2	−.004	2.0	−.004
3.1	26	1	−.005	3.0	51	5	+.005	3.0	+.003
3.9	42	1	+.005	4.0	121	8	−.005	4.0	−.004
4.8	4	–	+.024	5.0	73	5	−.011	5.0	−.010

Cambridge 1830 $\Delta'\alpha_m = +^s.002$ $\Delta\alpha_m = -^s.006$

M	**	p	$\Delta\alpha_m$ (s)	M	**	p	$\Delta\alpha_m$ (s)	M	$\Delta\alpha_m$ (s)
				0.9	8	2	−.005	0.9	−.005
				2.0	13	2	−.020	2.0	−.020
				3.0	28	3	+.003	3.0	+.003
				4.0	56	5	−.003	4.0	−.003
				5.1	62	5	−.001	5.1	−.001

Cape 1840 $\Delta'\alpha_m = {}^s.000$ $\Delta\alpha_m = -^s.008$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
0.9	8	4	−.004					0.9	−.004
2.1	9	4	.000	2.1	17	2	+.007	2.1	+.002
2.9	31	10	+.001	3.0	40	4	+.005	2.9	+.002
4.0	48	10	−.001	4.0	48	4	−.005	4.0	−.002
4.9	40	2	−.021	4.9	28	2	+.001	4.9	−.010
5.8	8	1	+.023					5.8	+.023

Greenwich 1840 $\Delta'\alpha_m = +^s.009$ $\Delta\alpha_m = +^s.001$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
				0.9	8	3	−.027	0.9	−.027
				2.1	11	4	−.006	2.1	−.006
				2.9	35	10	−.005	2.9	−.005
				4.0	59	10	+.005	4.0	+.005
				4.9	55	6	+.014	4.9	+.014
				5.9	6	1	+.009	5.9	+.009

Greenwich 1845 $\Delta'\alpha_m = +^s.007$ $\Delta\alpha_m = -^s.001$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
				0.9	8	8	−.015	0.9	−.015
1.9	11	2	−.016	2.0	12	10	−.007	2.0	−.008
3.1	17	2	+.020	2.9	38	25	−.003	2.9	−.002
3.9	31	1	−.020	4.0	37	25	+.003	4.0	+.002
4.7	2	–	+.069	4.9	61	15	+.013	4.9	+.016
				5.9	8	2	+.022	5.9	+.022

Radcliffe 1845 $\Delta'\alpha_m = -^s.008$ $\Delta\alpha_m = -^s.016$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
				0.9	8	2	.000	0.9	.000
1.9	14	1	+.022	2.1	10	2	+.029	2.0	+.026
3.1	16	1	−.001	2.9	36	6	+.007	2.9	+.006
3.9	33	2	+.001	4.0	69	8	−.007	4.0	−.006
4.7	4	–	−.017	5.0	53	5	−.017	5.0	−.017

Pulkowa 1845 $\Delta'\alpha_m = +^s.002$ $\Delta\alpha_m = -^s.005$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
				0.9	8	6	−.003	0.9	−.003
1.9	15	11	−.004	2.0	14	16	−.001	2.0	−.002
3.1	27	17	−.006	3.0	51	50	−.001	3.0	−.002
3.9	41	25	+.006	4.0	112	96	+.002	4.0	+.003
4.8	3	1	+.004	5.0	21	8	−.001	5.0	.000

Paris 1845 $\Delta' a_m = +^s.003$ $\Delta a_m = -^s.005$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	5	−.006	0.9	−.006
1.9	15	8	−.007	2.0	14	15	−.011	2.0	−.010
3.1	24	14	+.007	3.0	48	43	−.004	3.0	−.001
3.9	35	6	−.007	4.0	119	56	+.004	4.0	+.003
4.8	5	1	−.006	4.9	72	28	+.002	4.9	+.002
				6.1	12	3	−.006	6.1	−.006

Santiago 1850 $\Delta' a_m = -^s.004$ $\Delta a_m = -^s.012$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	3	+.008	0.9	+.008
				2.1	10	3	+.002	2.1	+.002
				2.9	29	9	+.006	2.9	+.006
				4.0	39	6	−.006	4.0	−.006
				5.0	25	2	−.008	5.0	−.008

Greenwich 1850 $\Delta' a_m = +^s.003$ $\Delta a_m = -^s.005$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	7	−.003	0.9	−.003
1.9	13	3	−.001	2.0	14	9	−.010	2.0	−.009
3.0	23	3	+.004	3.0	50	27	−.003	3.0	−.002
3.9	27	3	−.004	4.0	107	34	+.003	4.0	+.002
				4.9	61	14	+.009	4.9	+.009
				6.0	14	4	+.001	6.0	+.001

Cape 1860 $\Delta' a_m = -^s.003$ $\Delta a_m = -^s.011$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
0.9	8	4	+.016					0.9	+.016
2.1	10	3	+.003	2.1	16	3	+.003	2.1	+.003
2.9	28	9	+.003	3.0	37	4	−.006	2.9	.000
4.0	46	12	−.003	4.0	46	2	+.006	4.0	−.002
5.1	52	10	.000	4.8	24	2	−.011	5.1	−.002

Washington 1860 $\Delta' a_m = +^s.002$ $\Delta a_m = -^s.006$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	6	.000	0.9	.000
1.8	14	3	−.024	2.0	13	8	+.003	2.0	−.004
3.0	24	3	+.019	3.0	48	27	+.002	3.0	+.004
3.9	31	3	−.019	4.0	112	42	−.002	4.0	−.003
4.8	4	–	−.048	4.9	70	28	+.007	4.9	+.006
				6.0	14	5	+.009	6.0	+.009

Greenwich 1860 $\Delta' a_m = +^s.001$ $\Delta a_m = -^s.007$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	9	−.001	0.9	−.001
1.9	11	5	+.014	2.0	14	11	+.001	2.0	+.005
3.0	13	4	+.002	3.0	43	40	.000	3.0	.000
3.9	22	5	−.002	4.0	98	63	.000	4.0	.000
4.7	5	–	+.027	4.9	67	34	+.005	4.9	+.005
				6.0	14	8	+.001	6.0	+.001

Radcliffe 1860 $\Delta' a_m = -^s.005$ $\Delta a_m = -^s.013$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	2	−.002	0.9	−.002
1.9	15	1	+.005	2.0	14	2	+.002	2.0	+.003
3.0	20	1	−.011	3.0	47	9	+.005	3.0	+.004
3.9	24	1	+.011	4.0	101	15	−.005	4.0	−.004
4.8	5	–	.000	4.9	61	8	−.018	4.9	−.017
				6.0	12	2	−.013	6.0	−.013

Melbourne 1860 $\Delta' a_m = -^s.002$ $\Delta a_m = -^s.009$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
0.9	8	5	−.004					0.9	−.004
2.1	10	6	+.002	2.2	10	2	−.007	2.1	.000
3.0	33	17	+.003	3.0	19	4	+.002	3.0	+.003
4.0	42	15	−.003	4.0	19	2	−.002	4.0	−.003
5.0	29	9	−.007	4.8	8	1	−.007	5.0	−.007

Paris 1860 $\Delta' a_m = +^s.004$ $\Delta a_m = -^s.003$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	10	−.008	0.9	−.008
1.9	15	7	−.006	2.0	13	15	−.010	2.0	−.009
3.1	22	7	−.002	3.0	50	45	−.002	3.0	−.002
3.9	25	6	+.002	4.0	106	70	+.002	4.0	+.002
4.9	4	–	+.015	4.9	69	29	+.006	4.9	+.006
				6.0	14	5	+.013	6.0	+.013

Greenwich 1864 $\Delta' a_m = -^s.001$ $\Delta a_m = -^s.009$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	14	+.001	0.9	+.001
1.9	11	5	+.002	2.0	13	16	+.003	2.0	+.003
3.1	17	4	+.002	3.0	45	50	.000	3.0	.000
3.9	19	5	−.002	4.0	94	75	.000	4.0	.000
4.7	3	–	−.012	4.9	62	41	−.003	4.9	−.003
				6.0	12	7	.000	6.0	.000

Cape 1865 $\Delta' a_m = -^s.006$ $\Delta a_m = -^s.014$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
0.9	8	4	+.011					0.9	+.011
2.0	13	5	+.010	2.1	13	2	+.011	2.0	+.010
3.0	40	15	+.004	3.0	31	3	+.016	3.0	+.006
4.0	71	25	−.004	4.0	21	1	−.016	4.0	−.005
4.9	60	18	−.008	4.9	15	1	+.006	4.9	−.007
6.1	11	3	−.016					6.1	−.016

Brussels 1865 $\Delta' a_m = +^s.004$ $\Delta a_m = -^s.004$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	5	−.004	0.9	−.004
1.9	14	3	+.006	2.0	13	6	+.003	2.0	+.004
3.1	25	4	−.003	3.0	50	22	−.001	3.0	−.001
3.9	38	4	+.003	4.0	119	34	+.001	4.0	+.004
4.8	6	1	+.030	4.9	70	20	+.012	4.9	+.013
				5.9	13	4	+.010	5.9	+.010

Harvard 1865 $\Delta' a_m = -^s.005$ $\Delta a_m = -^s.013$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
1.9	9	2	+.024					1.9	+.024
3.0	17	3	−.002	2.9	37	14	.000	2.9	.000
3.8	17	3	+.002	4.0	68	25	.000	4.0	.000
4.8	4	–	+.013	5.0	27	10	−.003	5.0	−.002

Pulkowa 1865 $\Delta' a_m = -^s.002$ $\Delta a_m = -^s.010$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	18	+.004	0.9	+.004
1.9	15	18	.000	2.0	14	25	+.009	2.0	+.004
3.1	27	27	.000	3.0	46	71	.000	3.0	.000
3.9	41	42	.000	4.0	105	150	.000	4.0	.000
4.8	3	2	−.010	4.8	9	10	−.008	4.8	−.008

Melbourne 1870 $\Delta' a_m = {}^s.000$ $\Delta a_m = -^s.007$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
0.9	8	10	−.008					0.9	−.008
2.1	10	12	−.004	2.1	15	4	−.001	2.1	−.003
2.9	29	34	−.002	3.0	36	6	+.002	2.9	−.002
3.9	28	31	+.002	4.0	34	3	−.002	3.9	+.002
4.9	16	15	−.006	4.8	21	3	−.007	4.9	−.006
6.0	4	4	−.007					6.0	−.007

Greenwich 1872 $\Delta' a_m = -^s.002$ $\Delta a_m = -^s.009$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	16	+.006	0.9	+.006
1.9	15	11	+.009	2.0	14	20	+.007	2.0	+.007
3.1	26	11	+.010	3.0	49	62	+.001	3.0	+.003
3.9	37	12	−.010	4.0	108	98	−.001	4.0	−.002
4.8	4	–	−.043	4.9	67	51	+.005	4.9	+.004
				6.0	12	9	−.006	6.0	−.006

Washington 1875 $\Delta' a_m = -^s.001$ $\Delta a_m = -^s.008$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	14	+.003	0.9	+.003
1.9	15	8	−.003	2.0	14	19	+.002	2.0	.000
3.1	24	10	+.008	3.0	50	60	+.001	3.0	+.002
3.9	38	11	−.008	4.0	108	72	−.001	4.0	−.002
4.7	4	–	−.019	4.9	55	36	.000	4.9	.000
				6.0	10	6	+.001	6.0	+.001

Pulkowa 1875 $\Delta' a_m = +^s.003$ $\Delta a_m = -^s.005$

M	**	p	Δa_m	M	**	p	Δa_m	M	Δa_m
				0.9	8	7	−.005	0.9	−.005
1.9	15	6	−.015	2.0	14	8	−.002	2.0	−.008
3.1	27	8	−.006	3.0	45	27	.000	3.0	−.001
3.9	42	12	+.006	4.0	111	57	.000	4.0	+.001
4.8	6	1	+.035	4.9	36	11	−.002	4.9	+.002
				6.1	8	2	+.007	6.1	+.007

Harvard 1875 $\Delta' a_m = +^s.001$ $\Delta a_m = -^s.007$

M	**	p	Δa_m (s)	M	**	p	Δa_m (s)	M	Δa_m (s)
				0.9	8	11	.000	0.9	.000
1.9	15	11	+.005	2.1	11	14	−.002	2.1	+.001
3.1	27	14	−.006	3.0	45	44	+.001	3.0	−.001
3.9	40	22	+.006	4.0	98	83	−.001	4.0	+.001
4.7	3	2	+.005	4.9	46	33	.000	4.9	+.005
				6.1	3	2	−.012	6.1	−.012

Cordoba 1875 $\Delta' a_m = -^s.002$ $\Delta a_m = -^s.010$

0.9	8	7	+.008					0.9	+.008
2.0	13	9	+.005	2.1	18	3	+.005	2.0	+.005
3.0	41	30	+.001	3.0	42	5	+.002	3.0	+.001
4.0	66	30	−.001	4.0	53	4	−.002	4.0	−.001
5.0	50	17	−.002	4.8	31	2	+.011	5.0	−.001

Paris 1875 $\Delta' a_m = +^s.003$ $\Delta a_m = -^s.005$

				0.9	8	7	−.010	0.9	−.010
1.9	14	3	−.019	2.0	14	7	−.002	2.0	−.007
3.0	21	5	−.005	3.0	47	37	.000	3.0	−.001
3.9	24	6	+.005	4.0	96	51	.000	4.0	+.001
5.1	2	1	−.006	4.9	67	30	+.006	4.9	+.006
				6.0	13	7	.000	6.0	.000

Cape 1880 $\Delta' a_m = -^s.003$ $\Delta a_m = -^s.011$

0.9	8	6	+.013					0.9	+.013
2.0	14	7	+.005	2.1	18	2	+.016	2.0	+.008
3.0	43	21	.000	3.0	41	4	−.004	3.0	−.001
4.0	86	25	.000	4.0	53	3	+.004	4.0	.000
4.9	54	15	−.008	4.8	29	2	+.022	4.9	−.004
6.1	8	3	−.006					6.1	−.006

Melbourne 1880 $\Delta' a_m = +^s.003$ $\Delta a_m = -^s.005$

0.9	8	5	−.008					0.9	−.008
2.1	12	7	+.001	1.9	9	1	−.009	2.1	−.001
3.0	45	22	−.003	2.9	12	2	−.011	3.0	−.004
4.0	34	29	+.002	4.1	13	–	+.011	4.0	+.002
4.9	46	14	+.005	4.9	11	1	+.004	4.9	+.005
5.9	6	3	+.007					5.9	+.007

Greenwich 1880 $\Delta' a_m = +^s.001$ $\Delta a_m = -^s.007$

				0.9	8	14	−.005	0.9	−.005
1.9	15	7	+.006	2.1	13	17	−.006	2.0	−.003
3.1	24	6	−.005	3.0	50	54	−.001	3.0	−.001
3.9	38	9	+.005	4.0	115	88	+.001	4.0	+.001
4.8	5	1	−.040	4.9	61	48	.000	4.9	−.001
				6.0	14	8	.000	6.0	.000

Pulkowa 1885 $\Delta' a_m = ^s.000$ $\Delta a_m = -^s.008$

				0.9	8	18	−.002	0.9	−.002
1.9	15	18	+.002	2.0	14	25	+.001	2.0	+.001
3.1	27	26	−.003	3.0	45	72	+.001	3.0	.000
3.9	40	41	+.003	4.0	99	151	−.001	4.0	.000
4.7	2	2	−.015	4.7	6	10	.000	4.7	−.002

Cape 1885 $\Delta' a_m = ^s.000$ $\Delta a_m = -^s.008$

0.9	8	6	+.010					0.9	+.010
2.1	13	8	−.001	2.1	18	4	−.007	2.1	−.003
3.0	50	35	.000	3.0	40	6	−.001	3.0	.000
4.0	114	76	.000	4.0	34	6	+.001	4.0	.000
4.9	59	30	+.002	4.9	14	3	+.017	4.9	+.003
5.9	6	3	−.007					5.9	−.007

Strassburg 1885 $\Delta' a_m = +^s.001$ $\Delta a_m = -^s.006$

				0.9	8	14	−.004	0.9	−.004
1.9	15	10	−.003	2.0	13	16	−.007	2.0	−.005
3.1	19	8	+.002	3.0	45	47	−.002	3.0	−.001
3.9	19	8	−.002	4.0	85	70	+.002	4.0	+.001
4.7	2	1	+.015	4.9	45	34	+.002	4.9	+.002
				5.7	7	4	−.016	5.7	−.016

Radcliffe 1890 $\Delta' a_m = ^s.000$ $\Delta a_m = -^s.008$

M	**	p	Δa_m (s)	M	**	p	Δa_m (s)	M	Δa_m (s)
				0.9	8	6	+.006	0.9	+.006
1.9	10	1	−.012	2.1	13	9	+.006	2.1	+.004
3.1	19	2	+.013	3.0	47	29	−.001	3.0	.000
3.9	23	2	−.013	4.0	107	47	+.001	4.0	.000
4.9	3	–	+.019	4.9	68	26	+.003	4.9	+.003
				6.0	13	5	+.008	6.0	+.008

Cape 1890 $\Delta' a_m = -^s.005$ $\Delta a_m = -^s.013$

0.9	8	5	+.013					0.9	+.013
2.0	14	9	+.007	2.1	18	4	−.005	2.0	+.003
3.0	51	33	+.003	3.0	42	8	+.001	3.0	+.003
4.0	120	70	−.003	4.0	52	10	−.001	4.0	−.003
4.9	59	26	−.009	4.8	24	4	+.001	4.9	−.008
6.2	6	3	−.017					6.2	−.017

Greenwich 1890 $\Delta' a_m = +^s.002$ $\Delta a_m = -^s.006$

				0.9	8	7	−.014	0.9	−.014
1.9	15	11	.000	2.0	14	21	−.003	2.0	−.002
3.1	27	16	+.002	3.0	51	73	−.002	3.0	−.001
3.9	41	18	−.002	4.0	125	128	+.002	4.0	+.001
4.8	5	1	−.022	4.9	73	70	.000	4.9	.000
				6.0	14	12	+.003	6.0	+.003

Madison 1890 $\Delta' a_m = -^s.005$ $\Delta a_m = -^s.013$

				0.9	8	9	+.014	0.9	+.014
1.9	14	8	+.012	2.1	10	11	−.005	2.1	+.002
3.0	25	13	+.006	3.0	37	47	+.003	3.0	+.004
3.9	39	21	−.006	4.0	80	96	−.003	4.0	−.004
4.6	3	2	+.006	4.7	18	23	−.008	4.7	−.007
				6.0	2	2	−.015	6.0	−.015

Berlin 1890 $\Delta' a_m = +^s.004$ $\Delta a_m = -^s.003$

				0.6	5	4	−.008	0.6	−.008
2.0	13	10	−.010	2.1	10	15	−.003	2.1	−.006
3.1	25	18	−.002	3.0	35	51	−.003	3.0	−.003
3.9	42	26	+.002	4.0	83	111	+.003	4.0	+.003
4.7	3	2	+.001	4.8	14	21	+.005	4.8	+.005
				6.1	3	5	+.012	6.1	+.012

Lisbon 1890 $\Delta' a_m = -^s.001$ $\Delta a_m = -^s.008$

				0.9	8	16	−.004	0.9	−.004
1.7	11	10	+.004	2.0	13	27	+.002	2.0	+.003
3.1	20	19	−.003	3.0	48	100	+.002	3.0	+.001
3.9	33	29	+.003	3.9	96	168	−.002	3.9	−.001
4.7	2	1	+.019	4.8	17	36	−.001	4.8	.000
				6.0	3	5	−.009	6.0	−.009

Berlin 1895 $\Delta' a_m = ^s.000$ $\Delta a_m = -^s.008$

				0.9	7	8	+.010	0.9	+.010
1.9	14	10	−.001	2.0	10	16	+.004	2.0	+.002
3.0	21	13	+.003	3.0	41	56	.000	3.0	+.001
3.9	40	27	−.003	3.9	68	98	.000	3.9	−.001
4.7	3	2	−.013	5.0	16	24	+.006	5.0	+.005
				6.0	2	4	+.016	6.0	+.016

Mt. Hamilton 1895 $\Delta' a_m = -^s.003$ $\Delta a_m = -^s.011$

1.6	3	1	+.017	1.6	6	6	+.003	1.6	+.006
3.0	4	2	−.011	3.0	21	41	+.002	3.0	+.002
3.9	4	2	+.011	4.0	44	44	−.002	4.0	−.002
5.1	2	1	+.014	4.8	32	32	−.005	4.8	−.004
				6.0	4	4	−.003	6.0	−.003

Albany 1898 $\Delta' a_m = -^s.004$ $\Delta a_m = -^s.012$

				0.9	8	10	+.004	0.9	+.004
1.7	9	4	+.019	2.1	10	11	+.007	2.1	+.010
3.0	6	1	+.008	3.0	27	26	+.003	3.0	+.003
3.9	2	1	−.008	3.9	28	24	−.003	3.9	−.003
				4.9	11	7	−.008	4.9	−.008

Comparison of Eye-and-Ear with Chronographic Transits.

The residuals, $\Delta\alpha_m$, of Table I, so far as they have any meaning, evidently depend upon the difference between the magnitude-equation of each individual catalogue and that of the Standard Catalogue. Therefore, for nearly contemporaneous catalogues at least, we may eliminate the magnitude-equation of the Standard Catalogue in finding the difference of equations between two or more catalogues of observation. This was the first use of Table I that occurred to me.

After 1850 there are still many series of right-ascension determinations which depend upon eye-and-ear transits, which may be compared with the nearly contemporaneous results from chronographic registry. Accordingly I divided the observations into two series: Series A, 1850–1870; Series B, 1875–1890. Washington 1860 and Pulkowa 1865 contain a few observations by the eye-and-ear method, but not in sufficient number to impeach their character as representatives of the method of chronographic registry. The residuals of the equatorial zone (Zone II) alone are used in this differential comparison, in order to avoid any possible suspicion that the final differences may be due to any other form of systematic error than that due to magnitude-equation.

Relative Magnitude Effect; Eye-and-Ear Transits.

Series A.

		$0^M.9$	$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$
		s	s	s	s	s	s
Stgo.	50	+.008	+.002	+.006	—.006	—.008	-
Greenw.	50	—.003	—.010	—.008	+.003	+.009	+.001
Cape	60	+.016	+.003	+.003	—.003	.000	-
Radcl.	60	—.002	+.002	+.005	—.005	—.018	—.013
Paris	60	—.008	—.010	—.002	+.002	+.006	+.013
Bruss.	65	—.004	+.003	—.001	+.001	+.012	+.010
Means		+.0012	—.0017	+.0013	—.0013	+.0002	+.0028

Series B.

		$0^M.9$	$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$
Pulk.	75	—.005	—.002	.000	.000	—.002	+.007
Paris	75	—.010	—.002	.000	.000	+.006	.000
Cape	80	+.013	+.005	.000	.000	—.008	—.006
Cape	85	+.010	—.001	.000	.000	+.002	—.007
Radcl.	90	+.006	+.006	—.001	+.001	+.003	+.008
Means		+.0028	+.0012	—.0002	+.0002	+.0002	+.0004

Relative Magnitude Effect; Chronographic Transits.

Series A.

		$0^M.9$	$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$
		s	s	s	s	s	s
Wash.	60	.000	+.003	+.002	—.002	+.007	+.009
Greenw.	60	—.001	+.001	.000	.000	+.005	+.001
Melb.	60	—.004	+.002	+.003	—.003	—.007	-
Greenw.	64	+.001	+.003	.000	.000	—.003	.000
Cape	65	+.011	+.010	+.004	—.004	—.008	—.016
Pulk.	65	+.004	+.009	.000	.000	—.008	-
Melb.	70	—.008	—.004	—.002	+.002	—.006	—.007
Means		+.0004	+.0034	+.0010	—.0010	—.0029	—.0026

Series B.

		$0^M.9$	$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$
Wash.	75	+.003	+.002	+.001	—.001	.000	+.001
Harv.	75	.000	—.002	+.001	—.001	.000	—.012
Cord.	75	+.008	+.005	+.001	—.001	—.002	-
Melb.	80	—.008	+.001	—.003	+.002	+.005	+.007
Greenw.	80	—.005	—.006	—.001	+.001	.000	.000
Pulk.	85	—.002	+.001	+.001	—.001	.000	-
Strassb.	85	—.004	—.007	—.002	+.002	+.002	—.016
Madn.	90	+.014	—.005	+.003	—.003	—.008	—.015
Berlin	90	—.008	—.003	—.003	+.003	+.005	+.012
Lisb.	90	—.004	+.002	+.002	—.002	—.002	-
Means		—.0006	—.0012	.0000	—.0001	.0000	—.0033

Evidently we may compare the mean results from eye-and-ear observations with those from chronographic registry in the same series without fear that the variation of the magnitude-equation, or of other forms of systematic correction, for the standard catalogue will be of any importance whatever in the short interval between the mean epochs of the groups within each of the series. Thus, if we take the magnitude-equation of the eye-and-ear observations as standard we shall find that the chronographic observations require the following corrections:

Eye-and-Ear minus Chronographic Transits; Summary.

Magn.	Series A	Series B.	Mean
M	s	s	s
0.9	—.0008	—.0034	—.0021
2.0	+.0051	—.0024	+.0014
3.0	—.0003	+.0002	.0000
4.0	+.0003	—.0003	.0000
5.0	—.0031	—.0002	—.0016
6.0	—.0054	—.0037	—.0046

From the last column we may conclude that if the chronographic transits had been corrected by $-0^s.0008\ (M-3.5)$ in the mean there would have been close agreement in the results from the two methods of observation. This difference is exceedingly small; and even at the ninth magnitude its effect would be less than $0^s.005$. We seem to be justified in the opinion that, so far as it relates to stars brighter than the sixth magnitude, the mean difference between the magnitude-equations for eye-and-ear and for chronographic transits is wholly insignificant. This conclusion appears to be based upon an amount of testimony which ought to be sufficient to establish it beyond question; yet the history of the question indicates that there are many instances in which the magnitude-equation for eye-and-ear transits is a small, if not vanishing, quantity; and there seems to me to be a greater proportion of such instances in the employment of the eye-and-ear method than in that of chronographic registry. Nevertheless, if the existence of this state of facts were to be admitted, it does not necessarily conflict with the conclusion that the mean magnitude-equation for chronographic transits is only slightly larger than for those wherein the eye-and-ear method is employed.

This conclusion seems to justify two inferences of importance.

1. The magnitude-equation is essentially a subjective phenomenon; and its true explanation belongs to the field of experimental psychology.

2. When many star-catalogues are employed the determination of the proper motions of the brighter stars is pratically independent of the magnitude-equation; since there is no appreciable difference between the average equations pertaining to the two methods, and therefore the mean magnitude-equation of the observations previous to 1850, when all transits were obtained by the eye-and-ear method, must be nearly the same as for those subsequent to that date.

On the Law of Magnitude-Equation.

Although the effective range of the star-magnitudes involved is not very great (since the weights outside of the limits $2^M.0$ and $5^M.0$ are comparatively small) it seemed that it might be possible to secure some evidence as to the general law of variation of magnitude-equation for diminishing magnitudes. There seems to be no reason why the progression should be strictly proportional to the magnitude; and there does seem to be a very natural reason why the rate of change in the equation should increase with diminishing steps of brightness, if it is associated in any very close way with the relative difficulty of seeing. Accordingly the means were collected in nearly contemporaneous groups for those catalogues which seemed to have equations smaller than that of the standard, for comparison with those which appeared to have equations decidedly larger than that of the standard. If the individual equations are linear, according to star-magnitude, then the differences of the two groups of catalogues should be likewise linear; and if, in the individual equations there are very sensible terms depending on the second or higher powers of ΔM, then such terms should appear in the mean difference between large and small equations. The material is selected from Zone II only.

For Small minus Large Equations.

Small Equations.

		$0^M.9$	$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$
		s	s	s	s	s	s
Paris	60	—.008	—.010	—.002	+.002	+.006	+.013
Pulk.	75	—.005	—.002	.000	.000	—.002	+.007
Paris	75	—.010	—.002	.000	.000	+.006	.000
Melb.	80	—.008	+.001	—.003	+.002	+.005	+.007
Berlin	90	—.008	—.003	—.003	+.003	+.005	+.012
Means		—.0078	—.0032	—.0016	+.0014	+.0040	+.0078

Large Equations.

		$0^M.9$	$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$
Cape	65	+.011	+.010	+.004	—.004	—.008	—.016
Pulk.	65	+.004	+.009	.000	.000	—.008	-
Cord.	75	+.008	+.005	+.001	—.001	—.002	-
Cape	80	+.013	+.005	.000	.000	—.008	—.006
Cape	90	+.013	+.007	+.003	—.003	—.009	—.017
Madn.	90	+.014	—.005	+.003	—.003	—.008	—.015
Alb.	98	+.004	+.007	+.003	—.003	—.008	-
Means		+.0096	+.0054	+.0020	—.0020	—.0073	—.0135

If the line of means for the "Small Equations" be subtracted from the line of means for "Large Equations" we shall have the observed values of $\Delta'\alpha_m$ necessary to reduce the catalogues of the latter category to harmony with those of the former.

Small minus Large Equations; Values of $\Delta'\alpha_m$.

Magn.	p	Obs'd $\Delta'\alpha_m$	Computed I	Computed II
s		s	s	s
0.9	2	+.0174	+.0187	+.0167
2.0	2	+.0086	+.0108	+.0102
3.0	3	+.0036	+.0036	+.0036
4.0	3	—.0034	—.0036	—.0037
5.0	3	—.0113	—.0108	—.0117
6.0	1	—.0213	—.0180	—.0204

If the residual at magnitude, 2.0, for Madison 90 be rejected, the second of the observed values of $\Delta'\alpha_m$ becomes $+0^s.0104$ instead of $+0^s.0086$.

The weights are estimated for use in the solutions. It is evident at once from an inspection of the observed residuals, $\Delta'\alpha_m$, that these numbers are very nearly proportional to the difference of magnitudes. They testify very strongly both for the reality of the phenomenon of magnitude-equation, and for the stability of its effects upon the work of the observers. The numbers in column I are computed from the formula,

$$-0^s.0072\ (M-3.5)\,;$$

and those in column II from the formula,

$$-0^s.0073\ (M-3.5)\ -0^s.00034\ (M-3.5)^2$$

Were it not for the very high precision of the observed quantities and their freedom from the imputation of systematic error there would be very slight ground for preferring one of these formulas over the other. However this may be, we seem to be justified in the conclusion that, for the brighter stars, the magnitude-equation is very nearly proportional to star-magnitude, — so nearly so that a rigid investigation of the adopted magnitude-scale itself would be necessary before a more definite conclusion can be maintained.

When the equation is extended to fainter stars we possess some further evidence on this point. Some of the best determinations of magnitude-equation with screens indicate very slight, or no, increase in the rate of the effect with diminishing brightness. In other instances, however, notably in the case of A.G. Cambridge (Eng.) (*M.N.* Vol. LX, Turner), and A.G. Leipsic (*A.N.* 3854) the rate of magnitude-equation appears to increase rapidly with diminishing brightness of the star, so that even a higher power of ΔM than the second is needed to indicate the change of rate.

On the whole we may conclude that for stars brighter than the seventh magnitude the rate of the equation in its relation to magnitude is usually very nearly linear, with a tendency in some cases to increase with diminishing brightness of the star.

The evident trustworthiness and precision of the means upon which the foregoing conclusions were based induced me next to examine the question as to the absolute magnitude-equation of the Standard Catalogue. Contrary to the impressions I had formed from following the published papers on magnitude-equation during more than twenty years, — impressions which had nearly deterred me from making this investigation at all, — I found the individual determinations of magnitude-equation consistent in a degree fully as high as could have been anticipated from a hopeful estimate of their value. That such determinations have hitherto been regarded by many as untrustworthy, if not

entirely illusory, is abundantly evident from positive comments to that effect, from inconsistent conclusions reached in various investigations, from the various devices invented to get rid of the magnitude-equation, from the fact that many observers of skill and experience make no attempt to determine this equation for their own observations, and from the further fact that no one, as yet, has ventured to publish a series of right-ascensions actually corrected for magnitude-equation, although an intention to do this in the future has been expressed by more than one observer.

Every catalogue of observation for which the magnitude-equation is known by observation with screens, or otherwise, furnishes a means of determining the absolute magnitude-equation of the Standard Catalogue. Let us consider these in turn.

Pulkowa 1875. ROMBERG did not determine his magnitude-equation; but as his observations are nearly contemporary with those of the Albany and Berlin (BECKER) A.G. zone-observations for which absolute magnitude-equations were determined, it is possible to utilize the comparison with these zone-observations to determine the absolute equation of ROMBERG's right-ascensions. In order to utilize these comparisons to good advantage it is desirable to know the relations, Berl.—Pulk. and Alb.—Pulk., at the magnitude 4.0. Theoretically, since all of these observations are based upon the same standard catalogue, the *Fundamental-Catalog* of Dr. AUWERS, these differences should be zero. Practically, they may differ sensibly from this value; and the determination of this quantity is the weakest point in the comparison. After most careful attention to all the evidence, direct and indirect, I have adopted the following:

$$\text{At } 4^{\text{M}}.0: \quad \text{Berlin}-\text{Pulkowa} = \overset{s}{0}.000$$
$$\text{Albany}-\text{Pulkowa} = +0.003$$

Combining with these results the comparison, Berlin—Pulk., contained in the Berlin A.G. Catalogue [*Int.*, p. (10)], and of Albany—Pulk. (*A.J.*, XVIII, p. 118) I find the absolute equation of Pulk. 75 to be:

through Berlin,	$-\overset{s}{0}.0052$
through Albany,	-0.0037
Pulk. 75, adopted,	-0.0045, per magnitude.

Cape 1885. The magnitude-equation of this catalogue is given by Sir DAVID GILL as (*Int.*, p. XLI),

$$-0^s.011$$

and this is recommended for adoption. Since the observations were made by the eye-and-ear method this determination of magnitude-equation is probably entitled to much smaller weight than would be assignable to one depending on chronographic registry, because of the great difficulty in securing a number of observations sufficiently great to overcome the casual errors of determination.

Berlin 1890. For KÜSTNER's magnitude-equation we have his statement (*A.N.* 142, p. 118) in connection with the remark of Dr. AUWERS [*A.G.* Berlin (AUWERS), p. (133)], and also the explanation of KÜSTNER in the Publications of the Bonn Observatory (*Heft* 4, pp. 41 and 42), from which I assume as the equation of Berlin 90,

$$-0^s.0040$$

This depends upon the differential places in a heliometric triangulation.

Cape 90. The details of the determination of magnitude-equation for Cape 90 are given in the introduction to the Catalogue (pp. XL and XLII), and

$$-0^s.0139$$

is there adopted and recommended for application to the right-ascensions of the catalogue.

Berlin 1895. BATTERMANN determined his magnitude-equation in several series. Following is a summary of his determinations taken from the introduction to his catalogue.

1894, usual method,	$-\overset{s}{0}.0100\,(M-4)$	$-\overset{s}{0}.0008\,(M-4)^2$
1897, usual method,	-0.0103	-0.0005
By alternate transits,	-0.0110	-0.00146
Reduced aperture,	$-0.0070\,(M-4)$	

The latter value he seems to consider the most trustworthy, but doubts whether it is applicable to transits with full aperture. In view of all the evidence I adopt as BATTERMANN's equation for bright stars,

$$-0^s.009$$

Mt. Hamilton 1895. In the introduction of his catalogue for 1895 (Pub. Lick Obs., Vol. IV, p. 22) Professor TUCKER states that his equation from screen-observations is

$$-0^s.009$$

but he appears to entertain grave doubts as to the reality of the effect. The result is based upon 75 single determinations, and I see no reason why it should not be entitled to confidence.

Albany 1898. The determination of the magnitude-equation for this series of observations rests upon a large number of observations, and has been adopted in the computations for the Albany Catalogue now in progress. This equation is (*A.J.* XXII, p. 99),

$$-0^s.0132\,(M-4)\ -0^s.00019\,(M-4)^2$$

Bonn 1900. KÜSTNER has given special attention to the determination of his magnitude-equation, and in a paper of great interest (*A.N.* 3778) determines the magnitude-equation of the *Fundamental-Catalog* of Dr. AUWERS to be

$$-0^s.005$$

Much evidence bearing on this point is contained in the several introductions of the recent publications of the Bonn Observatory (Nos. 4, 5 and 6). From these sources we get the following determinations of KÜSTNER'S magnitude-equation, together with the corresponding equation of F.C.

Zone	Küstner	F.C.
0° to $+18^\circ$	$+0^s.0070$	$-0^s.0043$
$+18$ to $+36$	-0.0041	-0.0053
$+36$ to $+51$	-0.0036	-0.0034

One is struck with the exceptional equation of the observer for zone 0° to +18°. His previous equation at Berlin had been $-0^s.004$, as already stated, and to this value he returned again in the second zone observed at Bonn. But the consistency of the equation for F. C. in the first zone with the other values found in later zones seems to establish the reality of the abnormal change. Since the revised *Fundamental-Catalog* depends essentially on the same modern authorities upon which my Standard Catalogue is based I shall assume that the equation which KÜSTNER found for the former is equally applicable to the latter.

We are now ready to combine the various determinations of absolute magnitude-equation to ascertain the absolute equation of the Standard Catalogue. The process is very simple, and needs merely to be indicated. For instance, we find the magnitude-equation of Pulkowa 75 to be $-0^s.0045\,(M-3.5)$. If we apply this to the right-ascensions of ROMBERG'S Catalogue, and then compare the corrected right-ascensions with those of the Standard Catalogue, we shall arrive at the same result that we shall have if we alter the numbers in the last column of the sub-table for Pulk. 75 in Table I by the addition of $+0^s.0045\,(M-3.5)$. Then reversing the signs of these quantities, so corrected, to make them applicable as corrections to the Standard Catalogue, we shall have the observed corrections given by Pulk. 75 to the Standard Catalogue for the magnitude-effect. Proceeding in this manner we have the following observed values of the magnitude-effect as determined through the respective catalogues of observations.

OBSERVED VALUES OF $\Delta\alpha_m$ FOR DETERMINATION OF THE ABSOLUTE MAGNITUDE-EQUATION OF THE STANDARD CATALOGUE.

Magnitude through	$0^M.9$	$2^M.0$	$3^M.0$	$4^M.0$	$4^M.9$	$6^M.0$	Wt.
	s	s	s	s	s	s	
Pulk. 75	+.017	+.015	+.003	−.003	−.009	−.018	2
Cape 85	+.019	+.018	+.006	−.006	−.018	−.019	1
Berl. 90	+.020	+.012	+.005	−.005	−.010	−.022	3
Cape 90	+.023	+.018	+.004	−.004	−.011	−.021	3
Berl. 95	+.013	+.012	+.004	−.003	−.019	−.038	1
Mt.H.95	–	+.014	+.002	−.002	−.017	−.020	1
Alb. 98	+.028	+.008	+.004	−.002	−.011	–	2
Bonn 97	+.013	+.007	+.002	−.002	−.007	−.012	4

Collecting the means by weight of the vertical columns (assigning weight, 2, to the residual at $0^M.9$ for Berlin 90 which depends on only five stars) we have in the third column of the following table the mean observed magnitude-equation of the Standard Catalogue.

SUMMARY OF OBSERVED MAGNITUDE-EQUATION FOR THE STANDARD CATALOGUE.

Magn.	p	$\Delta\alpha_m$ (O)	$\Delta\alpha_m$ (C)	C − O
M		s	s	s
0.9	1	+0.0185	+0.0199	+0.0014
2.0	3	+0.0122	+0.0115	−0.0007
3.0	6	+0.0036	+0.0038	+0.0002
4.0	6	−0.0033	−0.0038	−0.0005
4.9	3	−0.0109	−0.0107	+0.0002
6.0	0.5	−0.0193	−0.0192	+0.0001

Employing the relative weights in the second column I find the magnitude-equation of the Standard Catalogue to be

$$-0^s.0077\,(M-3.5)$$

and this is adopted. The fourth column is derived from this equation, and the last column exhibits the discrepancies, C—O. It will be seen that a large weight has been attributed to the determinations of KÜSTNER both at Berlin and Bonn. The Berlin determination rests upon the observation of 37 stars, the relative places of which were determined by the heliometric triangulation of GILL. Of these, 31 stars are between the magnitude $7^M.4$ and $8^M.5$, and only 6 stars are brighter than $7^M.0$, of which the brightest is $5^M.7$. Yet, on account of the difference of method, together with the high precision and homogeneity of KÜSTNER'S Berlin observations, it seemed to me that the assigned weight was not relatively too great. It so happens that the result of this determination is very near the mean of all. As to the Bonn determination, it represents the most elaborate series of determinations of magnitude-equations which has been attempted, and were it not for the anomalous result in the first zone (*Heft.* 4) there could be no question as to the weight to which it is entitled.

On the whole there seems to be little likelihood that the magnitude-equation of my Standard Catalogue is less than $-0^s.007$, or more than $-0^s.009$. The probable error of the coefficient, $-0^s.0077$, I estimate to be about $\pm 0^s.0005$; though its nominal, or computed, probable error is only $\pm 0^s.00014$, which, in the present instance, has no real meaning.

Provided with the coefficient of magnitude-equation for the Standard Catalogue, determined with a precision apparently very great, the suggestion follows of itself that we should attempt to determine the absolute magnitude-equations of the individual catalogues of observation. In the last column of each sub-table in Table I, under the caption, $\Delta\alpha_m$, are the observed corrections necessary to reduce the individual catalogues into systematic conformity with the Standard Catalogue in so far as star-magnitude is the

argument for systematic correction. By means of conditional equations the systematic difference of magnitude-equation, $\Delta'\alpha_m$, between each catalogue and the Standard Catalogue has been determined, and the result is shown in full-faced type over the middle of each sub-table. These are the equations which, applied to the right-ascensions of the several catalogues of observation, are supposed to bring them into systematic harmony with those of the Standard Catalogue. Adding to these the adopted magnitude-equation of the Standard Catalogue, $-0^s.0077$, we derive the absolute magnitude-equation, $\Delta\alpha_m$, for each catalogue, as shown by the second of the quantities in full-faced type over each of the sub-tables contained in Table I.

For many of the older catalogues the determination of $\Delta'\alpha_m$ is very uncertain, both because of the small range of magnitude involved and also because of the small weight of the observed residuals. At most, the values of $\Delta\alpha_m$ for these are but rude approximations to the true result. For some of the later catalogues the results are entitled to a fair degree of confidence, as appears from the following schedule of those for which it is possible to compare with the magnitude-equations determined by the respective observers.

COMPARISON OF OBSERVED AND COMPUTED EQUATIONS.

	Comp.	Obs.	C − O
	s	s	s
Pulk. 75	−0.0050	−0.0045	−0.0005
Cape 85	−0.0078	−0.0110	+0.0032
Cape 90	−0.0125	−0.0139	+0.0014
Berlin 90	−0.0033	−0.0040	+0.0007
Berlin 95	−0.0076	−0.0090	+0.0014
Mt. H. 95	−0.0106	−0.0090	−0.0016
Alb. 98	−0.0115	−0.0130	+0.0015
Bonn 99	...	...	−0.0027

The C−O for Bonn 1899 is inferred through comparison of its determination of the absolute equation of the standard, $-0^s.005$, with the adopted value, $-0^s.0077$. The mean difference between the observed and computed values without regard to weight is $0^s.0016$, and, assuming that the error is equally due to the observed and computed values, it follows that the average error of the computed equation is $\pm 0^s.0011$.

Madras 35 (DOWNING's reduction) and Madras 75, though each is of very low weight, and subject to many anomalous errors, have been investigated for magnitude-equation. The observations extend into three zones. We have,

Madras 35.

Magn.	Zone I		Zone II		Zone III		Means
		s		s		s	s
0.9			8–1	−.001			−.001
2.0	14–1	−.003	14–2	+.004	16–1	−.001	+.001
3.0	27–1	−.005	51–5	−.001	39–1	+.001	−.001
4.0	41–1	+.005	123–9	+.001	50–1	−.001	+.001
4.9			71–5	−.031	30–1	−.021	−.028
6.0			13–1	−.040			−.040

From the final means I find for the absolute magnitude-equation, $-0^s.016$; but if a term depending on the second power of ΔM be introduced the equation becomes:

$$-0^s.016\,(M-3.5)\;-0^s.0044\,(M-3.5)^2$$

The extension of the present work to stars of fainter magnitude will afford a test of the interesting question whether the apparently rapid increase of the equation for fainter stars is merely due to the systematic instability of the catalogue, or to a real deviation of the form indicated. To some extent this same peculiarity appears in Madras 75; and this raises the question whether this phenomenon may be attributed to a racial peculiarity. The table for Madras 75 follows.

Madras 75.

Magn.	Zone I		Zone II		Zone III		Means
		s		s		s	s
0.9			8– 3	−.006			−.006
2.0	15–1	+.004	14– 4	+.006	18–1	−.018	+.001
3.0	27–1	−.019	51–15	−.004	42–2	−.006	−.005
4.0	39–1	+.019	123–22	+.004	48–1	+.006	+.005
4.9			71–11	−.012	27–1	−.035	−.014
5.9			11– 2	−.020			−.020

The absolute magnitude-equation may be taken as $-0^s.010$; but if a term depending on $(\Delta M)^2$ be introduced it becomes,

$$-0^s.010\,(M-3.5)\;-0^s.0024\,(M-3.5)^2$$

The assumption that the magnitude-equation of the Standard Catalogue does not sensibly vary from one epoch to another rests wholly on the soundness of the conclusion that, in the mean, the equation for eye-and-ear transits is the same as for chronographic registry. As a further check upon this deduction it may be of interest to institute a comparison between the absolute equations as determined in Table I, for the period 1850 to 1890.

MAGNITUDE-EQUATIONS COMPARED.

Eye-and-Ear		$\Delta\alpha_m$	p	Chronographic		$\Delta\alpha_m$	p
		s				s	
Stgo.	50	−.012	1	Washn.	60	−.006	2
Gch.	50	−.005	2	Gch.	60	−.007	2
Cape	60	−.011	2	Melb.	60	−.009	1
Radcl.	60	−.013	1	Gch.	64	−.009	2
Paris	60	−.003	4	Cape	65	−.014	1
Bruss.	65	−.004	1	Harv.	65	−.013	1
Pulk.	75	−.005	1	Melb.	80	−.007	2
Paris	75	−.005	3	Gch.	72	−.009	2
Cape	80	−.011	2	Washn.	75	−.008	5
Cape	85	−.008	3	Harv.	75	−.007	1
Radcl.	90	−.008	1	Cord.	75	−.010	2
				Melb.	80	−.005	1
				Gch.	80	−.007	4
				Pulk.	85	−.008	2
				Strass.	85	−.006	2
				Gch.	90	−.006	4
				Madn.	90	−.013	1
				Berlin	90	−.003	1
				Lisbon	90	−.008	1

The relative weights are based partly upon the trustworthiness of the comparisons, and partly upon the number of observers who took part in the various series. The means by weights are presented in the last column of the subjoined statement, and the arithmetical means are in the preceding column.

	Series	Arith. Means	Weighted Means
Eye-and-ear	11	$-0^s.0077$	$-0^s.0070$
Chronographic	19	-0.0082	-0.0078

The result of the direct comparison is confirmed.

Another question arises as to the application of the equations to observations at a great distance from the equator. The testimony from Zones I and III in Table I is entirely insufficient to afford an answer to this question. But since the amount of the equation seems to be practically independent of the magnifying power employed we may infer that the equations apply in full force to 60° of declination at least. It would be very difficult to test this question for higher declinations by actual observation without an expenditure of labor which might be regarded as disproportioned to the end to be attained. Very little harm can result from the assumption that the equation applies to all declinations; since for the higher declinations its actual effect upon the right-ascension is in proportion to cos δ.

It is evident that all our conclusions from this discussion are strictly applicable to observations of the brighter stars only. There is no reasonable doubt, however, that in most instances the inferences may be extended to the observations of stars somewhat fainter than the sixth magnitude, — perhaps to the magnitudes $7^M.0$ or $8^M.0$. As a rule the determination of magnitude-equation by the employment of screens indicates a linear relation of the magnitude-effect down to stars as faint as the eighth, or ninth, magnitude, so that most observers appear to doubt whether any term involving $(\Delta M)^2$ has any real existence. On the other hand there are doubtless exceptions to this rule, as in the cases of the Cambridge and Leipsic A.G. zones already cited. Modifying the result of Professor TURNER's paper on the right-ascensions of the Cambridge A.G. zone (*M.N.*, LX), taking into account the comparison by which he demonstrates the equality of magnitude-equations for Greenwich 80 and Cambridge relative to the brighter stars, and accepting the result of the present discussion which assigns $-0^s.007$ as the magnitude-equation of Greenwich 80, we may tabulate the magnitude-effect of Cambridge A.G. approximately as in the following table.

It is very probable that other equally striking examples of the non-linear effect of the magnitude-equation will be discovered hereafter, so that, whatever the general rule may be, its application cannot be considered as inflexible.

CORRECTION OF CAMBRIDGE A.G. FOR MAGNITUDE-EQUATION.

$1^M.0$	$+0^s.021$	$6^M.0$	$-0^s.020$
2.0	+0.014	7.0	−0.040
3.0	+0.007	8.0	−0.071
4.0	0.000	9.0	−0.116
5.0	−0.007	10.0	−0.182

Allusion should be made to one source of uncertainty which has not been mentioned in the foregoing discussion. This relates to the fact that many of the transits upon which the comparisons depend were observed in daylight. These, of course, form but a small part of the entire material, and their influence is limited to the fourth magnitude. It may reasonably be doubted whether these follow the same law as the transits obtained at night. To have separated the two classes of observations would have been impracticable in many cases, and a task of forbidding proportions in others. We may suppose, however, that much the same differential effects obtain in the daylight transits as in those of the night, and, therefore, that the commingling of results may not have produced, on the whole, serious anomalies.

In this same connection it should be observed that the determination of equinox-correction is involved in this difference between daylight and night observations; though it is extremely probable that the errors to be feared from this source are not important in comparison with constant errors in observed transits of the sun.

The Catalogue of 627 Standard Stars may readily be freed from the effect of magnitude-equation by the application to the right-ascensions of the correction, $-0^s.0077$ $(M-3.5)$. This will not disturb the position of the adopted equinox to any sensible degree; and in the case of an observer who has thoroughly determined his magnitude-equation, the adoption of this course will doubtless lead to a sensible improvement in the accordance of results for the correction of the clock. This course has been adopted in the reduction of the transits for the Albany catalogue.

The determination of absolute magnitude-equation should be regarded as an indispensable requisite in all observations aiming at precision. There is every reason to believe that this can be accomplished successfully by the use of wire-gauze screens to produce artificial diminution of magnitude, provided the amount of absorption is at least $2^M.5$. The gain in the value of the results more than compensates the comparatively small amount of extra observing and computing involved, even when the determination is, as it should be, much more complete than has usually been the case.

CONTENTS.

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THE ASTRONOMICAL JOURNAL.—SUPPLEMENT TO 536.

OBSERVATIONS OF COMET *a* 1903 (*GIACOBINI*),

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

BY C. W. FREDERICK.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1903 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Jan. 21 6 39 31	1	9, 4	+0 58.85	+1 24.0	23 0 6.61	+ 2 47 46.0	9.574	0.724	−0.29	+2.7
22 6 33 43	2	9, 4	−1 53.79	−0 49.5	23 1 10.86	+ 3 2 7.8	9.570	0.722	−0.28	+2.6
23 6 51 4	3	20, 8	+3 21.25	−0 59.6	23 2 17.39	+ 3 16 55.0	9.595	0.723	−0.30	+2.7
30 6 46 22	4	*d*10, 10	+0 31.59	+9 4.7	23 10 41.63	+ 5 7 13.6	9.611	0.716	−0.27	+2.2
Feb. 5 6 46 29	5	40, 8	+1 15.73	−4 17.7	23 18 53.17	+ 6 53 3.2	9.627	0.711	−0.29	+1.8
6 6 40 54	6	*d* 8, 10	−0 11.94	−3 18.8	23 20 21.02	+ 7 11 40.9	9.624	0.708	−0.26	+1.8
9 7 8 26	7	30, 8	+1 46.03	+4 0.4	23 24 55.10	+ 8 9 52.9	9.649	0.715	−0.25	+1.7
12 7 0 14	8	30, 6	+3 16.97	−3 13.3	23 29 42.19	+ 9 10 23.0	9.650	0.711	−0.23	+1.6
17 7 17 23	9	*d*10, 10	−0 13.45	−6 25.0	23 38 17.60	+10 57 26.9	9.663	0.716	−0.23	+1.1
19 7 15 19	10	15, 3	+7 57.27	+2 22.0	23 41 55.34	+11 41 49.5	9.665	0.716	−0.24	+1.1
20 6 42 55	11	*d*10, 10	−0 13.45	−1 44.3	23 43 44.40	+12 3 52.9	9.654	0.698	−0.20	+0.9
22 6 50 58	12	*d*10, 10	−0 25.19	−7 0.2	23 47 33.20	+12 49 23.7	9.661	0.702	−0.19	+0.8
23 7 3 13	13	19, 4	+3 4.13	−1 15.8	23 49 30.53	+13 12 21.5	9.667	0.709	−0.19	+0.8
25 7 10 53	14	*d*10, 10	+0 26.81	−2 17.2	23 53 28.22	+13 57 59.9	9.671	0.713	−0.18	+0.6

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority

THE

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FOUNDED BY B. A. GOULD.

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THE TRUE RADII OF CONVERGENCE OF THE EXPRESSIONS FOR THE RATIOS OF THE TRIANGLES WHEN DEVELOPED AS POWER-SERIES IN THE TIME-INTERVALS,

By F. R. MOULTON.

1. Introduction. The computation of the elements of an unknown orbit from three complete observations consists of two distinct parts: (*a*) the determination of the heliocentric coordinates of the body in question at the epochs of the observations; and (*b*) the determination of the elements from the time-intervals and the data given by (*a*). Of these two parts, (*b*) has given most of the theoretical and practical difficulties. In the *Astronomical Journal*, No. 510, the author has given a solution* of (*b*) which from both a practical and a theoretical stand-point leaves little to be desired.† There is one limitation to the application of the method, however, arising from the fact that it depends upon the use of infinite series which converge only for sufficiently small values of the heliocentric angular motions. But their true radii of convergence were exactly determined, and it was remarked‡ that this is not an objection of importance since the same sort of limitation occurs in (*a*).

In this paper it is proposed to find the true radii of convergence for the series employed in (*a*) in all the cases which can arise.§ It will be shown that the series developed in (*b*) always may be used when the time-intervals are such that those occurring in (*a*) are of practical value.

Tables will be inserted to make the theoretical conclusions arrived at of immediate value to the computer without labor on his part. Finally, it will be shown to what extent the question of convergence enters in some of the other principal methods of determining orbits.

2. Expressions for the Ratios of the Triangles. Let the epochs of the three observations be t_1, t_2, and t_3, and let the corresponding heliocentric distances of the body be r_1, r_2, and r_3. The triangles in question are the projections on the planes of reference of those contained between the r_j, taken two at a time, and the chord joining their extremities. There are three of these triangles arising from the three combinations of r_1, r_2, and r_3. The ratios of the projections of the triangles are equal to the ratios of the triangles themselves, the case where the plane of the orbit coincides with one of the fundamental planes being excluded. Therefore the problem may be considered in the plane of the orbit without any loss of generality.

Take the plane of motion as the xy-plane, and suppose the positive end of the x-axis is directed toward the perihelion point. Let the rectangular heliocentric coordinates at the epoch t_j be x_j, y_j, $j = 1, 2, 3$. Then the expression for the ratio of the triangles may be written in the form

$$\frac{x_i y_j - y_i x_j}{x_k y_l - y_k x_l} \tag{1}$$

where $i \neq j$, $k \neq l$, and $i, j, k, l = 1, 2,$ or 3.

It follows from well-known theorems respecting the multiplication and addition of power-series that, if x_j, y_j, etc., are developable into power-series with a common realm of convergence, then the numerator and denominator of (1) are separately developable into power-series which converge in this common realm. Therefore the first problem is to find the conditions under which x and y may be expressed as convergent power-series in $t-t_0$. Since the motion is by hypothesis undisturbed these variables are defined by the differential equations

**A General Method of Determining the Elements of Orbits of All Eccentricities from Three Observations.*

† This occasion is taken to make note of the following errors:

On p. 43, column 2, line 5, read "geocentric" instead of "heliocentric."

On p. 45, column 2, at bottom, read "r^2" instead of "r_2."

On p. 47, column 2, line 12, read

"$\cos^{-1}\left(\frac{-1}{e}\right)$" instead of "$\cos\left(\frac{-1}{e}\right)$"

‡ *loc. cit.* p. 44, column 1.

§ Dr. W. A. Hamilton has given the complete solution of the problem in the case of parabolic orbits in a memoir in *A. J.*, No. 533.

$$(2)\quad \begin{cases} \dfrac{d^2x}{dt^2} = -k^2(1+m)\dfrac{x}{r^3} \\ \dfrac{d^2y}{dt^2} = -k^2(1+m)\dfrac{y}{r^3} \end{cases}$$

Suppose $x = x_0$, $y = y_0$ at $t = t_0$. If the right members of (2) are regular for $x = x_0$, $y = y_0$, it follows that x and y are expansible as power-series in $t-t_0$ which converge so long as the modulus of this argument is sufficiently small.* Consequently, the time-intervals may always be taken small enough in the actual case so that the processes are valid, but the methods of both CAUCHY and HARZER prove only the existence of the limit without giving its true value with any approximation. For example, when the eccentricity of the orbit equals zero,

$$(3)\quad \begin{cases} x = a\cos nt \\ y = a\sin nt \end{cases}$$

where a is the major semi-axis and n is the angular velocity. In this case x and y are expansible as permanently converging power-series in $t-t_0$ for any initial t_0, while the methods of CAUCHY and HARZER give only a small value of $|t-t_0|$. In the case of parabolic orbits HARZER's existence formula† gives results which are 90% in error, the true limit being found by HAMILTON's formula.‡

Before the expression (1) is used in practice the quotient of the two power-series into which they are expanded is taken, and hence the radius of convergence is limited still further by the poles which are defined by the vanishing of the denominator. Therefore two problems are to be solved. (*A*) To find the conditions under which the rectangular coordinates may be expanded as convergent power-series in the time, and (*B*) to find the conditions under which the denominator of (1) may vanish.

3. THE SINGULAR POINTS OF THE RECTANGULAR COORDINATES CONSIDERED AS FUNCTIONS OF THE TIME. Let the following notation be adopted:

a = major semi-axis,
e = the eccentricity,
p = the parameter,
n = the mean motion,
T = the time of perihelion passage,
M = the mean anomaly,
v = the true anomaly,
E = the eccentric anomaly in elliptic motion,
F = the corresponding auxiliary in hyperbolic motion.

* This theorem, which has been the basis for a very large part of the rigorous developments of the theory of differential equations, was given by CAUCHY in *Comptes Rendus, July 4, 1842*, (*Coll. Works, 1st Series, Vol. VII, p. 5, et seq.*). HARZER, in the *Publications of the Kiel Observatory, Vol. XI, p. 24, et seq.*, has arrived at the same results in the special case under present discussion by successive simplifications of the actual series.

† *loc. cit.*, p. 30.

‡ *Astronomical Journal*, No. 533, Eq. (18).

It is necessary to treat elliptic, parabolic, and hyperbolic motion separately, and they will be discussed in this order.

(*a*) *Case of Elliptic Motion.* In elliptic motion the following equations are true:*

$$\left.\begin{aligned} p &= a(1-e^2) \\ n &= \frac{k\sqrt{1+m}}{a^{\frac{3}{2}}} \\ m &= \text{mass of body} = 0 \text{ with sufficient approximation} \\ M &= n(t-T) = E-e\sin E \\ x &= r\cos v = a\cos E - ae \\ y &= r\sin v = a\sqrt{1-e^2}\sin E \end{aligned}\right\}\quad (4)$$

Suppose the value of E at t_0 is E_0, and write $E = E_0+E_1$. Then the last two equations of (4) give

$$\left.\begin{aligned} x &= a\cos(E_0+E_1) - ae \\ y &= a\sqrt{1-e^2}\sin(E_0+E_1) \end{aligned}\right\}\quad (5)$$

from which it follows that x and y may be expanded into permanently converging power-series in E_1, where E_1 is real or complex, for every finite value of E_0. These series may be written

$$\left.\begin{aligned} x &= \sum_{i=0}^{\infty} a_i E_1^i \\ y &= \sum_{i=0}^{\infty} b_i E_1^i \end{aligned}\right\}\quad (6)$$

where the radius of convergence is infinite.

It follows from the fourth equation of (4) that E is finite for every finite value of M. Consequently x and y may be expanded into converging power-series in E_1, where E_1 is defined by the third equation of (4), for all finite values of M.

Suppose E_1 is expansible as a power-series in $M-M_0$, where M_0 is the value of M at $t = t_0$, for all $|M-M_0|<R$. If this series is substituted in the right members of (6) and the terms are rearranged with respect to powers of $M-M_0$, then power-series will be obtained which, in accordance with a theorem given by WEIERSTRASS,† converge for all $|M-M_0|<R$. It follows, therefore, that to find the conditions under which x and y may be expanded as converging power-series in $t-t_0$, or in $M-M_0$, which is essentially the same because of the relation between M and t, it is only necessary to find the conditions under which $E_1 = E-E_0$ may be expanded as a converging power-series in $M-M_0$.

Consider E as a function M. This function has singular points for certain values of M, and the expansion as a power-series in $M-M_0$ will converge, in accordance with CAUCHY's well-known theorem, for all values of $M-M_0$

* *Introduction to Celestial Mechanics*, pp. 152, 153.

† *Functionen Lehre*, p. 73. The theorem is given for LAURENT series, but this may be thought of as containing the ordinary power-series as a special case.

whose moduli are less than the distance from M_0 to the nearest singular point. The quantity E is defined in terms of M by the equation

$$E - e \sin E = M$$

The problem is to locate the singular points of E considered as a function of M from this transcendental relation. To do this consider the differential equation which relates E and M,

$$\frac{dE}{dM} = \frac{1}{1 - e\cos E} \tag{7}$$

The right member of this differential equation is analytic in E, and, in accordance with CAUCHY's theorem quoted in Section 2 of this paper, in the vicinity of every E_0 for which it is regular $E - E_0$ is developable as a power-series in $M - M_0$ which has a positive radius of convergency. Consequently, at all of those points for which the right member of (7) is regular, E considered as a function of M is regular. Conversely, for every singularity of the right member of (7), E considered as a function of M has a singular point.

The right member of (7) is uniform, and the only singularities are the poles defined by

$$1 - e\cos E = 0 \tag{8}$$

Since E is to be considered as a function of the complex variable M it will in general be complex. Suppose it has the form

$$E = u + \sqrt{-1}\, w$$

where u and w are real. Then (8) becomes

$$\begin{aligned} 1 - e\cos(u + \sqrt{-1}\, w) = 1 - e\cos u \cos(\sqrt{-1}\, w) \\ + e\sin u \sin(\sqrt{-1}\, w) = 0 \end{aligned} \tag{9}$$

Since $\cos(\sqrt{-1}\, w)$ is real while $(\sin\sqrt{-1}\, w)$ is pure imaginary this equation is equivalent to the two equations

$$\left\{ \begin{aligned} \sin u \sin(\sqrt{-1}\, w) &= 0 \\ 1 - e\cos u \cos(\sqrt{-1}\, w) = 1 - e\cos u \cosh w &= 0 \end{aligned} \right. \tag{10}$$

The first of these equations is satisfied by $w = 0$, or $u = \nu\pi$, where ν is zero or any integer. The second equation becomes in these respective cases

$$\left\{ \begin{aligned} 1 - e\cos u &= 0 \\ 1 - (-1)^{\nu} e\cosh w &= 0 \end{aligned} \right. \tag{11}$$

Since $0 \leqq e < 1$ the first of these equations is impossible, and since $\cosh w$ is positive for all values of w the second can be satisfied only if ν is even. Hence the conditions for singular points of the right member of (7) are given by

$$\left\{ \begin{aligned} &u = 2\nu\pi\ , \quad (\nu = \text{zero or any integer}) \\ &1 - e\cosh w = 0 \end{aligned} \right. \tag{12}$$

As w^2 varies from zero to infinity $\cosh w$ is an increasing monotonic function whose limits are unity and infinity; hence, for every $0 < e < 1$, the second equation of (12) is satisfied by one, and but one, value of w^2. The singular points are therefore given by

$$\left. \begin{aligned} u &= 2\nu\pi \\ w &= \pm\cosh^{-1}\left(\frac{1}{e}\right) = \pm\log\left(\frac{1+\sqrt{1-e^2}}{e}\right) \end{aligned} \right\} \tag{13}$$

where the logarithm is taken to the Naperian base.

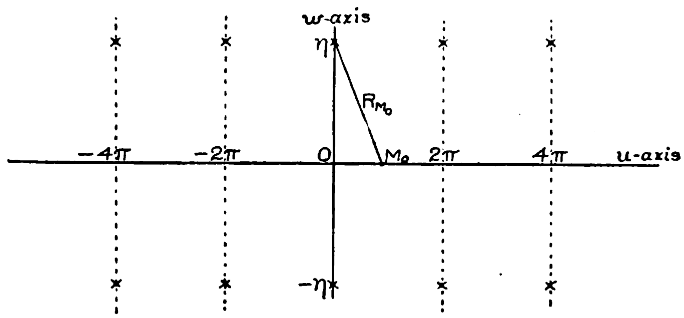
From this point on let $\bar{u}$ and $\bar{w}$ represent those values determined by (13). The real problem is to find for what values of M singular points occur, but so far only the values of E for which they occur have been found. KEPLER's equation at once enables us to solve the problem. Let $M = \xi + \sqrt{-1}\,\eta$; then

$$\xi + \sqrt{-1}\,\eta = E - e\sin E = \bar{u} + \sqrt{-1}\,\bar{w} - e\sin(\bar{u} + \sqrt{-1}\,\bar{w})$$

Expanding the last term, putting $\bar{u} = 2\nu\pi$, and equating real and imaginary parts, it is found that

$$\left. \begin{aligned} \xi &= 2\nu\pi \\ \bar{\eta} &= \bar{w} - \frac{e\cos\bar{u}\,\sin(\sqrt{-1}\,\bar{w})}{\sqrt{-1}} = \bar{w} - e\sin h\bar{w} \end{aligned} \right\} \tag{14}$$

Since $\bar{\eta}$ is an odd function of $\bar{w}$, which is two-valued, $\bar{\eta}$ is two-valued. The singular points are located at the x in the following figure.



Since E is finite for all finite values of M, these singular points are branch points.

It follows from (13) and (14) that the location of the singular points depends upon e alone, and it is easily seen that as e varies from 0 to 1 the singular points travel from infinity along the lines $2\nu\pi$ toward the real axis. In practice M_0 is always real, and, because of the symmetry of the orbit with respect to the major axis, it is sufficient in discussing the radius of convergence to suppose that $-\pi \leqq M_0 \leqq \pi$. The true radius of convergence of E when expressed as a power-series in $M - M_0$ is (see fig.)

$$R_{M_0} = \sqrt{M_0^2 + \bar{\eta}^2} \tag{15}$$

where η is defined by the second equations of (13) and (14).

It follows also from the chain of arguments given above that R_{M_0} is the true radius of convergence of the rectangular coordinates and the expressions for the triangles when expanded as power-series in $M-M_0$.

If t_0 is the value of t corresponding to M_0 the true radius of convergence for all these functions when expanded as power-series in $t-t_0$ is

$$(16) \qquad R_{t_0} = \frac{R_{M_0}}{n} = \frac{a^{\frac{3}{2}} R_{M_0}}{k}$$

(*b*) *Case of Parabolic Motion.* Dr. Hamilton treated this case in his memoir giving not only the location of the singular points and the true radius of convergence, but also a complete discussion of the Riemann surface associated with the function. His formula for the true radius of convergence is,* in the notation of this paper,

$$(17) \qquad R_{t_0} = \frac{p^{\frac{3}{2}}}{3k}\sqrt{1+\frac{9k^2(t_0-T)^2}{p^3}}$$

It can also be found more simply by the methods employed in this paper. The rectangular coordinates are expressed in terms of the true anomaly by the equations

$$(18) \qquad \left\{ \begin{aligned} x &= r\cos v = \frac{p\cos v}{1+\cos v} = \frac{p}{2}\left(1-\tan^2\frac{v}{2}\right) \\ y &= r\sin v = \frac{p\sin v}{1+\cos v} = p\tan\frac{v}{2} \end{aligned} \right.$$

The true anomaly is related to the time by the well-known equation

$$(19) \qquad \tfrac{1}{3}\tan^3\frac{v}{2}+\tan\frac{v}{2} = \frac{2k(t-T)}{p^{\frac{3}{2}}}$$

and the differential equation which relates them is

$$\frac{d\left(\tan\frac{v}{2}\right)}{dt} = \frac{2k}{p^{\frac{3}{2}}\left(\tan^2\frac{v}{2}+1\right)}$$

In accordance with the principles employed in the elliptic case $\tan\frac{v}{2}$ is a regular function of t at all points except at those in which

$$\tan^2\frac{v}{2}+1 = 0$$

The solutions of this equation are

$$(20) \qquad \tan\frac{v}{2} = \pm\sqrt{-1}$$

Substituting in (19), the values of t for which the function is singular are found to be

$$(21) \qquad (t'-T) = \pm\frac{p^{\frac{3}{2}}\sqrt{-1}}{3k}$$

* *Astronomical Journal*, No. 533, Eq. (18).

Then the true radius of convergence for any real t_0 is

$$R_{t_0} = \sqrt{(t'-T)^2+(t_0-T)^2} = \frac{p^{\frac{3}{2}}}{3k}\sqrt{1+\frac{9k^2(t_0-T)^2}{p^3}}$$

agreeing with Dr. Hamilton's results.

(*c*) *Case of Hyperbolic Motion.* In the case of hyperbolic orbits the coordinates, elements, and auxiliaries are related by the following equations: *

$$\left. \begin{aligned} p &= a(e^2-1) \quad , \quad e>1 \\ M &= \frac{k}{a^{\frac{3}{2}}}(t-T) = -F+e\sinh F \\ r &= a(-1+e\cosh F) \\ \cos v &= \frac{e-\cosh F}{-1+e\cosh F} \\ \sin v &= \frac{\sqrt{e^2-1}\sinh F}{-1+e\cosh F} \\ x &= r\cos v = ae-a\cosh F \\ y &= r\sin v = a\sqrt{e^2-1}\sinh F \end{aligned} \right\} \qquad (22)$$

It follows from the last two equations that x and y are expansible into permanently converging power-series in $F-F_0$ for any finite F_0. From the second equation it follows that F is finite for all finite values of M; therefore x and y are expansible into power-series in $F-F_0$ which converge for all finite values of M. By virtue of the theorem of Weierstrass the conditions under which $F-F_0$ may be expanded as a convergent power-series in $M-M_0$ must be found in order to solve the problem.

The problem is treated precisely as in the elliptic case. The differential equation relating F and M is

$$\frac{dF}{dM} = \frac{1}{-1+e\cosh F} \qquad (23)$$

The singular points of F considered as a function of M are located by

$$-1+e\cosh F = -1+e\cosh(u+\sqrt{-1}\,w) = 0 \qquad (24)$$

Expanding the right member and equating the real and imaginary parts to zero, it is found that

$$\left. \begin{aligned} \sinh u\sin w &= 0 \\ -1+e\cosh u\cos w &= 0 \end{aligned} \right\} \qquad (25)$$

The first of these equations is satisfied by $w = \nu\pi$, or $u = 0$. The second equation becomes in these respective cases

$$\begin{aligned} -1+(-1)^\nu e\cosh u &= 0 \\ -1+e\cos w &= 0 \end{aligned}$$

Since e is greater than unity, and $\cosh u$ is greater than unity for all values of u, the first equation cannot be satis-

* *Introd. to Cel. Mec.*, pp. 155 and 156.

fied. The singular points are consequently given by the equations

(26) $u = 0 \quad , \quad -1+e\cos w = 0$

The solutions of the second equation are

(27) $$\pm w = 2\nu\pi + \cos^{-1}\left(\frac{1}{e}\right)$$

where ν is zero or any integer, and $\cos^{-1}\left(\frac{1}{e}\right)$ is the primitive value of this multiple valued function. Hence, substituting $\pm\sqrt{-1}\,w$ for F, where w is defined by (27), in the second equation of (22), the values of M for which the functions have singular points are found to be

$$M = \xi + \sqrt{-1}\,\eta = \mp\sqrt{-1}\,w + e\sinh(\pm\sqrt{-1}\,w)$$
$$= \mp\sqrt{-1}\,w \pm \sqrt{-1}\,e\sin w$$

Therefore,

(28) $\xi = 0 \quad , \quad \pm\eta = -w + e\sin w$

Let $\bar{w} = 2\nu\pi + \cos^{-1}\left(\frac{1}{e}\right)$; then (28) becomes

(29) $\xi = 0 \quad , \quad \pm\bar{\eta} = -w - 2\nu\pi + e\sin w$

The singular points are, therefore, all on the imaginary axis, distributed symmetrically on each side of the real axis, and occurring at intervals of 2π out to infinity. The only values of M_0 used in practice are real. Hence the true radius of convergence of the expansions of F, and consequently of x, y, and the triangles, as power-series in $M-M_0$, when the motion is hyperbolic, is given by the equation

(30) $$R_{M_0} = \sqrt{M_0^2+\eta^2}$$

The true radius of convergence for expansions in powers of $t-t_0$ is found from the relation between M and t to be

(31) $$R_{t_0} = \frac{a^{\frac{3}{2}}}{k} R_{M_0}$$

4. *Numerical Results.* The preceding formulas are comparatively simple, and the numerical results can be obtained in any special case without much labor, nevertheless tables of results with convenient intervals for the arguments are desirable. The true radii of convergence when the argument is $M-M_0$ are capable of being tabulated very simply since they depend upon the two parameters e and M_0. They are given by the formulas (15) and (30). In these equations η depends upon e alone. They are defined by (14) and (29) respectively. For convenience the formulas used will be collected here.

When $0 \leqq e < 1$, then

(32) $$\left\{\begin{array}{ll} \bar{w} = \log\left(\frac{1+\sqrt{1-e^2}}{e}\right) & , \quad R_{M_0} = \sqrt{M_0^2+\bar{\eta}^2} \\ \bar{\eta} = \bar{w} - e\sinh\bar{w} & \quad R_{t_0} = \frac{a^{\frac{3}{2}} R_{M_0}}{k} \end{array}\right.$$

If a quantity M is introduced into the theory of parabolic motion for the sake of uniformity by the equation

$$M = \frac{2k(t-T)}{p^{\frac{3}{2}}}$$

the corresponding equations become, when $e = 1$,

$$\left.\begin{array}{l} R_{M_0} = \sqrt{\frac{4}{3}+M_0^2} \\ R_{t_0} = \frac{p^{\frac{3}{2}}}{2k}R_{M_0} = \frac{p^{\frac{3}{2}}}{3k}\sqrt{1+\frac{9k^2(t_0-T)^2}{p^3}} \end{array}\right\} \quad (33)$$

When $e>1$

$$\left.\begin{array}{ll} \bar{w} = \cos^{-1}\left(\frac{1}{e}\right) & R_{M_0} = \sqrt{M_0^2+\bar{\eta}^2} \\ \bar{\eta} = -\bar{w} + e\sin\bar{w} & , \quad R_{t_0} = \frac{a^{\frac{3}{2}}}{k}R_{M_0} \end{array}\right\} \quad (34)$$

From these equations Table I has been computed.

TABLE I. $\log\bar{\eta}$.

e	$\log\bar{\eta}$	e	$\log\bar{\eta}$
0	∞	.95	8.0294
.1	0.3006	1.05	8.0000
.2	0.1181	1.1	8.4624
.3	9.9637	1.2	8.8865
.4	9.8130	1.3	9.1367
.5	9.6539	2.0	9.8357
.6	9.4749	5.0	0.5479
.7	9.2584	10.0	0.9283
.8	8.9690	100.0	1.9931
.9	8.4942	1000.0	2.9993

From this table and the third, first, and third formulas of (32), (33) and (34) respectively the following table of values of $\log\frac{R_{M_0}}{k}$ has been computed.

TABLE II. $\log\frac{R_{M_0}}{k}$.

e	$M_0=0$	$M_0=60°$	$M_0=120°$	$M_0=180°$
0	∞	∞	∞	∞
.1	2.0651	2.1178	2.2260	2.3351
.2	1.8826	1.9895	2.1574	2.2965
.3	1.7281	1.9086	2.1238	2.2794
.4	1.5774	1.8553	2.1055	2.2709
.5	1.4183	1.8213	2.0953	2.2660
.6	1.2394	1.8014	2.0898	2.2635
.7	1.0228	1.7907	2.0871	2.2623
.8	0.7334	1.7862	2.0859	2.2618
.9	0.2586	1.7846	2.0855	2.2616
.95	9.7938	1.7844	2.0854	2.2616
1.0	1.5883	1.8583	2.1064	–
e	$M_0=0$	$M_0=30°$	$M_0=60°$	$M_0=90°$
1.05	9.7794	1.4835	1.7794	1.9605
1.1	0.2185	1.4840	1.7846	1.9606
1.2	0.6509	1.4880	1.7856	1.9610
1.3	0.9011	1.4978	1.7881	1.9622
2.0	1.6001	1.7000	1.8618	1.9983
5.0	2.3123	2.3171	2.3307	2.3515
10.0	2.6927	2.6935	2.6960	2.7000
100.0	3.7575	3.7575	3.7575	3.7576
1000.0	4.7637	4.7637	4.7637	4.7637

The irregularities in the numbers at $e = 1$ arise from the fact that M bears quite different relations to the true anomalies in the three kinds of conic sections.

From Table II the true radii of convergence in t may be computed from the formulas

$$(35)\quad \left\{\begin{aligned} R_{t_0} &= \frac{R_{M_0}}{k}a^{\frac{3}{2}} \quad \text{if} \quad e<1 \text{ or } e>1 \\ R_{t_0} &= \frac{R_{M_0}}{k}\frac{p^{\frac{3}{2}}}{2} \text{ if } \quad e = 1 \end{aligned}\right.$$

These formulas are so very simple that it is not worth the labor from a practical point of view to construct tables with different values of a and p. But it is of interest to know about what the time intervals are for typical cases. The average major axis in the case of asteroid orbits, where the elliptic theory has application, is about 2.65, and the following table has been computed from the first part of Table II and the first equation of (35), using this value of a.

TABLE III. R_{t_0} in days when $a = 2.65$.

e	For $M_0 = 0$	For $M_0 = 60°$	For $M_0 = 120°$	For $M_0 = 180°$
0	∞ days	∞ days	∞ days	∞ days
.1	501.2	553.0	726.0	933.7
.2	329.3	421.1	620.0	854.0
.3	230.7	349.6	573.7	821.0
.4	163.1	302.2	550.1	805.1
.5	113.0	285.9	537.3	796.0
.6	74.9	273.1	530.5	791.5
.7	45.5	266.5	527.3	789.3
.8	23.4	263.7	525.8	788.4
.9	7.7	262.7	525.3	788.0
.95	2.8	262.6	525.2	788.0

It follows from this table that the expansions of the rectangular coordinates and the triangles in power-series in the time are, in the case of the asteroid orbits where e rarely equals .3, valid for more than 200 days whatever position the body may have in its orbit. The time-interval for $M_0 = 0$ decreases very rapidly as e approaches unity, while the change is much less for other values of M_0. The physical reason for this is that the angular velocity changes most rapidly at the perihelion as the eccentricity varies. Since the major axis is kept constant the limit of the orbit as e approaches unity is a straight line of length $2a$.

Perhaps a more satisfactory idea of the effect of a change in the eccentricity upon the radius of convergence can be obtained by keeping the perihelion distance constant. The physical changes in the vicinity of the perihelion point are differences in curvature in the orbit, and the velocity of the body in its orbit. The curvature decreases continually as e varies from zero to infinity, while the velocity continually increases. The decrease in curvature tends to increase the radius of convergence, but the increase of velocity decreases it much more. The major axis is expressed in terms of q and e by

$$\left.\begin{aligned} a &= \frac{q}{1-e} \quad \text{for} \quad e<1 \\ p &= \;\; 2q \quad \text{for} \quad e=1 \\ a &= \frac{q}{e-1} \quad \text{for} \quad e>1 \end{aligned}\right\} \quad (36)$$

Taking $q = 1$ as an example, which will be fairly representative of many comets' orbits, the following table has been computed, taking $M_0 = 0$, or $t_0 = T$. For a different value of t_0 all of the corresponding radii, which are expressed in days, are larger.

TABLE IV. $\log R_{t_0}$ and R_{t_0} in days for $t_0 = T$ and $q = 1$.

e	$\log R_{t_0}$	R_{t_0}	e	$\log R_{t_0}$	R_{t_0}
0	∞	∞ days	1.0	1.7388	54.8
.1	2.1338	136.1	1.05	1.7309	53.8
.2	2.0281	106.7	1.1	1.7185	52.3
.3	1.9606	91.3	1.2	1.6994	50.1
.4	1.9101	81.3	1.3	1.6853	48.5
.5	1.8698	74.1	2.0	1.6001	39.9
.6	1.8364	68.6	5.0	1.4093	25.7
.7	1.8070	64.1	10.0	1.2614	18.3
.8	1.7819	60.5	100.0	0.7641	5.8
.9	1.7586	57.4	1000.0	0.2643	1.8
.95	1.7453	55.6			

5. *Zeroes of the Triangles.* The conditions under which the coordinates and triangles may be expanded into converging series in the time have been determined for all classes of conics. But in practice the ratios of the expansions of two triangles, instead of the expansions of single triangles, are used. Hence poles of the functions exist for all those values of the argument for which the denominator vanishes. It should be remarked that the expressions for the triangles contain factors which vanish with the time intervals. In the ratios of the triangles the quotient of these factors is taken separately, and the remaining series in the denominator divided into the corresponding one in the numerator. Thus, t_2 being the origin for the expansions,

$$\frac{x_2 y_3 - y_2 x_3}{x_1 y_2 - y_1 x_2} = \frac{t_3-t_2}{t_2-t_1} \times \qquad (37)$$
$$\left\{1 - \frac{k^2(t_3-t_2)^2 - k^2(t_2-t_1)^2}{6r_2^3} + \frac{k^2(t_3-t_2)^2 + k^2(t_2-t_1)^2}{4r_2^4}\frac{dr_2}{dt} + \ldots\right\}$$

Hence the problem is to find the zeroes of $\dfrac{x_i y_j - y_i x_j}{t_j - t_i}$. These singularities do not cancel those which were previously found, for the former were all branch points.

It is most convenient in finding the zeroes of the expression above to find them first in terms of the true anomaly. This is easily anticipated, since the triangle vanishes when

the heliocentric motion in the interval is any multiple of π. Of course, this would be the answer to the problem if it were proved that the triangles could not vanish for any complex values of the argument which have smaller moduli. Instead of using $t_j - t_i$ in the denominator $v_j - v_i$ will serve the purpose equally well, for these two quantities vanish simultaneously. The expression in question becomes in polar coordinates

(38)
$$\frac{x_i y_j - y_i x_j}{v_j - v_i} = r_i r_j \frac{(\cos v_i \sin v_j - \sin v_i \cos v_j)}{v_j - v_i} = r_i r_j \frac{\sin (v_j - v_i)}{v_j - v_i} = 0$$

Now $r_i = \dfrac{p}{1 + e \cos v_i}$ which cannot vanish for any finite value of v_i, either real or complex. A similar statement applies to r_j. Hence the condition for poles of the ratios of the triangles becomes

(39)
$$\frac{\sin (v_j - v_i)}{v_j - v_i} = 0$$

The variable v, depending on the complex variable t, will in general be complex. Suppose it has the form

$$v = \theta + \sqrt{-1}\, \varphi$$

then (39) becomes

$$\frac{\sin \{(\theta_j - \theta_i) + \sqrt{-1}\,(\varphi_j - \varphi_i)\}}{\theta_j - \theta_i + \sqrt{-1}\,(\varphi_j - \varphi_i)} = \frac{(\theta_j - \theta_i) \sin (\theta_j - \theta_i) \cosh (\varphi_j - \varphi_i)}{(\theta_j - \theta_i)^2 + (\varphi_j - \varphi_i)^2}$$
$$+ \frac{(\varphi_j - \varphi_i) \cos (\theta_j - \theta_i) \sin h\, (\varphi_j - \varphi_i)}{(\theta_j - \theta_i)^2 + (\varphi_j - \varphi_i)^2}$$
$$+ \frac{\sqrt{-1}\,(\theta_j - \theta_i) \cos (\theta_j - \theta_i) \sin h\, (\varphi_j - \varphi_i)}{(\theta_j - \theta_i)^2 + (\varphi_j - \varphi_i)^2}$$
$$- \frac{\sqrt{-1}\,(\varphi_j - \varphi_i) \sin (\theta_j - \theta_i) \cosh (\varphi_j - \varphi_i)}{(\theta_j - \theta_i)^2 + (\varphi_j - \varphi_i)^2} = 0$$

Equating the real and imaginary parts separately to zero, the conditions for poles are found to be

$$\left.\begin{aligned} \frac{(\theta_j - \theta_i) \sin (\theta_j - \theta_i) \cosh (\varphi_j - \varphi_i) + (\varphi_j - \varphi_i) \cos (\theta_j - \theta_i) \sinh (\varphi_j - \varphi_i)}{(\theta_j - \theta_i)^2 + (\varphi_j - \varphi_i)^2} &= 0 \\ \frac{(\theta_j - \theta_i) \cos (\theta_j - \theta_i) \sin h\, (\varphi_j - \varphi_i) - (\varphi_j - \varphi_i) \sin (\theta_j - \theta_i) \cosh (\varphi_j - \varphi_i)}{(\theta_j - \theta_i)^2 + (\varphi_j - \varphi_i)^2} &= 0 \end{aligned}\right\} \quad (40)$$

From these equations it is found that

(41)
$$\left\{\begin{aligned} \cos (\theta_j - \theta_i) \ \sinh (\varphi_j - \varphi_i) &= 0 \\ \sin (\theta_j - \theta_i) \ \cosh (\varphi_j - \varphi_i) &= 0 \end{aligned}\right.$$

after dividing out the factor

$$\frac{(\theta_j - \theta_i)^2 + (\varphi_j - \varphi_i)^2}{(\theta_j - \theta_i)^2 + (\varphi_j - \varphi_i)^2}$$

The solutions of the first equation are

$$\theta_j - \theta_i = (2\nu + 1)\frac{\pi}{2} \quad \text{or} \quad \varphi_j - \varphi_i = 0$$

The second equation becomes in these respective cases

$$\cosh (\varphi_j - \varphi_i) = 0 \quad , \quad \sin (\theta_j = \theta_i) = 0$$

The first equation cannot be satisfied, and the solutions of the second are

$$\theta_j - \theta_i = \nu\pi$$

Consequently the solutions of (41) are

(42)
$$\theta_j - \theta_i = \nu\pi \quad , \quad \varphi_j - \varphi_i = 0$$

In the first equation ν is any integer except zero, for equations (40) are indeterminate for $\theta_j - \theta_i$ and $\varphi_j - \varphi_i$ simultaneously zero.*

The problem is to find the relation between t_j and t_i, or more simply between M_j and M_i, for which these poles occur. The solution of this problem is simple, since t is expressible in terms of v by finite equations in each of the three classes of conics which must be considered separately.

(*a*) *Case of Elliptic Motion.* The eccentric anomaly is expressed in terms of the true anomaly by means of the equation

$$\tan \frac{E}{2} = \sqrt{\frac{1-e}{1+e}} \tan \frac{v}{2}$$

Therefore

$$\left.\begin{aligned} \tan \frac{E_i}{2} &= \sqrt{\frac{1-e}{1+e}} \tan \tfrac{1}{2} (\theta_i + \sqrt{-1}\, \varphi_i) \\ \tan \frac{E_j}{2} &= \sqrt{\frac{1-e}{1+e}} \tan \tfrac{1}{2} (\theta_i + \nu\pi + \sqrt{-1}\, \varphi_i) \end{aligned}\right\} \quad (43)$$

If ν is odd

$$\tan \frac{E_j}{2} = -\sqrt{\frac{1-e}{1+e}} \cot \tfrac{1}{2} (\theta_i + \sqrt{-1}\, \varphi_i) = -\frac{(1-e)}{1+e} \cot \frac{E_i}{2}$$

If ν is even

$$\tan \frac{E_j}{2} = \tan \frac{E_i}{2}$$

Then the mean anomalies are computed from the equations

$$\left.\begin{aligned} M_i &= E_i - e \sin E_i \\ M_j &= E_j - e \sin E_j \end{aligned}\right\} \quad (44)$$

and the time interval from

$$t_j - t_i = \frac{M_j - M_i}{n} = \frac{a^{\frac{3}{2}} (M_j - M_i)}{k} \quad (45)$$

* Dr. HAMILTON gave these results for parabolic orbits, *loc. cit.*, p. 54.

In practice t_i and t_j represent two of t_1, t_2, t_3, and t_2 is taken as the origin of time for the expansions. Since t_2 is real it follows that v_2 is real, and then from second equation of (42) that if one of the epochs t_i, t_j is t_2 the other is real. The limit on the time-interval is that the heliocentric motion of the body during it shall be less than 180°. If $t_j - t_i = t_3 - t_1$ there are two quantities limited by but a single equation, and one of them may be taken arbitrarily. From (42) it follows that if one is real they both are, and the modulus becomes a minimum. Hence in all cases $\varphi_i = \varphi_j = 0$. The time-interval must be such that the heliocentric motion during it shall be less than 180°. This leads to the following interesting result. Suppose an observation has been made at t_1, and that two more are to be made; the radius of convergence of the reciprocal of the triangle does not depend upon the way in which the second observation divides the whole interval, at least except so far as the intervals $t_2 - t_1$ and $t_3 - t_2$ are limited by the branch points previously discussed.

(*b*) *Case of Parabolic Motion.* The formula for parabolic orbits is

(46)

$$t_j - t_i = \frac{p^{\frac{3}{2}}}{2k}\left\{\tan\frac{(v_i+\pi)}{2} - \tan\frac{v_i}{2} + \tfrac{1}{3}\left(\tan^3\frac{(v_i+\pi)}{2} - \tan^3\frac{v_i}{2}\right)\right\}$$

(*c*) *Case of Hyperbolic Motion.* The formulas for hyperbolic orbits are

$$(47)\quad\left\{\begin{aligned}
\tan\frac{F_i}{2} &= \sqrt{\frac{e-1}{e+1}}\tan\frac{v_i}{2}\\
\tan\frac{F_j}{2} &= \sqrt{\frac{e-1}{e+1}}\tan\left(\frac{v_i+\pi}{2}\right) = -\sqrt{\frac{e-1}{e+1}}\cot\frac{v_i}{2}\\
M_i &= -F_i + e\sinh F_i\\
M_j &= -F_j + e\sinh F_j\\
t_j - t_i &= \frac{M_j - M_i}{n} = \frac{a^{\frac{3}{2}}(M_j - M_i)}{k}
\end{aligned}\right.$$

In case the triangle between r_1 and r_3 occurs in the denominator the expansions of the ratios the triangles are never valid when the whole heliocentric motion in the interval of time $t_3 - t_1$ is as much as 180°. Notwithstanding this fact much space has been used in explaining the solutions of certain equations depending on these expansions in the case where this limit is exceeded.* If the heliocentric motion in the whole interval is less than 180°, the series are always convergent unless the intervals are still further limited by the branch points of the functions. The formulas given in section 3, or the tables of section 4, show when this will occur.

6. *Numerical Results.* For practical use a table giving $\log\left(\frac{M_3 - M_1}{k}\right)$ will be most servicable since it depends only upon the arguments e and v_1. The time in days can be found by multiplying this quantity by $a^{\frac{3}{2}}$ in the case of elliptic and hyperbolic orbits, and by $\frac{p^{\frac{3}{2}}}{2}$ in the case of parabolic orbits. It will be sufficient to give the results for $v_1 = -90°$, $v_1 = 0$, and $v_1 = +90°$ in elliptic orbits, and for $v_1 = -90°$ in the case of parabolic and hyperbolic orbits.

TABLE V. $\log\left(\frac{M_3 - M_1}{k}\right)$.

e	$v_1 = -90°$	$v_1 = 0$	$v_1 = +90°$	e	$v_1 = -90°$
0	2.2616	2.2616	2.2616	1.0	2.1904
.1	2.2026	"	2.3132	1.05	0.3835
.2	2.1350	"	2.3593	1.1	0.8273
.3	2.0568	"	2.4000	1.2	1.2646
.4	1.9647	"	2.4362	1.3	1.5158
.5	1.8540	"	2.4679	2.0	2.2444
.6	1.7161	"	2.4958	5.0	2.9594
.7	1.5358	"	2.5197	10.0	3.3476
.8	1.2789	"	2.5397	100.0	4.4199
.9	0.8349	"	2.5542	1000.0	5.4266
.95	0.3857	"	2.5595	-	-

While the preceding table enables one to compute very simply the limit with sufficient accuracy in any case that can arise, it fails to give one at a glance the way the limit changes with the eccentricity. For this purpose the following table was computed keeping the perihelion distance equal to unity.

TABLE VI. $t_3 - t_1$ in days for $q = 1$, $v_1 = -90°$.

e	$t_3 - t_1$	e	$t_3 - t_1$
0	182.6	1.0	219.2
.1	186.8	1.05	216.3
.2	190.8	1.1	212.5
.3	194.7	1.2	205.6
.4	198.4	1.3	199.5
.5	202.1	2.0	175.6
.6	205.7	5.0	113.9
.7	209.1	10.0	82.5
.8	212.5	100.0	26.7
.9	215.9	1000.0	8.5
.95	217.4		

These intervals for $v_1 = -90°$ are the shortest for which any of the three triangles can vanish when the perihelion distance is unity. If the interval between the first and last observations is less than the number given in the table no trouble can arise from this singularity.

The real question is whether the radius of convergence of the quotient of two triangles is determined by the poles or the branch points. To settle this question for that part of the orbit where the radius of convergence is least compare Tables IV and VI. It is necessary to make some assumption regarding the way the second observation di-

* OPPOLZER, *Bahnbestimmung*, pp. 79, 93. WATSON, *Theoretical Astronomy*, pp. 186-7.

vides the whole interval. For simplicity suppose it divides it into two equal parts. Then, to get the whole interval as determined by the branch points, it is necessary to multiply the numbers of Table IV by two, while those in VI are to be taken as they stand. It is at once seen that so long as $e \overline{\overline{<}} .2$ the true radius is determined by the poles defined by the vanishing of the triangle in the denominator; and so long as $e \overline{\overline{>}} .3$ the true radius of convergence is determined by the branch points which were found in section 3. This is true for all orbits as well as for $q = 1$ since the limits in both cases depend upon the linear dimensions of the orbit in the same manner.

If the triangle contained between r_1 and r_3 does not occur in the denominator the numbers in Table VI should be multiplied by two, for the whole interval will be about twice as long as $t_3 - t_2$ or $t_2 - t_1$. In this case the branch points determine the true radius of convergence if $e \overline{\overline{>}} .1$. In no case does the convergence of the series persist until the whole heliocentric motion gets near 180° except when $e < .2$.

7. *Comparison with the Series used in the Astronomical Journal*, No. 510. In the paper in *loc. cit.*, solving the second part of the problem of determining the elements of an unknown orbit, certain series were employed which were shown to converge if the heliocentric motion in the intervals $t_2 - t_1$ and $t_3 - t_2$ were not too great. The precise radius of convergence for various values of the eccentricity was given in Table I, p. 48. The question of interest is whether the series used in that paper converge so long as those which express the ratios of the triangles. In the determination of orbits the whole interval of heliocentric motion must be less than 180° as has been shown. Consequently, so long as the radius of convergence is greater than 90° for the series used in *A.J.* No. 510, the second part of the problem is solvable by the method developed there if the first part is by the usual method. It is seen from Table I, *loc. cit.* p. 48, that for the middle observation in any part of the orbit the series are valid over an interval greater than 180° if $e \overline{\overline{<}} .1$; and, when $t_2 = T$, the interval is greater than 180° for all finite values of e. For $e = 1$, $t_2 = T$, the limit is precisely 360°.

It was shown in the preceding section that the true radius of convergence is defined by the poles as long as the eccentricity has a value less than .3. It follows from Table I, *A.J.* No. 510, that the series developed there for all $e < .3$ are valid for greater intervals than those giving the ratios of the triangles except when $v_2 > 120°$, in which case the limit is a little less for $e = .2$ and $e = .3$. Therefore no trouble can arise in the asteroid orbits in using the method of solving the second part of the problem of orbits which was developed in the former paper. It also follows from Table I, *loc. cit.*, and the results of this paper that in case of parabolic orbits the proposed series converge for greater intervals than those for the ratios of the triangles so long as the true anomaly at the time of the second observation does not exceed 90°. This is taking into consideration only the poles, and the branch points show that the radius of convergence of the series of the former paper are greater than those in this for $e = 1$ for values of v_2 considerably greater than 90°.

This is all that is desired from a practical point of view, for observations of comets are not usually made when the true anomaly is very great. The conclusion is that whenever the series for the ratios of the triangles are of practical use those developed in *A.J.* No. 510 for finding the elements may also be used. It was shown in the former paper that three terms of the series used in finding the elements give results accurate to the sixth place when $e = 1$, $q = 1$ and $t_3 - t_2 < 13$ days, $t_2 - t_1 < 13$ days. In the expression for the ratio of the triangle between r_3 and r_2 to that between r_2 and r_1 the term of the fourth degree contains $\frac{k^4 (t_3 - t_2)^4}{36\, r_2^{\,6}}$, the remainder of it vanishing if $t_3 - t_2 = t_2 - t_1$. If $t_3 - t_2 = 13$ days, $q = 1$, and $v_2 = 0$ the numerical value of this term is .00007, giving an error of seven units in the fifth place if it is omitted.

8. *The Convergence of the Series used in Other Methods.* The preceding investigations exhibit in a conspicuous manner the weakness of the Olbers and Gauss methods, which can be used only when the intervals of time between the observations are comparatively short. On the other hand the elements are better determined when the points on the orbit are not very near together. Thus, the computer finds himself limited in both directions, and in most cases he can secure satisfactory agreement between theory and observations only by differential corrections based on errors in an ephemeris. A thorough discussion of what intervals are most advantageous in the various possible cases which can arise in practice is much to be desired, and the results contained in this paper furnish a solid foundation for such an investigation.

Another question of much interest and practical importance is whether some of the other methods, which are short enough to be of practical value, are not free from the limitations to which the method of Gauss is subject. Most conspicuous of these is that first developed by Laplace.*

The same general ideas have been followed out by Villarceau in an exhaustive memoir,† which for some unexplained reason is not referred to by later writers using the same fundamental ideas. The process has been carried out to terms of higher order, but at the price of great complexity, by Harzer.‡ More recently Leuschner has

* *Mécanique Céleste*, Vol. I, Part I, Book II, Chap. IV.
† *Annales de l'Observatoire Impérial de Paris*, Vol. III.
‡ *Astronomische Nachrichten*, No. 3371.

developed it* so as to make its practical application at every point as simple as possible. In a very suggestive preface to TISSERAND's *Leçons*, POINCARÉ has commented on the fundamental ideas in the methods of GAUSS and LAPLACE, and has shown that they are essentially the same in the first approximation.

In all of these expositions developments of the geocentric polar coordinates as power-series in the time are used. The present question relates to their convergence. Let the geocentric rectangular coordinates be ξ, η, ζ and the polar coordinates ρ, α, δ. The rectangular coordinates are related to the rectangular heliocentric coordinates by linear equations with constant coefficients, and they have, therefore, the same radius of convergence. The rectangular and polar coordinates are related by the equations

$$(48)\quad \left\{ \begin{aligned} \xi &= \rho\cos\alpha\cos\delta \\ \eta &= \rho\sin\alpha\cos\delta \\ \zeta &= \rho\sin\delta \end{aligned} \right.$$

From these equations it follows that

$$(49)\quad \left\{ \begin{aligned} \rho &= \sqrt{\xi^2+\eta^2+\zeta^2} \\ \alpha &= \tan^{-1}\left(\frac{\eta}{\xi}\right) \\ \delta &= \tan^{-1}\frac{\zeta}{\sqrt{\xi^2+\eta^2}} \end{aligned} \right.$$

The singular points of these equations are defined by

$$(50)\quad \left\{ \quad \xi^2+\eta^2+\zeta^2=0 \quad , \quad \frac{\eta}{\xi}=1 \quad , \quad \frac{\zeta}{\sqrt{\xi^2+\eta^2}}=1 \right.$$

If these equations were expressed in terms of the elements of the body and the earth, and of the time, and if the resulting equations were solved for the time (using complex values) the singular points of the functions would be found. There would be no serious practical difficulty in the matter, since these coordinates are expressible linearly in terms of the heliocentric coordinates which are expressed in terms of the time for all conics by means of well-known equations. It is not intended to enter into the details of this matter here. It is sufficient to point out that when $e \overline{\gtrless} .3$ the developments of ξ, η, and ζ, and consequently of ρ, α, and δ converge only so long as the expressions for the ratios of the triangles converge. In this case the LAPLACIAN method has no advantage from this point of view over the GAUSSIAN. It is to be noted further that the singularities (50) depend not only upon the eccentricity and parameter of the orbit in question, and upon the position of the body in its orbit, but also upon the elements which define the plane of the orbit, and the coordinates of the earth.

It is evident that these many parameters might occur in such a manner that one of equations (50) would be fulfilled for a time-interval of very small modulus, when the method would fail for even short intervals between the observations. For example, the expansions would soon fail if the body were near the pole. This is only a fault of the method, and not an inherent difficulty, for a change to ecliptic coordinates will avoid it. When the eccentricity is less than .3 the LAPLACIAN expansions may converge longer than the GAUSSIAN, but there is no guarantee of it in general. The conclusion is that the two methods are subject to the same general restrictions, though in special cases each may be better than the other.

There are three problems worthy of solution by the more powerful methods of modern mathematics. (*a*) To find under what conditions the data furnished by three observations are *essentially* insufficient to define the elements of the orbit; (*b*) to find under what conditions the same data are insufficient to define the elements by the LAPLACIAN method; and (*c*), the same problem for the GAUSSIAN method. Perhaps the answer in the three cases is the same. Dr. HAMILTON has answered (*a*) in many, if not all, cases of parabolic orbits in a memoir still unpublished, by a direct discussion of the Jacobian of the coordinates with respect to the elements.

* *Publications of the Lick Observatory*, Vol. VII, Part I.

The University of Chicago, 1903 *March* 28.

OBSERVATION OF TURNER'S "*NOVA*," (2387 — *GEMINORUM*),*

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By C. W. FREDERICK.

[Communicated by Captain COLBY M. CHESTER, U.S.N., Superintendent.]

1903 W.M.T.		Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	Red. to App. Place	
March 31	8^h	29 , 6	$+1^m\ 17^s.54$	$-2'\ 20''.9$	$6^h\ 38^m\ 1^s.73$	$+30°\ 2'\ 21''.8$	$+1^s.18$	$-6''.9$

Mean Place of Comparison-Star for the beginning of the year.

α	δ	Authority
$6^h\ 36^m\ 43^s.01$	$+30°\ 4'\ 49''.6$	½ [Leiden A.G. 2783 + Cambridge, Eng., A.G. 3447]

The comparisons in α were made by transits.

The position of the *Nova* reduced to 1903.0 is: $6^h\ 38^m\ 0^s.54\ \ +30°\ 2'\ 28''.7$.

* *From Supplement to No. 536.*

PHOTOGRAPHIC OBSERVATION OF THE MINOR PLANET, (60) *ECHO*,*

OBTAINED WITH THE 6-INCH STAR-CAMERA AT THE U. S. NAVAL OBSERVATORY,

By G. H. PETERS.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent.]

The minor planet (60) *Echo*, which was discovered by FERGUSON at this Observatory, was picked up in 1899 by photography. Before its elements were determined it was considered a new discovery, but subsequently was identified as *Echo*.

This asteroid was photographed at the Naval Observatory on April 17, 1903, on a plate exposed from 12^h to $13^h\,15^m$ W.M.T., and the following correction to the position in the *Berliner Jahrbuch* determined.

Correction $\Delta\alpha\ +1^m.7$ $\Delta\delta\ -6'$.

* *From Supplement to No. 536.*

PHOTOGRAPHIC OBSERVATIONS OF MINOR PLANETS,

OBTAINED WITH THE 6-INCH STAR CAMERA AT THE U.S. NAVAL OBSERVATORY,

By G. H. PETERS.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent.]

The appended corrections to the *Berliner Jahrbuch* positions were determined from photographic trails of the asteroids given below. In the case of (236) *Honoria* no observations are noted since 1890.

Asteroid	Date	Correction	
(83) *Beatrix*	April 27, 1903	$\Delta\alpha$ $-2^m.3$	$\Delta\delta$ $+13'$
(236) *Honoria*	April 28, 1903	-6.6	$+28$
(335) *Roberta*	April 28, 1903	-2.0	$+3$

OBSERVATIONS OF COMET *a* 1903 (GIACOBINI),*

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By C. W. FREDERICK.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1903 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
Jan. 21 6 39 31 (h m s)	1	9, 4	+0 58.85 (m s)	+1 24.0 (′ ″)	23 0 6.61 (h m s)	+ 2 47 46.0 (° ′ ″)	9.574	0.724	−0.29 (s)	+2.7 (″)
22 6 33 43	2	9, 4	−1 53.79	−0 49.5	23 1 10.86	+ 3 2 7.8	9.570	0.722	−0.28	+2.6
23 6 51 4	3	20, 8	+3 21.25	−0 59.6	23 2 17.39	+ 3 16 55.0	9.595	0.723	−0.30	+2.7
30 6 46 22	4	*d*10, 10	+0 31.59	+9 4.7	23 10 41.63	+ 5 7 13.6	9.611	0.716	−0.27	+2.2
Feb. 5 6 46 29	5	40, 8	+1 15.73	−4 17.7	23 18 53.17	+ 6 53 3.2	9.627	0.711	−0.29	+1.8
6 6 40 54	6	*d* 8, 10	−0 11.94	−3 18.8	23 20 21.02	+ 7 11 40.9	9.624	0.708	−0.26	+1.8
9 7 8 26	7	30, 8	+1 46.03	+4 0.4	23 24 55.10	+ 8 9 52.9	9.649	0.715	−0.25	+1.7
12 7 0 14	8	30, 6	+3 16.97	−3 13.3	23 29 42.19	+ 9 10 23.0	9.650	0.711	−0.23	+1.6
17 7 17 23	9	*d*10, 10	−0 13.45	−6 25.0	23 38 17.60	+10 57 26.9	9.663	0.716	−0.23	+1.1
19 7 15 19	10	15, 3	+7 57.27	+2 22.0	23 41 55.34	+11 41 49.5	9.665	0.716	−0.24	+1.1
20 6 42 55	11	*d*10, 10	−0 13.45	−1 44.3	23 43 44.40	+12 3 52.9	9.654	0.698	−0.20	+0.9
22 6 50 58	12	*d*10, 10	−0 25.19	−7 0.2	23 47 33.20	+12 49 23.7	9.661	0.702	−0.19	+0.8
23 7 3 13	13	19, 4	+3 4.13	−1 15.8	23 49 30.53	+13 12 21.5	9.667	0.709	−0.19	+0.8
25 7 10 53	14	*d*10, 10	+0 26.81	−2 17.2	23 53 28.22	+13 57 59.9	9.671	0.713	−0.18	+0.6

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	22 59 8.05 (h m s)	+2 46 19.3 (° ′ ″)	Albany, A.G. 7960	8	23 26 25.45 (h m s)	+ 9 13 34.7 (° ′ ″)	Leipzig II, A.G. 11676
2	23 3 4.93	+3 2 54.7	Albany, A.G. 7980	9	23 38 31.28	+11 3 50.8	Leipzig I, A.G. 9412
3	22 58 56.44	+3 17 51.9	Albany, A.G. 7957	10	23 33 58.31	+11 39 26.4	Leipzig I, A.G. 9387
4	23 10 10.31	+4 58 6.7	Albany, A.G. 8028	11	23 43 58.05	+12 5 36.3	Leipzig I, A.G. 9451
5	23 17 37.73	+6 57 19.1	Leipzig II, A.G. 11622	12	23 47 58.58	+12 56 23.1	Leipzig I, A.G. 9469
6	23 20 33.22	+7 14 57.9	Leipzig II, A.G. 11641	13	23 46 26.59	+13 13 36.5	Leipzig I, A.G. 9461
7	23 23 9.32	+8 5 50.8	Leipzig II, A.G. 11652	14	23 53 1.59	+14 0 16.5	Leipzig I, A.G. 9505

The first observation by W. W. DINWIDDIE. Comparisons in *a* were directly determined by micrometer when marked *d*.

* *From Supplement to No. 536.*

OBSERVATIONS OF COMET *d* 1902 (GIACOBINI),*

MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER MCCORMICK OBSERVATORY, UNIVERSITY OF VIRGINIA,

By T. McN. SIMPSON, Jr.

1903 Charl. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 18 10 59 8	1	–, 9	. . .	–0 53.5	. . . .	+19 22 53.6	. .	9.516	. .	–11.1
11 11 48	1	6, –	+2 21.66	. . .	6 36 37.06	. . . .	9.464	. .	+1.82	. .
20 11 28 40	2	–, 5	. . .	–4 51.9	. . . .	+19 58 46.5	. .	9.523	. .	–11.1
11 47 50	2	11, –	–0 46.52	. . .	6 36 30.78	. . . .	9.562	. .	+1.83	. .
21 11 18 37	3	–, 12	. . .	–1 36.0	. . . .	+20 16 15.9	. .	9.513	. .	–10.9
23 9 13 57	4	–, 8	. . .	+1 15.2	. . . .	+20 49 23.4	. .	9.420	. .	–10.7
9 15 10	4	d14, –	–0 5.96	. . .	6 36 33.66	. . . .	9.028	. .	+1.79	. .
24 9 41 12	5	8, 8	–1 26.92	+3 39.0	6 36 38.12	+21 6 44.4	9.228	9.428	+1.78	–10.7
25 11 20 38	6	8, 8	+3 7.98	+2 34.3	6 36 44.34	+21 24 44.1	9.552	9.513	+1.75	–10.4
26 10 17 17	7	d16, 12	–0 20.32	–0 22.3	6 36 51.64	+21 40 40.3	9.407	9.499	+1.75	–10.3
Mar. 3 9 51 56	8	12, 8	–1 12.11	+4 59.6	6 37 54.23	+23 0 59.1	9.390	9.414	+1.67	– 9.9
4 11 33 31	9	16, 10	–0 49.22	–1 48.2	6 38 12.32	+23 17 36.4	9.619	9.532	+1.66	– 9.8

Mean Places of Comparison-Stars for 1903.0.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	6 34 13.58	+19 23 58.2	A.G. Berlin, A, 2297	6	6 33 34.61	+21 22 20.2	A.G. Berlin, B, 2505
2	6 37 15.47	+20 3 49.5	A.G. Berlin, B, 2544	7	6 37 10.21	+21 41 12.9	A.G. Berlin, B, 2543
3	6 36 14.92	+20 18 2.8	A.G. Berlin, B, 2533	8	6 39 4.67	+22 56 9.4	A.G. Berlin, B, 2564
4	6 36 37.83	+20 48 18.9	A.G. Berlin, B, 2535	9	6 38 59.88	+23 19 34.4	A.G. Berlin, B, 2563
5	6 38 3.26	+21 3 16.1	A.G. Berlin, B, 2552				

NOTES: *d* refers to direct micrometrical measurements. March 4 — Comet faint, observations interrupted by clouds.

Charlottesville, Va.

* *From Supplement to No. 536.*

RESULTS OF OBSERVATIONS WITH THE ZENITH TELESCOPE, FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA,

By C. L. DOOLITTLE.

The following series is a continuation of that found in the *Astronomical Journal* of October 16, 1891 (No. 509).

The value of the constant of aberration resulting from this series is 20″.513 ±.009.

$\varphi = 39° \; 58' +$

1901		IV	No.	I	No.
		″		″	
Oct.	1	1.87	8	..	..
	3	2.01	9	1.97	8
	4	2.06	9	2.00	8
	5	1.87	9	1.96	6
	6	2.19	9	2.12	9
	7	2.03	9	1.87	10
	8	1.93	9	1.94	9
	10	1.93	9	..	..
	15	2.15	9	1.81	8
	16	1.75	9	2.02	10
	17	2.20	8	1.88	10
	18	1.97	9	..	..
	19	2.18	8	2.17	10
	21	2.04	9	2.15	10
	23	1.99	9	2.14	10
	24	2.06	9	1.94	10
	25	2.06	9	2.01	10
	26	1.90	9	2.14	10

1901		IV	No.	I	No.
		″		″	
Oct.	27	2.07	9	2.10	10
	29	1.97	9	1.92	10
	30	2.09	9	2.13	5
Nov.	1	..	..	2.06	10
	2	2.19	9	1.99	10
	3	2.19	9	2.00	10
	6	2.00	9	2.07	10
	7	1.88	8	..	..
	8	..	..	2.09	10
	9	1.96	8	2.23	10
	10	2.06	9	2.06	10
	12	2.10	9	..	..
	14	1.95	8	..	..
	15	..	..	1.98	10
	19	2.10	9	2.14	10
	20	1.98	9	1.98	10
	21	1.96	9	2.01	10
	22	1.90	9	2.19	6

1902		I	No.	II	No.
		″		″	
Jan.	25	2.18	9	..	..
	27	..	..	2.31	10
	28	2.39	7	..	..
	30	2.24	10	..	..
Feb.	2	2.20	10	2.15	10
	3	2.07	10	..	..
	4	2.20	10	2.13	9
	5	1.81	10	..	..
	6	2.11	10	..	..
	7	2.29	10	2.36	10
	8	2.21	10	2.06	10
	9	2.32	4	..	..
	10	2.35	10	2.39	10
	11	1.88	7	2.06	2
	13	2.28	10	2.19	10
	14	2.30	10	2.22	2
	15	2.47	10	2.24	10
	18	2.30	10	2.16	10
	19	2.49	10	2.31	10
	22	..	..	2.21	10
	23	2.24	10	..	..
	24	1.94	4	..	..
Mar.	1	1.95	5	..	..
	2	2.19	6	..	..
	3	2.22	10	..	..

1902		I	No.	II	No.
		″		″	
Mar.	6	2.21	10	2.27	10
	9	2.16	3	..	..
	10	2.33	10	2.41	10
	11	1.87	10	2.02	8
	14	..	..	2.23	5

1902		II	No.	III	No.
		″		″	
May	5	2.40	10	2.49	9
	8	2.08	9	2.14	10
	9	2.60	10	2.42	10
	11	2.34	10	2.35	10
	13	..	..	2.46	10
	14	2.44	10	2.32	10
	15	2.17	10	2.26	10
	17	2.26	6	..	..
	19	2.36	9	2.24	10
	21	..	..	2.00	10
	22	2.23	10	2.19	3
	24	2.06	10	..	..
	28	2.30	10	2.13	10
	29	2.09	10	2.03	10
	30	2.13	9	2.22	10
	31	1.98	10	2.18	10
June	1	2.03	10	2.07	9

1902		II	No.	III	No.
		″		″	
June	2	2.00	6	..	..
	4	2.51	7	2.45	9
	5	2.10	10	2.48	9
	6	2.10	9	2.02	7
	7	..	..	2.16	10
	8	2.48	10	2.45	8
	9	1.92	8	2.03	10
	10	2.14	5	..	..
	12	2.17	6	..	..
	17	..	..	2.23	10
	19	..	..	2.21	10
	21	..	..	2.25	5
	22	..	..	2.34	10
	23	..	..	2.36	10
	24	..	..	1.99	8
	27	..	..	2.32	2
July	4	..	..	2.04	2

1902		III	No.	IV	No.
		″		″	
July	8	2.18	10	2.16	7
	10	2.29	10	2.53	9
	11	2.01	10	2.02	8
	12	2.00	10	2.13	9
	13	2.02	10	1.98	9
	14	2.02	10	2.00	9
	15	1.99	8	2.16	9
	16	2.31	10	2.11	9
	17	2.00	10	2.02	9
	22	2.35	10	2.37	8
	23	2.15	2	..	..
	24	2.00	8	..	..
	27	2.02	10	1.94	9
Aug.	1	2.15	7	..	..
	2	2.16	8	2.00	9
	4	2.19	10	2.04	9
	6	..	..	2.37	9
	7	2.02	7	..	..
	8	2.10	1	2.24	9
	9	2.01	10	2.01	6
	11	2.12	10	1.98	9
	12	2.11	10	2.24	9
	14	2.14	10	1.97	9
	16	2.13	9	2.25	9
	17	2.20	10	2.01	9
	18	1.91	10	2.16	9

1902		IV	No.	I	No.
		″		″	
Aug.	30	1.89	9	..	..
Sept.	2	2.40	1	..	..
	4	2.19	9	..	..
	5	2.30	9	..	..
	6	2.05	9	..	..
	7	1.93	9	..	..
	10	2.12	9	..	..
	11	1.94	9	..	..
	13	2.18	9	..	..
	14	2.07	9	..	..
	15	2.28	9	..	..
	16	2.05	9	..	..
	17	2.18	3	..	..
Oct.	9	..	..	2.07	8
	12	1.98	9	..	..
	14	2.08	9	1.93	10
	15	2.07	9	1.91	10
	19	1.96	9	2.13	10
	20	2.17	8	1.98	10
	21	2.08	9	2.10	10
	22	1.95	9	1.89	10
	23	1.69	3	..	..
	25	2.17	9	..	..
	28	1.97	9	2.20	9
	29	..	..	1.88	10
	30	..	..	1.89	10
	31	2.06	9	1.95	10
Nov.	1	1.83	9	2.32	10
	2	1.92	9	1.90	9
	3	1.90	7	2.01	6
	7	2.13	7	2.00	7
	14	1.84	9	..	..
	15	2.06	8	..	..
	19	2.05	9	2.10	6
	20	1.97	9	..	..
	21	..	..	1.88	10
	23	1.85	9	1.83	10
	27	1.99	9	1.94	9
	28	1.88	9	1.91	9
	29	1.81	5	..	..
Dec.	1	1.90	8	2.06	10

Weighted Mean Date	φ	No.
1901-2	° ′ ″	
Oct. 9	39 58 1.974	184
Oct. 26	2.060	194
Nov. 13	2.040	192
Feb. 3	2.188	149
Feb. 16	2.264	145
Mar. 7	2.194	87
May 12	2.334	153
May 28	2.114	131
June 7	39 58 2.224	114
1902	° ′ ″	
June 21	39 58 2.234	57
July 16	2.116	213
Aug. 11	2.112	198
Sept. 9	2.098	103
Oct. 18	2.012	133
Oct. 28	2.008	130
Nov. 24	39 58 1.938	129
Whole number,		2312

MICROMETER OBSERVATIONS OF THE SATELLITE OF *NEPTUNE* IN 1901-1902 AND 1902-1903,

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY,

By E. E. BARNARD.

These measures of the satellite of *Neptune* are continuation of the observations previously printed in *A.J.* 508, etc.

The measures have all been made with a power of 700 diameters. It was thought there might be some gain in using this eyepiece, though a lower power would sometimes have shown the satellite better.

The season of 1902-1903 has been a very bad one, and the measures of the satellite in general have been difficult. An unusual amount of cloudy weather has cut down the number of nights on which observations could be made.

As in previous observations the center of *Neptune* was bisected; as the disc is not large, this can be done with great exactness, and it is believed that observations so made, in the case of this planet, are preferable to measures made from the limb or limbs.

The two sets of distance measures were made with the fixed and movable wires interchanged, so that they are essentially double distances.

OBSERVATIONS OF THE SATELLITE OF *Neptune*, 1901-1902.

1901 *August.*

90° time						Comp.
d	h m s		°	″		
27	15 57 16		92.11	. .	6	
	16 3 7		. . .	16.06	4	
	16 9 0		. . .	15.81	4	
			September.			
3	15 41 45		42.05	. .	6	
	15 47 59		. . .	12.81	5	
	15 53 0		. . .	13.00	5	
16	15 21 57		302.75	. .	7	
	15 30 23		. . .	12.89	6	
	15 36 16		. . .	12.85	6	
22	15 35 12		295.16	. .	7	
	15 40 7		. . .	13.60	5	
	15 43 34		. . .	13.51	5	

1901 *September* — Cont.

90° time d h m s	°	″	Comp.	
23 15 36 3	251.58	. .	6	
15 41 24	. . .	15.72	4	
15 45 19	. . .	15.67	4	
24 15 6 31	190.32	. .	9	
15 13 30	. . .	10.61	5	
15 17 28	. . .	10.51	5	
October.				
1 15 14 35	104.70	. .	5	
15 20 54	. . .	15.47	5	
15 25 44	. . .	15.50	5	
20 16 35 43	42.11	. .	6	
16 41 9	. . .	13.50	4	
16 45 7	. . .	13.09	5	
21 13 37 48	325.18	. .	7	
13 43 42	. . .	11.52	5	
13 47 33	. . .	11.32	5	
22 14 7 36	35.14	. .	6	The object observed,
14 14 39	. . .	9.31	5	not the satellite.
14 19 17	. . .	9.44	5	
29 13 14 22	218.76	. .	6	
13 22 28	. . .	12.46	4	
13 28 2	. . .	12.59	4	
November.				
12 16 19 58	68.03	. .	5	
16 24 19	. . .	16.36	4	
16 27 36	. . .	16.32	4	
18 14 44 37	65.43	. .	6	
14 49 5	. . .	15.67	4	
14 52 9	. . .	15.69	4	
19 10 57 0	11.77	. .	6	
11 2 29	. . .	11.20	4	
11 6 8	. . .	10.91	4	
26 15 21 28	275.95	. .	6	
15 26 43	. . .	16.31	4	
15 30 25	. . .	16.30	5	
December.				
10 11 27 30	148.65	. .	7	
11 33 8	. . .	10.90	4	
11 36 35	. . .	11.31	4	
11 41 24	148.17	. .	5	
15 9 42 0	229.16	. .	6	
9 49 6	. . .	13.94	4	
9 54 1	. . .	14.03	4	
16 11 26 4	138.90	. .	6	
11 31 18	. . .	12.38	4	
11 35 36	. . .	12.08	5	

1901 *December* — Cont.

90° time d h m s	°	″	Comp.	
17 10 9 2	88.45	. .	5	
10 14 51	. . .	16.90	5	
10 19 1	. . .	17.15	5	
22 8 11 13	138.63	. .	7	Satellite difficult.
8 19 19	. . .	12.48	5	Seeing excessively
8 24 24	. . .	10.86	5	bad.
23 8 40 1	85.59	. .	4	
9 33 4	84.65	. .	5	Question if the satel-
9 39 48	. . .	16.86	4	lite. There was a
9 42 42	. . .	16.96	4	fainter object 2″ s.f.
9 47 41	84.26	. .	6	this.
29 9 54 12	78.59	. .	6	
9 59 37	. . .	17.19	5	
10 4 49	. . .	17.10	5	
30 8 17 29	31.13	. .	7	
8 25 57	. . .	12.63	4	
8 29 25	. . .	12.92	5	
1902 *January.*				
2 8 10 43	206.83	. .	8	
8 17 39	. . .	11.78	4	
8 21 21	. . .	12.08	4	
5 9 42 50	19.16	. .	6	
9 47 41	. . .	11.34	4	
9 51 11	. . .	11.64	4	
10 9 32 44	69.51	. .	6	
9 38 11	. . .	16.45	4	
9 42 15	. . .	16.69	4	
12 8 24 5	291.85	. .	7	
8 26 54	. . .	14.01	4	
8 32 12	. . .	14.03	4	
13 8 2 32	249.83	. .	6	
8 7 21	. . .	16.44	4	
8 10 49	. . .	16.41	4	
18 7 52 7	286.27	. .	6	
7 55 57	. . .	14.85	4	
7 58 50	. . .	14.80	4	
24 11 41 50	273.06	. .	5	
11 46 53	. .	16.41	4	
11 50 44	. . .	16.33	4	
27 7 44 48	97.15	. .	6	Extremely difficult.
7 50 46	. . .	16.04	5	
7 54 12	. . .	16.56	5	
31 7 32 2	233.19	. .	5	
7 36 42	. . .	14.44	4	
7 40 30	. . .	14.59	4	

1902 *February.*

d	h m s (90° time)	°	″	Comp.	
2	7 25 23	93.12	. .	5	Excessively difficult.
	7 32 39	. . .	16.34	4	
	7 39 7	. . .	16.65	4	
7	7 13 16	145.63	. .	6	
	7 19 27	. . .	11.53	5	
	7 23 39	. . .	11.13	5	
8	6 49 19	88.66	. .	6	
	6 54 55	. . .	16.83	4	
	6 58 32	. . .	16.68	4	
15	7 0 6	37.53	. .	6	
	7 5 10	. . .	13.28	4	
	7 8 11	. . .	12.96	4	
17	6 42 2	261.50	. .	7	
	6 49 1	. . .	16.59	5	
	6 53 41	. . .	16.59	5	
24	6 48 50	204.25	. .	7	
	6 53 57	. . .	11.70	4	
	6 57 25	. . .	11.72	4	
25	6 44 52	120.49	. .	7	
	6 49 9	. . .	12.98	4	
	6 51 56	. . .	13.23	4	
		March.			
17	8 32 45	341.65	. .	9	
	8 39 34	. . .	10.14	5	
	8 42 54	. . .	10.31	5	
18	8 19 35	274.38	. .	6	
	8 24 35	. . .	15.59	4	
	8 28 24	. . .	15.74	4	
24	7 42 45	270.59	. .	7	
	7 48 59	. . .	15.81	5	
	7 53 4	. . .	16.17	5	
25	7 54 43	227.00	. .	9	
	8 2 40	. . .	14.28	4	
	8 7 19	. . .	13.69	5	
		April.			
6	7 42 4	213.17	. .	7	Single distances;
	7 49 1	. . .	12.53	5	very difficult, and lost in clouds.
8	7 54 55	78.46	. .	7	
	8 0 1	. . .	16.26	5	
	8 3 26	. . .	16.21	5	
13	7 51 17	119.12	. .	6	
	7 56 28	. . .	13.50	5	
	8 0 6	. . .	13.27	5	
14	7 36 33	73.84	. .	6	
	7 41 45	. . .	16.24	4	
	7 45 32	. . .	16.06	4	
15	7 45 35	19.58	. .	5	
	7 49 56	. . .	11.07	4	
	7 53 6	. . .	10.78	5	

Observations of the Satellite of *Neptune*, 1902–1903.

1902 *August.*

d	h m s (90° time)	°	″	Comp.	
25	16 7 47	183.85	. .	5	Exceedingly faint,
	16 12 55	. . .	10.21	5	clouds.
	16 17 18	. . .	10.25	5	
		September.			
1	15 22 18	104.17	. .	8	Difficult.
	15 27 38	. . .	15.56	4	
	15 31 26	. . .	15.32	4	
8	16 21 51	54.54	. .	8	Excessively difficult.
	16 29 56	. . .	14.05	5	
	16 34 2	. . .	14.20	5	
9	15 21 34	338.97	. .	7	
	15 28 39	. . .	10.75	4	
	15 32 47	. . .	10.77	4	
15	15 19 56	329.09	. .	7	
	15 27 44	. . .	11.18	5	Very faint and
	15 33 52	. . .	11.95	5	difficult; clouds.
16	15 12 6	273.21	. .	8	Satellite fairly well
	15 17 43	. . .	16.09	4	seen; observations
	15 22 3	. . .	15.79	4	good.
18	15 38 54	141.42	. .	7	Excessively difficult.
	15 47 17	. . .	11.46	4	
	15 51 52	. . .	11.97	4	
29	14 41 55	126.34	. .	6	Sky fogging.
	14 46 57	. . .	13.45	4	
	15 51 0	. . .	12.96	5	
		October.			
6	15 38 5	116.72	. .	6	
	15 43 4	. . .	14.20	4	
	15 46 57	. . .	14.51	5	
7	13 15 17	77.34	. .	6	Satellite very faint;
	13 23 12	. . .	16.39	4	seeing excessively
	13 30 16	. . .	16.43	5	bad.
13	15 1 27	69.44	. .	7	Satellite difficult.
	15 8 50	. . .	16.25	4	
	15 12 28	. . .	15.94	5	
14	14 46 51	5.37	. .	8	Satellite very faint.
	14 54 7	. . .	11.38	6	
	14 59 2	. . .	11.06	5	
27	17 5 13	273.98	. .	10	Satellite very faint.
	17 12 16	. . .	16.93	4	
	17 16 20	. . .	16.65	4	
		November.			
24	11 2 50	30.54	. .	6	Satellite fairly well
	11 8 12	. . .	12.27	4	seen.
	11 11 34	. . .	12.03	4	

1902 *December*.

d	90° time h m s	°	″	Comp.	
1	14 52 9	289.37	. .	7	Satellite well seen.
	14 57 15	. . .	14.99	5	
	15 1 11	. . .	15.02	5	
30	9 19 38	332.34	. .	8	Excessively difficult.
	9 27 33	. . .	11.98	4	
	9 34 32	. . .	11.73	6	

1903 *January*.

d	90° time h m s	°	″	Comp.	
12	9 52 57	261.86	. .	6	Question if the satellite; very faint.
	9 59 28	. . .	16.68	4	
	10 4 45	. . .	16.80	4	
19	9 28 24	202.34	. .	7	Very faint; in clouds.
	9 34 3	. . .	11.77	5	
	9 38 28	. . .	11.41	5	
20	7 49 38	123.08	. .	6	Satellite better seen than at any observations this season.
	7 55 16	. . .	13.22	5	
	7 58 58	. . .	13.16	5	

February.

d	90° time h m s	°	″	Comp.	
2	12 30 25	239.03	. .	6	Clouds stopp'd obsns. Excessively difficult.
9	9 24 58	344.22	. .	8	Satellite very faint.
	9 32 44	. . .	10.76	5	
	9 37 37	. . .	10.78	5	

1903 *February* — Cont.

d	90° time h m s	°	″	Comp.	
16	8 45 43	272.99	. .	8	Satellite very difficult.
	8 54 16	. . .	16.13	5	
	8 57 56	. . .	15.79	5	
17	8 35 29	231.00	. .	9	
	8 39 29	. . .	14.09	4	
	8 42 47	. . .	13.70	4	
23	8 42 21	223.58	. .	6	Satellite easy.
	8 48 11	. . .	13.31	5	
	8 52 33	. . .	13.19	5	
24	9 39 17	136.30	. .	8	Satellite well seen.
	9 45 33	. . .	12.00	5	
	9 49 8	. . .	12.00	5	

March.

d	90° time h m s	°	″	Comp.	
2	8 16 14	132.06	. .	10	Satellite faint.
	8 22 36	. . .	12.57	5	
	8 26 13	. . .	12.49	5	
25	8 19 27	175.82	. .	7	Satellite well seen.
	8 26 34	. . .	10.63	5	
	8 31 1	. . .	10.58	5	
30	10 2 0	235.44	. .	5	
	10 6 5	. . .	14.57	4	
	10 9 25	. . .	14.15	5	

The times are all six hours slow of Greenwich.

Yerkes Observatory, Williams Bay, Wis., 1903 *April* 15.

COMET *b* 1903.

[From RITCHIE'S Circular, No. 134, of May 8.]

A cable message from Dr. KREUTZ via Harvard College Observatory, received May 2, announced the discovery of a comet by GRIGGS of Thames, N.Z., on April 17, a position secured by Mr. TEBBUTT of Windsor, N.S.W., accompanying the announcement. The latter position is the following:

April 26.8617 Gr. M.T., R.A. $4^h\ 3^m\ 1^s.6$, Decl. $-16°\ 23'\ 25''$.

The daily motion of the object was given, 1° 26′ in R.A., and south 27′ in Declination.

A later message gives the following orbit, computed by Dr. KREUTZ from observations of April 26, 29 and May 1.

ELEMENTS.

T = 1903 March 25.51 Greenw. M.T.

$$\left.\begin{array}{l}\pi = 186°\ 41'\\ \Omega = 213\ \ 15\\ i = \ \ 66\ \ 30\end{array}\right\}\text{Mean Eq. 1903.0}$$

q = 0.5135

EPHEMERIS.

Gr. Midnight 1903	R.A. h m s	Decl. ° ′	Br.
May 9	5 14 44	−21 1	0.60
13	5 36 44	−22 3	
17	5 58 8	−22 54	
21	6 18 52	−23 35	0.38

Brightness at discovery = 1.

CONTENTS.

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ON THE PHOTOGRAPHIC EFFICIENCY OF A 13-INCH REFLECTOR OF 20-INCHES FOCUS,

By J. M. Schaeberle.

For certain lines of astronomical work the efficiency of the mirror described in this paper seems to be remarkably great. A brief review of the well-known theoretical and practical principles involved may be desirable before comparing the photographic results with the data given by powerful existing telescopes.

If I denotes the intensity of the image of a large celestial area as formed by a theoretically perfect lens or mirror having the focal-length F and diameter D, then according to the principles of geometrical optics we can write

$$I = a \cdot \frac{D^2}{F^2} \tag{1}$$

For a given celestial object the factor a in the above and following equations will be assumed to have the same value for all telescopes near the limit of photographic vision.

If I' is to be the intensity of the same surface in a second telescope whose known aperture is $D' = nD$, then the required focal length F' is given by

$$F' = nF\sqrt{\frac{I}{I'}} \tag{2}$$

When, however, the luminous area is very small the above formulas become sensibly inaccurate, and finally, when the image is that of a fixed star, they are no longer even approximately true.

Long ago Airy demonstrated mathematically that according to the undulatory theory of light the diameter of the spurious disk of a star, as formed on the optical axis of a theoretically perfect telescope, varies inversely as the diameter of the image-forming surface. So that if, for instance, the effective diameter of the lens or mirror is doubled the image contracts to one-half its former diameter, resulting in a four-fold increase in the light-intensity.

For fixed stars we therefore have the expression,

$$I = a\frac{D^4}{F^2} \tag{3}$$

In order that the intensity of the same star in a second telescope shall be I' the value of F' must now be

$$F' = n^2F\sqrt{\frac{I}{I'}} \tag{4}$$

Equations (1) and (3) show that the ratio of the intensities of the same star in any two telescopes is n^2 times as great as the surface-intensity ratios in the same telescopes. So far as the contrast (between a star and a surface against which it appears projected) depends upon these ratios it may be said to vary with the area of the aperture; for if we assume the aperture to remain constant and the focal length to change, both star and surface would vary according to the same law, so that the increase in contrast due to a diminution of the focal length results simply from the smaller scale of the image. In the former case the change in contrast may be said to be real, in the latter case only apparent.

In visual work there are a number of serious objections to the plan of securing greater intensity by decreasing the focal length indefinitely to a certain limit. If in addition the image-forming surface is composed of a system of lenses, apparently insurmountable errors, due mainly to chromatic aberration, are introduced. Nearly all of these objections are removed when the parabolic reflector is used in connection with the photographic plate, and only such work undertaken which deals with images near the optical axis. The obstacles to be overcome in the attempt to secure the best results on a *very* small scale seem to be almost wholly of a mechanical nature. If a mirror can be figured with such perfection that the microscopic images on the photographic plate have angular diameters not much greater than the corresponding photographic images formed by a powerful telescope, then, with the aid of a microscope it will be possible to study certain special problems with even greater facility than can be done with any visual telescope.

As an effort towards determining how far the power of

a telescope can be increased by diminishing the scale, and still have the theoretical and practical advantages for certain kinds of work outweigh the disadvantages, an extreme case, or rather a case believed to be extreme at the time, was decided on for trial. It may be of interest to give a brief description of the instrument.

Two parabolic mirrors were figured. One with an aperture of 12 inches, and a focal length of 46 inches is, with a power of 360 diameters, used as a Newtonian guiding telescope for the second or principal mirror. This has a clear aperture of 13 inches, and a focal length of 20 inches. The tubes containing these mirrors are bolted together, and are carried by an equatorial mounting of the old English style.

For focussing, a battery of three objectives of a compound microscope is placed so close to the three points of support of the photographic plate, that the latter, or the ground-glass, can just be slipped into position. A small rectangular prism back of the lenses throws the rays to the eye-piece (at the side of the tube). The magnifying power is somewhat greater than 400 diameters, and the diameter of the visual field of view is about three minutes of arc.

As the "expense" item is a rather serious matter in a private undertaking of this nature, a number of deviations from the usual plans were made, and parts not necessary for securing the highest possible degree of accuracy were not finished.

In long exposures a good driving-clock is almost as essential as a well-figured mirror, or a good focal adjustment of the photographic plate. For this purpose a simple governor was made for an old eight-day clock movement, and a few other parts added, the chief one being a carefully cut $\frac{7}{8}$-inch steel screw two feet long, with 24 threads to the inch. This screw revolves once in 14 seconds, and thus gives a horizontal motion to a Babbitt-metal nut. Two thin steel bands (whose ends are fastened to the nut and sector-arc of 43 inches radius respectively) make the connection between the clock and the hour axis. A third band on the sector is connected with a simple arrangement for producing a constant pull of about two pounds (a few ounces will move the telescope) to keep the other two bands taut. The clock has simply to overcome the friction in the nut due to the pull of two pounds. The clock-platform is mounted on three wheels, and perfect slow-motion in R.A. is secured by a slight pull on an endless rope, which turns a screw, and thus moves the clock horizontally along a tangent to the sector-arc. The clock can be made to run two hours without re-winding. These parts are all out of the way under the observing floor.

A small finder attached to the main tube serves its usual purpose well. Inclosing the whole is a cylindrical sheet-iron dome eight feet in diameter. The guiding star is always the brightest one to be found within a degree or two of the object to be photographed (the variations in the differential refraction etc. are practically insensible during the short time required for the exposure); the 12-inch mirror is tilted, and the eye-piece shifted laterally to suit each particular case. For more than two years I have been wholly occupied with these optical, mechanical and experimental efforts to increase the efficiency of the photographic telescope (along certain lines) without increasing its size.

The one great difficulty which limits the power of even the largest telescope long before its capabilities have been exhausted, results from the fact that near the limit of vision we have to deal with a luminous area (caused primarily by reflections in our atmosphere, and in a less degree by reflections in space, nebulous areas and faint stars. Instrumental defects tend to increase the trouble).

With a given illumination of the sky (due to any cause exterior to the instrument) it would seem that as soon as the image of this sky-background begins to show on the photographic plate the faintest stars which can ever be photographed with any telescope, under the same conditions of sky, have already made their impressions on the same plate, and are necessarily of greater intrinsic brightness than this background. To make these limiting impressions visible, the contrasts must be increased; assuming the exposure and development of the plate to be the best possible, this can only be done, it would seem, by increasing the diameter of the aperture.

Before the instrument was finished or even commenced, it seemed reasonable to admit that even if the theoretical requirements could be practically fulfilled, the large existing telescopes, of darker field, might in long exposures be able to reveal stars several magnitudes fainter than could be obtained with the contemplated instrument during the comparatively short time a plate could be exposed to advantage in its bright field.

When, therefore, the remarkable fact was made apparent that negatives exposed for less than five minutes with the 13-inch mirror revealed stars apparently beyond the reach of the 36-inch refractor of the Lick Observatory, and also revealed every star shown on a published photograph which had an exposure of two hours in the Crossley reflector (aperture 3 feet, focal length 17.5 feet), the result, although not wholly unlooked for, really exceeded expectations.

In experiments made for the purpose of finding some way to lessen the drawback of a bright field, photographic plates, both common and orthochromatic, varying in sensitiveness from the most rapid to the slowest, were employed, with the expected result that slow plates give the greatest contrasts, but always at the expense of increased exposure-time.

A serious objection to a very long exposure exists when, as in my case, the image of the guiding star is not made by the surface which forms the photographic image. In long exposures the varying stresses as the instrument revolves

on the hour axis may and often do cause a relative drift of images in the two telescopes sufficiently great to become sensible under high powers, thereby annulling to a certain extent the value of the result. The method of the sliding plate-holder used with so much success by COMMON, the Lick Observatory Astronomers, by RITCHIE of the Yerkes Observatory, and others, could evidently not be advantageously employed in the present case. Mr. ROBERTS, the English astronomer, who has done so much valuable work in celestial photography, uses a novel but costly method of his own. For making his remarkable photographs of the Milky-Way Prof. BARNARD used a small achromatic-guiding telescope strapped to a six-inch portrait-lens of about 30 inches focus, which gave a large field fairly well covered. The notable discoveries of Dr. MAX WOLF are made with similarly mounted, but larger, objectives.

The photographs taken with the 13-inch mirror referred to below are all made on commercial SEED plates, No. 27. A 5 x 7 plate is cut into 18 pieces, so that each plate is $\frac{5}{8}$ x $\frac{3}{4}$ inches; the negative proper is near the center of the plate, and ordinarily about 0.1 or 0.2 inch in diameter, which can be increased to 0.5 inch if so desired.

To determine the approximate magnitude of the faintest stars visible, a number of plates were exposed on certain regions covered by a chart which Prof. TUCKER made with the aid of the 36-inch refractor of the Lick Observatory. This chart shows stars down to the 17th magnitude; it is printed in the Publications of the Astronomical Society of the Pacific, No. 37.

In any given region of the chart following *α Leonis* all but the faintest stars are photographed with an exposure of one minute. In a two-minute exposure practically every star, within 4′ or 5′ of the optical axis, in any given region of the chart, is revealed, and near the center of the negative new ones are usually to be detected. Before the development of a plate, exposed for 2^m, is complete, the background image plainly shows on the negative, so that under the ordinary conditions existing here* no material advantage is to be gained by prolonging the exposure much beyond 15 or 20 minutes.

Under favorable circumstances, then, this instrument certainly photographs stars fainter than the 17th visual magnitude in less than five minutes. For determining the photographic magnitude of these fainter stars no general method, making any claim to accuracy, is known. For comparison with other results, the best available data seem to be Professor KEELER's photographs of the *Ring* nebula in *Lyra*, published in the *Astrophysical Journal*, Vol. 10, p. 193. I have photographed this nebula at various times with the 13-inch mirror, and give here the results secured under the most favorable conditions.

The nebula is just to be recognized on negatives exposed for 4^s. In 8^s-negatives the central star shows plainly, as does also the 13th magnitude (LASSELL 1) close following the nebula. In 16^s-negatives, the nebula and these two stars are quite conspicuous.

In negatives exposed for 32^s LASSELL's Star 3 is just to be recognized (this star according to KEELER is still invisible on his original negative exposed for two minutes in the 3-foot reflector). LASSELL's Star 2 is plainly seen, as is also the 10″ distant companion to the central star. In 64^s-negatives all the stars to be found on the published plate of KEELER's 10-minute negative can be recognized. In negatives of 128^s exposure quite a number of stars are visible which do not show on KEELER's two-hour-exposure plate; they are doubtless to be found on his original negative. In a one-minute exposure the background of the sky is already faintly to be seen on fully developed negatives. In two minutes, as already stated, it is quite strongly impressed upon the plate. As the exposure time is prolonged the density of all objects increases, and the faintest objects can be recognized with greater certainty. I have prolonged the exposure up to 60 minutes, but nearly every star shown on the resulting negative can be found on plates exposed for less than five minutes.

Perhaps the best illustration of the power of this comparatively small instrument is the fact that it has revealed the true form of the *Ring* nebula in *Lyra*. This is now *plainly* shown to be a *two-branched spiral* which starts at the central star, and in a clock-wise direction leaves it on opposite sides near the minor axis. With a spider-line micrometer attached to a compound microscope magnifying 100 diameters, I have measured the following distances from the central star to points where the thread-like arcs

* My little private experimental observatory is surrounded by residences, a dozen or more within a stone's throw. The combinations of smoke and nearby powerful arc lights (for street illumination) which burn at all hours of the night when the moon is not above the horizon, often produce great variations in the sky-background, amounting at times to several stellar magnitudes, in an otherwise clear sky.

The brightness of the field of view may be slightly affected by a series of scratches which (when well advanced with the work after long and patient figuring) were made through an accident to the polisher of my machine. The variation in the radius of curvature is so rapid in this mirror, that the slightest indentation, due either to bubbles or scratches, causes the polisher to change the surface in the immediate neighborhood of every defect. I was, on this account, forced to discontinue the figuring somewhat before it should have been done. As a result, a somewhat imperfect polish and small rings still exist.

The general curvature of the whole surface is very satisfactory, and the definition is much the best of all my past amateur efforts in the way of figuring parabolic surfaces. The 12-inch mirror has a good figure, and no scratches.

cross the major axis, beginning at the preceding end, $-45''$, $-36''$, $-28''$, $-19''$, $-10''$, $+12''$, $+21''$, $+28''$, $+36''$, $+44''$; certain of these arcs are plainly double, the mean position is in such cases given above. The one crossing at $+10''$ starts on the *south* side of the star; the one at $-12''$ is the first crossing of the *north* branch, etc. The distances on the minor axis from the central star to tangents parallel to the major axis, for the several individual arcs clearly to be distinguished, are approximately as follows: beginning on the north $-13''$, $-9''$, $+2''$, $+7''$, $+12''$, $+17''$. Those at $-9''$ and $+2''$ correspond to the innermost north and south tangents respectively, etc.

From the central star outward both branches can be traced continuously through arcs of at least 420°, after which they seem to run into each other in projection, forming heavier rings apparently corresponding to the inner edge of the main nebula; at certain other positions they are again farther apart, giving rise to darker areas, and producing the impression that two or three nearly circular non-concentric heavy rings form the main ring. The scale of the whole photographic image is

$$90'' \times 60'' = 0^{\text{in.}}.009 \times 0^{\text{in.}}.006$$

There seems to be real nebulosity near the 13^{M} star and in the area inclosed by the two tangents from this star to the extreme outer boundary of the nebula.

From the data already given, it appears that, for exposures of two minutes or less, the smaller instrument photographs stars in about one-fourth of the time required by the larger; or, at a given instant reveals stars 1.6 magnitudes fainter. According to equation (3) it should, theoretically, be only 0.8 magnitude, assuming that plates of equal rapidity were used at both instruments. The difficulties connected with the guiding of the Crossley telescope which Keeler mentions in another paper (*Ap. J.*, Vol. XI, No. 5) will almost wholly account for the difference. Also to be considered is the fact that the best rays, amounting to $\frac{1}{18}$ of the total light, are cut off by the diagonal. With the 13-inch no trouble is experienced in keeping the guiding image at the intersection of two spider lines with a power of 360 diameters on the guiding telescope; the plate holder stops only $\frac{1}{100}$ part of the light, and there is no second reflection. The difficulties of guiding should have less effect on the visibility of a nebula, and this is shown in the foregoing comparison. Keeler's negative just reveals the nebula in a 30-second exposure, but I have not yet succeeded in photographing it in less than 4 seconds, while according to equation (1) it should require but 2 seconds; here the purer sky of Mt. Hamilton is clearly in evidence.

As a result of these observations and comparisons, I am inclined to agree with the views expressed by Mr. Roberts, that stars much fainter than the 18th or 19th magnitude cannot be photographed with any instrument working near sea-level or at moderate altitudes.

Variations in the sky-illumination for different directions in space, independent of those caused by our atmosphere, could readily be made with a short-focus instrument of this kind placed in a favorable climate and 10,000 feet or so in altitude. By slightly changing the plate after each exposure fifty or more protected images, each a few minutes of arc in diameter, could be made on the same negative in a single night. The exposure time, the zenith distance (= colatitude?) and the development being the same for each image, direct comparisons would be possible. In work of this kind it would be just as necessary to have accurate following as in photographing a nebula. This is plainly evident from an examination of some experimental negatives where the field is two or three degrees in diameter. The aberrational spread of the star images causes the appearance of a sensibly increasing density of the photographic background with increasing distance from the optical axis.

Near the center of the negative an image may have great density, and yet be so small that it is wholly invisible to the naked eye. With poor guiding, however, this would no longer be true, and the resulting background would not represent the actual brightness of the area photographed. For the observations of variable stars the value of the saving in time where very faint stars are under observation can hardly be overestimated.

These faint stars give disks so small that they are far beyond the reach of naked-eye vision; near the optical axis they do not, under favorable conditions, exceed $2''$ in diameter (= 0.0002 inches) in exposure up to five minutes or more. (I have made instantaneous exposures on ϵ *Lyræ*, using Seed's lantern-slide plates; the resulting negatives show the individual components of each pair, as they are successively brought near to the optical axis by moving the telescope in declination.) At the edge of the field of view having a radius of, say $7'$, the brighter stars are considerably enlarged and elongated to the extent indicated in the table below. For the fainter stars, however, only that portion of the image which is near the vertex of the pattern is sufficiently strong to make a record on the plate, so that at a distance of $4'$ or $5'$ from the optical axis these fainter stars still appear quite small, though sensibly elongated.

For preliminary examinations of the negatives, lenses varying from about one-half inch focus to the shortest focus (one-eighth inch or less) conveniently available, should be at hand.

For approximate measures a simple position filar micrometer attached to a compound microscope with powers varying from 20 to 200 diameters (the higher powers for such objects as ϵ *Lyræ* mentioned above, and also for structural detail in nebulas, etc.) can be most advantageously used.

I find that the most satisfactory illumination is afforded by light reflected (or transmitted) by a rough surface which subtends a large solid angle at the negative.

A very effective method of examining the plates, especially those of short exposure, is to use a *dark* background, illuminating the negative with a strong side-light; by this procedure all the fainter objects become luminous, the negatives seems to turn into a positive. To make this method available, the development must be stopped almost immediately, after the sky-background begins to show on the plate. Negatives of this class are not so suitable for examination by the other method.

As several photographs showing stars down to the 17th magnitude can be taken in 10 or 15 minutes, doubtful impressions due to defects of the plates, or to minute dust particles (forming false stars, or obliterating real objects) can be decided by comparisons between the different plates.

The radial aberrations at the several angular distances from the optical axis of this instrument, as found by actual measurement of the images of a rather bright star, are approximately as follows:

Dist. from Optical Axis	Radial Aberr.	Dist. from Optical Axis	Radial Aberr.
′	″	′	″
7	30	35	170
14	70	42	200
21	100	49	230
28	130		

From which we learn that the radial aberration is about 0.08 times the angular distance from the optical axis; the *blurring factor* is therefore 1.08. The analytical expression for this factor (*Astr. Jour.*, No. 435) is

$$\frac{1}{\cos v \cos^2 \frac{1}{2} v}$$

which gives, as it should, the same value.

When delicate results are required at considerable distances from the optical axis, such a short focussed mirror is of course no longer to be considered. To obtain, for instance, a photograph of the moon's terminator,* which gives sensibly the same definition throughout, the ratio of focal length to aperture must be even greater than that ordinarily used in the case of the achromatic telescope.

To illustrate, visually, the character of the preliminary results, I send a paper print† of a 75-fold enlargement of the original negative, taken 1902 Oct. 30, 128^s exposure, of the Ring nebula in *Lyra*, plainly showing the spiral features which have already been described. The spiral form was first recognized on negatives taken 1903 May 3.

* Through the publication of his investigations in detail, and through kindly encouraging letters during his life-time to enquiring young astronomers, Professor Henry Draper probably did more to illustrate and bring out the practical possibilities of the parabolic reflector in the United States, than all other sources of information combined. The photographs of the moon's whole terminator taken with his reflector, having a focal length ten times the aperture, were among the best then existing. He was well aware that decidedly better definition would have resulted with increasing distance from the moon's center, had a greater focal length to the same aperture been conveniently available for this object.

† This has been enlarged two-fold in the plate accompanying this number. — Ed.

Ann Arbor, 1903 *May* 12.

PARALLAX OF SUN FROM PHOTOGRAPH OF *EROS*,

By F. P. LEAVENWORTH.

During the winter of 1900, some fifty photographs of *Eros* were made with the 10½-inch refractor of this observatory. My plan was to make all exposures of one night upon the same plate, and with as great a range of hour-angle as possible.

I have lately finished reducing the plate taken Nov. 29. Thirteen separate exposures were made on this plate with hour-angles running from -3^h to $+6^h$.

The measures were first reduced to the mean of the sixth and seventh exposures by the formulas,

$$ax + by + c + x = x_1$$
$$dx + ey + f + y = y_1$$

These in turn were reduced to right-ascension and declination by means of the standard stars published in the bulletins of the astrophotographic congress. Two methods of reduction were employed; in the first the refraction was computed and applied to the measures. This requires four constants of reduction. In the second method the refraction was not applied, but determined from the plate by the addition of two more constants of reduction. The latter method is easier to work with, and in this case gives a parallax somewhat nearer to the accepted value.

The thirteen right-ascensions and declinations of *Eros* so obtained were next corrected for motion of the planet by means of the ephemeris in Circular No. 9 of the photographic congress. Finally, from these the observation-equations were formed, and the solar parallax derived by a least-square solution.

The measurements were made with the Repsold measuring machine, in duplicate by Mr. Roy Ferner and myself. The images near the center, which include *Eros*, were

usually round and well defined. But toward the edge of the plate there was a decided elongation and tail. I have since found that images near the edge can be very much improved by capping down the object-glass to about nine inches, without greatly diminishing the distinctness of the fainter images. A marked constant difference, between Mr. FERNER's measures and my own appears in the distorted images, but does not show in the round stars.

The times and lengths of exposure are found in the following table:

No.	Minneapolis Sid. Time	Exp.	Bar. Ther.	No.	Minneapolis Sid. Time	Exp.	Bar. Ther.
	h m s	m	in.		h m s	m	°
1	22 8 19	2	29.18	8	3 24 49	12	− 2.0
2	22 13 34	2.5	+1°.0 C	9	3 38 34	2.5	–
3	22 21 19	4	–	10	6 36 35	2.5	–
4	0 41 34	2.5	–	11	6 41 34	2.5	–
5	0 49 4	5	–	12	7 25 34	2.5	29.25
6	1 28 34	2.5	–	13	7 43 11	4.5	− 2.7
7	1 38 49	6	–				

The coordinates are measured from the center of the plate, and are corrected for error of run and scale-error. In reducing them to the mean of exposures six and seven, the different stars were weighted according to the roundness of their images.

The coordinates of *Eros* resulting from this reduction are as follows:

No.	x	y	No.	x	y
1	+3.9712	−0.1133	8	+2.2331	−4.0826
2	3.9475	0.1707	9	2.1650	4.2636
3	3.9055	0.2630	10	1.2981	6.7032
4	3.1428	1.9742	11	1.2742	6.7701
5	3.0993	2.0666	12	1.0903	7.3808
6	2.8817	2.5620	13	+1.0119	−7.6224
7	+2.8232	−2.6959			

These places were next reduced to right-ascension and declination. Owing to a mistake in identification, two of the standard stars most favorably situated were not measured. The number of reduction stars were thus reduced to six, and they were not symmetrical with reference to *Eros*.

The stars used were,

Star		α	δ	x	y
	°	h m s	° ′ ″		
301	+50	1 26 59.882	+50 22 1.06	− 8.4971	−38.6269
331	51	27 58.688	51 19 13.54	+ 1.5515	+18.6538
334	51	28 33.784	51 38 28.98	+ 7.2321	+37.9071
338	51	29 22.497	51 39 7.70	+14.8256	+38.5047
339	51	30 24.334	51 14 15.93	+24.3804	+13.5078
314	+50	1 30 44.595	+50 44 59.45	+27.4953	−15.8812

The reduction-equations resulting from these are,

A, with refraction applied:

$$+0.00256\,x\sec\delta - 0.00125\,y + 0^s.1666 + x\sec\delta + \alpha_0 = \alpha$$
$$+0.11142\,x\sec\delta + 0.00257\,y + 0''.338 + y + \delta_0 = \delta$$

B, refraction not applied:

$$+0.00255\,x\sec\delta - 0.00125\,y + 0^s.1663 + x\sec\delta + \alpha_0 = \alpha$$
$$+0.11328\,x\sec\delta + 0.00257\,y + 0''.340 + y + \delta_0 = \delta$$

where $x\sec\delta$ is expressed in seconds of time, y in seconds of arc, and

$$\alpha_0 = 1^h\,27^m\,50^s.00 \quad , \quad \delta_0 = 51°\,0'\,37''.0$$

and α and δ are the right-ascension and declination of the star for 1900.0.

By means of these formulas the following right-ascensions and declinations of *Eros* were obtained:

Exp.	α_A	α_B	δ_A	δ_B	Motion α	Motion δ
	h m s	s	° ′ ″	″	s	′ ″
1	1 28 15.334	15.333	+51 0 33.19	33.24	−5.378	−2 36.86
2	28 15.187	15.186	51 0 29.75	29.79	−5.238	−2 32.86
3	28 14.928	14.927	51 0 24.20	24.25	−5.033	−2 26.95
4	28 10.211	10.210	50 58 41.40	41.44	−1.352	−0 39.88
5	28 9.942	9.941	50 58 35.85	35.88	−1.157	−0 34.14
6	28 8.597	8.596	50 58 6.08	6.12	−0.132	−0 3.92
7	28 8.236	8.236	50 57 58.04	58.08	+0.132	+0 3.93
8	28 4.598	4.596	50 56 34.74	34.77	+2.853	+1 25.20
9	28 4.179	4.178	50 56 23.87	23.90	+3.204	+1 35.76
10	27 58.870	58.867	50 53 57.42	57.44	+7.684	+3 52.73
11	27 58.724	58.722	50 53 53.40	53.42	+7.808	+3 56.58
12	27 57.604	57.602	50 53 16.76	16.78	+8.899	+4 30.52
13	1 27 57.126	57.124	+50 53 2.26	2.28	+9.334	+4 44.13

The table also contains the corrections due to the motion of the planet in orbit. From this table the equations of condition were formed. The general equation is represented by

$$1^h\,28^m\,8^s.452 + \Delta\alpha - 0^s.9324\,a - a\,\Delta\Pi - \alpha = 0$$
$$50^\circ\,57'59''.00 + \Delta\delta - 8''.80\,a' \quad - a'\Delta\Pi - \delta = 0$$

and the equations of condition are

Exposures		A	A v	B	B v	$\sqrt{p}$
1	$+1\,\Delta\alpha\ +1.63\,\Delta\Pi$	$+0''.16 = 0$	$+0''.13$	$+0''.17 = 0$	$+0''.14$	1
2	1 +1.60	−0.04	−0.07	−0.03	−0.06	1
3	1 +1.55	+0.10	+0.08	+0.10	+0.08	1.3
4	1 +0.43	−0.04	−0.03	−0.03	−0.02	1
5	1 +0.36	+0.05	+0.07	+0.06	+0.06	1.3
6	1 0.00	−0.14	−0.11	−0.13	−0.11	1
7	1 −0.10	−0.09	−0.05	−0.07	−0.05	1.3
8	1 −1.04	+0.30	+0.36	+0.32	+0.37	0.8
9	1 −1.15	−0.02	+0.05	0.00	+0.05	1
10	1 −2.08	−0.38	−0.28	−0.36	−0.28	0.7
11	1 −2.09	−0.27	−0.18	−0.25	−0.17	1
12	1 −2.14	−0.39	−0.30	−0.38	−0.30	1
13	1 −2.13	+0.06	+0.16	+0.07	+0.16	1.2
1	$1\,\Delta\delta\ -0.26$	+0.33	+0.32	+0.28	+0.31	1
2	1 −0.24	+0.03	+0.03	−0.01	+0.02	1
3	1 −0.20	+0.04	+0.03	0.00	+0.02	1.3
4	1 +0.28	−0.07	−0.09	−0.10	−0.09	1
5	1 +0.30	−0.06	−0.08	−0.10	−0.08	1.3
6	1 +0.32	−0.30	−0.32	−0.34	−0.33	1
7	1 +0.32	−0.13	−0.15	−0.17	−0.16	1.3
8	1 +0.11	+0.06	+0.05	+0.03	+0.05	0.8
9	1 +0.06	−0.06	−0.08	−0.10	−0.08	1
10	1 −0.97	+0.35	+0.36	+0.34	+0.38	0.7
11	1 −1.00	+0.22	+0.24	+0.20	+0.25	1
12	1 −1.32	+0.12	+0.15	+0.11	+0.17	1
13	1 −1.45	−0.11 = 0	−0.08	−0.12 = 0	−0.06	1.2

Equations 3, 5, 7, 13 have been given extra weight because two images of the planet were obtained in each of these exposures, and the mean of the measures of these images was used. Less weight was assigned to 8 and 10 because of imperfect images. On account of the long exposure of 8, the weight ought probably to be much less than it is, or the measure rejected altogether.

Normal Equations	A	B
$+14.64\,\Delta\alpha + 0.00\,\Delta\delta - 3.41\,\Delta\Pi$	$-0''.53 = 0$	$-0''.37 = 0$
$0.00\,\Delta\alpha + 14.64\,\Delta\delta - 3.95\,\Delta\Pi$	$+0.06 = 0$	$-0.43 = 0$
$-3.41\,\Delta\alpha - 3.95\,\Delta\delta + 36.32\,\Delta\Pi$	$+1.26 = 0$	$+1.21 = 0$

From which,

	A		B	
$\Delta\Pi =$	$-0''.034$	$\pm 0''.021$	$-0''.029$	$\pm 0''.021$
$\Delta\alpha =$	+0.028	±0.038	+0.019	±0.038
$\Delta\delta =$	−0.013	±0.038	−0.021	±0.038
$r =$		±0.122		±0.124
$\Pi =$	8.766	±0.021	8.771	±0.021

From the smallness of the absolute terms of the equations of condition, it is evident that the systematic error must be small. Evidently the abnormal size of 10, 11 and 12 explains the variation of the parallax from the accepted value 8″.80.

I was not able to investigate the effect of the trail of the asteroid because the latter was too faint to be used as a guiding star in the following telescope.

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ON THE FUNDAMENTAL ELEMENTS OF COMPUTATION IN THEIR RELATION TO SYSTEMATIC STELLAR MOTION,

By LEWIS BOSS.

The systematic drift of stellar motions, whether it be due to reflected solar motion, or to other causes, is often of the order of the systematic errors of the observations from which the motions are computed. The adopted precessional motion may play an important part. Attention to these points is, therefore, of primary importance.

Let us consider, for example, Sir DAVID GILL's suspected rotation of bright relative to fainter stars (*A.N.* 3800), upon the systematic basis, B, of the right-ascensions published in *A.J.* 531-2. Assuming that his zone-observations are based upon the standard right-ascensions of AUWERS we have (for $-40°$ to $-52°$) B $-$Cape 1900 $= +0^s.068$. If GILL used the time stars of the *Berliner Jahrbuch* as for Cape 90, and if $\Delta\delta_s$ for the instrument remained the same, we should have: B $-$Cape 1900 $= +0^s.065$, nearly as before.

From direct comparison with B (with 31 additional standards in manuscript), I find for this zone:

	Stars	$\Delta\alpha$
B — Taylor	69	$-0^s.132$
B — C 1880	72	$+0.077$

We may therefore put:

$$\text{C 1900} - \text{Tayl.} = -0^s.200;\ \text{C 1900} - \text{C 1880} = +0^s.009$$

Then GILL's comparisons would stand as in the subjoined statement, the numbers in the first three columns having been copied from GILL's article, *A.N.* 3800.

CAPE 1900 — TAYLOR.

	Stars	C—T.	S. Corr.	C—T., corr'd
$5^M.8$	218	$-0^s.188$	$+0^s.200$	$+0^s.012$
7.4	472	-0.315	$+0.200$	-0.115

CAPE 1900 — CAPE 1880.

	Stars	C—C 80	S. Corr.	C—C 80, corr'd
$6^M.8$	681	$+0^s.011$	$-0^s.009$	$+0^s.002$
7.9	813	-0.014	-0.009	-0.023

We have still several elements of error to consider before we reach the question of rotation.

1. There is the possible error of comparison, including the uncertainty of the fundamental system.

2. There is the possible effect of magnitude equation, which may amount to $-0^s.003\,(M-3.5) - 0^s.0044\,(M-3.5)^2$ in case we assume the equation for C 1900 the same as that for C 1890 (see *A.J.* 536). The corrected numbers for C 1900 — Taylor would then become: $5^M.8$, $+0^s.040$; $7^M.4$, $-0^s.039$. No particular stress can be laid upon this result; but it illustrates actual possibilities.

3. Small modifications may be due to the employment of NEWCOMB's precessions.

From the proper motions computed by AUWERS for the zone $+15°$ to $+20°$, Professor SEELIGER obtains a test of the supposed rotation (*A.N.* 3865, p. 9). This test shows the rotation to be non-existent; but it leaves an average mean motion for all stars of about $-0^s.10$. This is reduced to zero, if the proper motions are reduced to conformity with NEWCOMB's precessions and the right-ascensions of system, B.

I take this opportunity to refer to the very able and interesting paper of Professor KAPTEYN (*A.N.* 3859), concerning the apex of solar motion. A problem of such extreme difficulty seems to demand in the very first line a thorough investigation of the errors to which the various series of star-positions are liable; and, in the second place, the employment of all the observations that can readily be brought to bear. In no other astronomical investigation do these requirements seem to me more indispensable. An attempt to apply these principles in the discussion of this problem for the brighter stars is in progress at Albany. The recent publication of a catalogue of standard stars (*A.J.* 531-2) is a step in this work. For some time to come our chief anxiety will be to learn what are actually the proper motions. Under these circumstances I wish to defer extended comment on KAPTEYN's paper at present, and will merely refer to certain points.

1. KAPTEYN's criticism (*A.N.*, 3859, pp. 328, 352) upon the systematic corrections employed by me in the discussion of L. STRUVE's Bradley-Pulkowa values of $100\mu_\delta$ (*A.J.* 501) is well taken. There was an oversight. The numbers, as I computed them, were for the zones $-7°.5$, $+7°.5$ and $+22°.5$, respectively, $-0''.86$, $-1''.14$ and $-1''.22$. How the wrong numbers came to be used is a mystery for which I find no explanation.

2. The determination of a systematic correction of all the proper motions through the discussion of the solar motion itself as KAPTEYN has attempted it (*A.N.* 156, pp. 1-20) seems to me an inadmissible procedure. This delicate element is thereby made to depend upon the mere fortuitous differences in the actual apical positions for the several restricted groups of stars.

3. Of existing determinations of the solar apex those which are based upon proper motions between $0''.1$ and $1''.0$ seem to be less open to objection than the others. Those based on smaller motions are untrustworthy from lack of thorough and satisfactory treatment of the systematic errors. The very large proper motions are very possibly abnormal.

4. The habit of some computers of setting the limits of proper motion to be employed not according to the total motion, but according to one standard for μ and another μ', is justly criticized by KAPTEYN; but I think he has exaggerated the effect in concrete instances. Taking into consideration the works of BISCHOF, STUMPE, and PORTER (second computation) I cannot think of the declination of the solar apex as much less than $+40°$.

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METHOD OF FORMING THE SYSTEM OF DECLINATIONS FOR THE CATALOGUE OF 627 STANDARD STARS (*A. J.* 531-2),

By LEWIS BOSS.

The general principle adopted in the formation of declinations for the Catalogue of 627 Standard Stars was the same as for the right-ascensions. First, we endeavor to form a system of declinations of individual stars which may be accepted as sufficiently homogeneous within each zone which is to be treated as a whole for correction. The immediate end desired is that the positions and motions of all the stars within the zone shall require the same corrections, whatever these may be. Thus, the zone-corrections (n, Table IV) at various epochs given by the several star-catalogues will be systematically correct relatively to each other, although these catalogues may be very far from having all the stars in common. This zone is now treated as a representative star for which the most probable correction of its computed declination and motion is to be ascertained from the evidence afforded by those star-catalogues which are entitled to weight as independent determinations.

In this work it was assumed that the writer's *Declinations of 500 Stars*, B_1, with its extension southward in *Standard Stars south* of $-20°$ (*A.J.* 448-450), B_2, offered a suitable basis for correction. But in preparing B_1 for this use the declinations of 50 stars were revised by means of recent observations reduced to systematic conformity with B_1. The list of stars forming the basis of operations then consisted of the 210 stars best determined in B_1 with the 179 stars of B_2.

Corrections of the Form $\Delta\delta_a$.

It turned out that the system, B_1, does not appear to require any material correction of the form $\Delta\delta_a$ in order to make it consistent with observations published since that system was formed. It is scarcely necessary to exhibit the original values of $\Delta\delta_a$ which were adopted throughout the computations. As a substitute for these Table I contains the values of $\Delta\delta_a$ which were computed for each catalogue of observation through comparison with the adopted declinations of the Catalogue of 627 Standard Stars, B. In general these values of $\Delta\delta_a$ are in close conformity with those which were computed in the preliminary operations. For one-half of the catalogues the maximum difference between the preliminary and adopted values of $\Delta\delta_a$ does not rise above 0".02 at any hour of right-ascension.

TABLE I. ADOPTED VALUES OF $\Delta\delta_a$.

		sin	cos			sin	cos
		"	"			"	"
Grw.	16	+0.07	−0.09	Bonn	66	+0.37	−0.14
Kön.	21	−0.27	+0.05	Leips.	67	+0.02	−0.01
Dpt.	24	+0.15	−0.02	Leid.	67	−0.02	−0.02
Åbo.	29	+0.25	−0.12	Melb.	68	+0.01	−0.11
Grw.	30	+0.16	+0.08	Pulk.	69	0.00	+0.04
St. H.	32	−0.08	+0.50	Grw.	72	−0.01	0.00
Cape	33	−0.06	−0.16	Madr.	75	+0.32	+0.17
Camb.	34	+0.04	−0.19	Harv.	75	0.00	0.00
Cape	37	−0.02	+0.01	Pulk.	76	+0.03	+0.06
Grw.	38	−0.05	−0.17	Cord.	76	−0.07	0.00
Radcl.	45	−0.08	+0.05	Paris	76	+0.25	−0.13
Grw.	45	+0.02	−0.06	Cape	76*	0.00	−0.05
Paris	45	−0.03	−0.07	Melb.	77	−0.06	−0.07
Pulk.	47	−0.03	+0.07	Washn.	77	+0.03	−0.02
Stgo.	51	+0.20	−0.21	Cape	83	−0.04	+0.12
Cape	51	−0.18	−0.03	Pulk.	84	−0.02	+0.02
Grw.	51	−0.03	−0.07	Radcl.	85	+0.12	+0.07
Pulk.	55	+0.02	+0.03	Str.	86	−0.01	+0.03
Washn.	56	−0.12	−0.07	Cape	89	−0.01	−0.06
Grw.	57	+0.05	+0.01	Berl.	90	−0.02	+0.06
Radcl.	57	−0.10	+0.08	Mun.	92	−0.01	+0.01
Cape	59	+0.11	+0.03	Grw.	94	+0.03	0.00
Paris	60	+0.14	−0.08	Mt. H.	95	+0.05	+0.05
Grw.	64	−0.02	+0.04	W.-Ott.	97	+0.23	+0.07
Bruss.	65	+0.08	−0.05	Alb.	98	−0.04	−0.06
Cape	65	0.00	−0.02				

		"	"	"	"
Madr.	35	−.07 sin α	−.22 cos α	+.00 sin 2α	+.14 cos 2α
Melb.	62	+.06	−.32	+.11	+.23
Grw.	82	+.04	+.08	−.07	−.02
Madn.	90	+.11	−.21	+.10	.00

* For stars north of −20° only. Zone corrections for stars further south.

In dealing with the individual star-declinations in this investigation, the work was performed as a continuation of the manuscript computations for B_s which have been preserved. It became simply necessary to add the results of later observations to the "star-sheets" prepared for B_s, and then to proceed to the revision by the zone-method as already described in connection with the right-ascensions. In the original work, and consequently in this, some of the older series of declinations were first corrected for terms of the form, $\Delta\delta_\alpha$, required in order to correct for difference from the STRUVE–PETERS values of nutation, precession, etc., adopted in the construction of the several catalogues. For example, corrections on account of nutation, etc., were first applied to the declinations of Königsberg 21 (DÖLLEN's reduction, *Recueil de Mém., Obs. Cent. de Russie*), Åbo 29, and Cape 33, before employing them in the operations for deduction of the normal system. These corrections were:

Königsberg	21,	$-0''.24 \sin\alpha$	$-0''.03 \cos\alpha$
Åbo	29,	$+0.24$	-0.04
Cape	33,	-0.01	-0.07

The additional corrections, $\Delta\delta_\alpha$, apparently necessary in order to reduce the respective catalogues to the system, B, are:

Königsberg	21,	$-0''.03 \sin\alpha$	$+0''.08 \cos\alpha$	A
Åbo	29,	$+0.01$	-0.08	
Cape	33,	-0.05	-0.09	

The combination of these two sets of corrections makes up the respective values of $\Delta\delta_\alpha$ given in Table I. Similar remarks apply to several other of the earlier catalogues, for which the details are given in B_s.

In computing $\Delta\delta_\alpha$ the observations were divided into zones: $+80°$ to $+40°$; $+39°$ to $-21°$; and $-22°$ to $-70°$. Usually the values of $\Delta\delta_\alpha$ from the separate years, for a given catalogue, were sufficiently consistent. The following notable differences were found, however:

		Zone	sin	cos
		° °	''	''
St. Helena	32,	+39 to −21	+.06	+.51
		−22 to −70	−.23	+.50
Cambridge	34,	+80 to +40	−.32	−.21
		+39 to −21	+.17	−.19
Washington	56,	+80 to +40	−.27	−.10
		+39 to −21	−.14	−.07
		−22 to −42	+.17	−.02
Melbourne	68,	+39 to −21	+.05	−.02
		−22 to −70	−.02	−.20
Madras	75,	+80 to +40	+.56	+.19
		+39 to −21	+.32	+.10
		−22 to −60	+.21	+.33

It would scarcely be advisable, however, to take these differences into account, since they do not, in general, much exceed the limit of uncertainty admissible according to the theory of probable error. It is proposed, however, to put $\Delta\delta_\alpha = 0$ for declinations south of $-20°$ in YARNALL's Washington Catalogue for 1860.

In the instances where terms in 2α have been introduced, it is for the reason that terms of single period give an unsatisfactory representation of the residuals. With the term of double period taken into account the representation of the observed residuals for Greenwich 82 is very good. A like term ($-0''.04 \sin 2\alpha$) is indicated for Greenwich 94, but the improvement in representation of the residuals is not marked, and the term is not adopted.

Effect of Variation of Latitude.

In founding the system as to $\Delta\delta_\alpha$ no special account has been taken of the effect of variation of latitude upon the observed declinations. In the time which could be allotted to the present discussion, it did not seem practicable to investigate the correction required on that account, since to accomplish this in a precise form would have required as much labor as for all the other operations put together. Certain items of testimony on this point are, however, readily available. Some of the older catalogues, like those of Kön. 21, Dpt. 24, Åbo 29, and Cape 33, were based upon polar points ascertained from the observation of close circumpolar stars, or of zenithal stars referred to an arbitrary zero. These are technically free from the effect of variation of latitude. Dr. CHANDLER's discussion of POND's observations with two mural circles has resulted in declinations which are freed from the effects of latitude-variation by a most exhaustive discussion which seems to leave nothing to be desired (*A.J.* XVI, p. 3). Dr. CHANDLER has also computed the corrections which are required in order to free the Pulkowa vertical circle observations, of mean date 1869, from this effect (*A.J.* 402). Dr. NYRÉN computed and applied this effect for the vertical circle observations of 1885; but Dr. CHANDLER points out that the annual term was neglected. GROSSMAN (*Abh. Kön. Sächs. Ges. d. Wiss.*, Vol. XXVII, p. 206) states that he applied in his reductions corrections for variation of latitude after ALBRECHT's researches. To these I have added another from the following combination, derived from Table I.

Corrections of B Having the Form $\Delta\delta_\alpha$.

Greenwich	64,	$+0''.02 \sin\alpha$	$-0''.04 \cos\alpha$
"	72,	$+0.01$	0.00
"	82,	-0.04	-0.08
Mean		0.00	-0.04
Washington	77,	-0.03	$+0.02$
W. Long. 38° 31′		-0.02	-0.01
Melb. 68 and 77, W. Long. 215° 1′		$+0.02$	$+0.09$
Mean corr. of B,		$0.00 \sin\alpha$	$+0.04 \cos\alpha$

Reversing the signs of formulas A, and of $\Delta\delta_\alpha$, for Pulkowa 84 and W.-Ott. 97 in Table I, and adding CHANDLER's corrections to Pulkowa 69, we have the following list of corrections to B on account of variation of latitude.

OBSERVED CORRECTIONS OF B, OF THE FORM $\Delta\delta_\alpha$.

Königsberg	21	$+0\overset{''}{.}03 \sin\alpha$	$-0\overset{''}{.}08 \cos\alpha$
Dorpat	24	-0.15	$+0.02$
Åbo	29	-0.01	$+0.08$
Greenw'h (CH.)	29	-0.04	$+0.07$
Cape	33	$+0.05$	$+0.09$
Pulkowa	69	-0.08	0.00
Combination	74	0.00	$+0.04$
Pulkowa	84	$(+0.02)$	(-0.02)
Wien-Ottakring	97	-0.23	-0.07

From various considerations the first and last values appear to be entitled to small weight. Those from Greenwich 29 and Pulkowa 69 are the only ones which depend on a thorough investigation that takes into account the annual term of latitude variation,— the only term which can have introduced a serious uncertainty in B as to terms in $\Delta\delta_\alpha$. If this annual term tends to have a constant value throughout the period of observation, then it is probable that the proper motions of B are virtually free from any sensible inequality of the form $\Delta\delta_\alpha$. This is the particular end desired by the writer in this investigation.

Several circumstances conspire to eliminate a part of the annual term in the observed declinations; so that the full effect of that term may not have appeared in B. Some of the older observations depend upon zenithal, or polar, points derived from observation of stars. Difference of longitude of the observatories tends to diminish the resultant effect. In many observatories the observation of brighter stars is extended over long periods of time; and when observations are made at all hours of the night, as at some of the principal observatories, the resultant effect of the annual term would be somewhat diminished.

When all the elements which conspire to produce errors of the form, $\Delta\delta_\alpha$, are considered, one can scarcely fail to be surprised at the very small discordances of this form that appear in the catalogues of large weight. For a considerable percentage of those catalogues $\Delta\delta_\alpha$ is comparable with the probable error of its determination through comparison with B.

Corrections of the Form, $\Delta\delta_\delta$.

In proceeding to obtain a normal system which may be considered to give the most probable representation of the testimony of observation, we must employ in the first instance only those catalogues which are supposed to give independent declinations. Since the astronomical refraction plays such an important part in the determination of the zenith-distances of stars, a series of observed declinations can scarcely be regarded as independent unless it can be shown that the adopted latitude and refraction constant are consistent with the results of extensive and sufficient observations of circumpolar stars. The strict application of this criterion would reduce the series of independent declinations to a comparatively small number. But the method of comparing catalogues resulting from nearly contemporaneous observations in the two hemispheres appears to offer a legitimate means for increasing the number of virtually independent determinations. This method of deducing conclusions as to the most probable value of the refraction constant for each of the sets of observations compared has been recognized ever since the time when accurate observations were first made in the Southern hemisphere. If we put

ρ'_n = the mean adopted refraction at the pole for the northern observatory,

ρ'_s = the same for the southern observatory,

ρ_n and ρ_s = the respective adopted mean refractions for a star common to the two catalogues — considered positive north of the zenith,

k_n and k_s = 100 times the factors by which the adopted refractions should be multiplied in order to find their corrections,

then each comparison between the observed declinations at a northern and southern observatory, respectively, furnishes an equation of the form,

$$\frac{\rho_n-\rho'_n}{100}\,k_n - \frac{\rho_s-\rho'_s}{100}\,k_s = \delta_s-\delta_n$$

This equation affords a sufficient approximation only when the values of k are relatively small; and it assumes that the true polar point has been already found in the reductions for the catalogue. It is also implied that the error of the adopted instrumental corrections may be neglected. In general, for the better class of catalogues, it may be assumed that the polar point has been well determined, in the sense that the observed declinations above and below the pole, in its vicinity, are the same; and that this has been established by an adequate number of observations. As to the instrumental corrections, it may be said that no determination of them can be regarded as free from sensible error. Errors depending on $\sin z$, however, are partly taken up in k, resulting, perhaps, in a spurious value of that quantity, but in a value which best represents the error of the catalogue for moderate zenith-distances. Errors of the form, $\cos z$, are not apt to be of serious consequence in these comparisons. For instance, in the case of Greenwich this error is zero at declination $+13°$, while, at the Cape, the length of the arc between $+22°$ and the south pole is independent of the term in $\cos z$. As in all other classes of meridian-observation for star-declinations, the error of graduation remains as a very uncertain ele-

ment; and for immunity from the ill effects of this uncertainty we should endeavor to bring to bear upon this problem the results from many different instruments.

Therefore, before forming the first set of zone-equations, several northern and southern catalogues, not otherwise absolutely independent, were compared for the determination of k. Omitting the voluminous details the results for k are exhibited in Table II.

TABLE II. REFRACTION FACTORS FROM CATALOGUE COMPARISONS.

		k	
Greenwich	30	−0.140	+Redn. to BESSEL's refr.
St. Helena	32	−0.190	+Redn. to BESSEL's refr.
Cambridge	34	+0.642	
Cape	33	−0.193	
Greenwich	42	+0.068	
Cape	37	−0.231	
Greenwich	57	−0.035	
Cape	59	−0.205	
Greenwich	64	−0.167	
Melbourne	68 (S.)	−0.257	
	(N.)	−0.257	+0.542 ρ
Washington	77	−0.125	
Cordoba	76	−0.412	
Melbourne	77 (S.)	−0.207	
	(N.)	−0.207	+0.542 ρ
Greenwich	82	−0.217	GILL's comparison, I
Cape	83	−0.218	GILL's comparison, I
Greenwich	94	−0.108	
Cape	89	−0.014	

In making the comparison of Greenwich 30 (POND) with St. Helena 32, the zonal means for POND's declinations were first revised to bring them into substantial conformity with the results derived from CHANDLER's reduction of POND's reciprocal observations with two mural circles (*A.J.* XVI, 3). There are 33 stars in common with B. We have:

POND (CH.) — B.

δ	**	$-\Delta\delta_0$
°		″
+65.4	7	+0.06
+40.6	4	+0.22
+25.6	8	+0.09
+11.4	12	+0.20

The range over which these determinations extend is not sufficient for a good determination of k through comparison with a southern catalogue. CHANDLER shows by means of POND's observations of four circumpolar stars culminating at a low altitude that BESSEL's refractions employed in the reductions produce consistent results (*A.J.* XIV, 4).

It seems to me that the extension of Dr. CHANDLER's reduction of POND's declinations to include all of the zenith-distances observed by POND (1825–1835) would be a work surpassing in importance any other of this nature which could be undertaken. Washington 77 was compared both with Cordoba 76 and Melbourne 77, and the results were so adjusted as to produce what was regarded as the best practicable reconciliation of the three catalogues. Some such combination as this was necessary in the case of the Cordoba observations, which were not carried much beyond 60° north of the zenith.

It should be remembered that the corrections, k, are applicable to the refractions employed in the reductions of the respective catalogues; so that for the Melbourne catalogue for 1870 we are to employ as the corrected refraction on both sides of the zenith, BESSEL's × 0.9937; and for Melb. 80, BESSEL's × 0.9942. Likewise for Cape 89, where Pulkowa refractions were employed in the reductions, we have as the corrected refraction, approximately, BESSEL's × 0.9972. The general result indicated in Table II is in favor of refractions virtually equivalent to those of the Pulkowa tables. In the mean, it appears that the Greenwich observations are best satisfied by the refraction, BESSEL's × 0.9987.

The instrumental corrections required by the Greenwich and Cape transit circles have been investigated very thoroughly. This is especially true as to errors of graduation. Consequently the values of k derived from the mutual comparisons of this series of observations appear to deserve great confidence. So much cannot be said for the other comparisons embraced in Table II; so that, for these, the quantities k, in each individual case ought not to be attributed to refraction alone. However, we may still hope that, in the mean, this process has resulted in the elimination of the larger part of the errors due to the employment of imperfect refractions in the reductions; and that, large as they are, there will also be a tendency toward the elimination of uncorrected instrumental errors. Accordingly the catalogues of Table II, as corrected, together with Kön. 21, Dpt. 24, Åbo 29, Pulk. 47, Paris (LAUGIER) 53, Leiden 67, Pulk. 69, Pulk. 84, and Strassb. 86, have been employed in deducing the fundamental system. The circumpolar observations of LAUGIER do not indicate any material correction of the adopted refraction (CAILLET's) so that this series can be regarded as fairly independent. A similar conclusion applies to the declinations of Strassburg 86, where the circumpolar observations are quite well satisfied by the adoption of BESSEL's refractions.

BAUSCHINGER's declinations (Munich 92), though independent, contain few stars outside the circumpolar region, and these are not well distributed for our purpose; so that no use is made of this series in the operations for establishing the system.

GROSSMANN's observations at Wien-Ottakring, 1897, contain a large number of observations of circumpolar stars, and were given weight as independent determinations from

zone $+40°$ northward. A somewhat careful analysis of the declinations leads to the suspicion that anomalies exist in this series of observations which will receive attention further on.

The method of forming the zone-equations from which the observed systematic corrections of B_s were ascertained for each zone of $5°$ need not be presented here. In place of this the process is illustrated in Table IV of the present paper, which is intended for an independent test of B. Here the zones are $15°$ instead of $5°$ in width; but the method of computation is the same. Following the columns in which the catalogues with their estimated mean dates of observation are given, the estimated weight of each in the fundamental sense appears under the designation, p'_0. Then, under n, in each zone, is given the mean correction of B indicated by each catalogue of observation, with the weight, p, which represents the precision of n when corrected for supposed systematic error. The unit of p is supposed to have a probable error of $\pm 0''.1$. The residuals, n, corrected for the effect of adopted k (Table II) are entered under the caption n'. These represent the means for correcting the system, B, in order to arrive at an absolute normal system, based upon the best testimony readily available for that purpose. This is accomplished by means of zone-equations of the form,

$$\Delta\delta_0 + T\Delta\mu'_0 = n',$$

T having been reckoned in units of a century from 1875. The weights have been taken from the column headed, p'_0, as already explained, but they have been modified in a few instances when p is very small, and in all cases where the zenith-distance of the stars, upon which the n of a given observatory is based, is greater than $65°$. The weight becomes zero when the zenith-distance is greater than $75°$. For the corrected catalogues enumerated in Table II, however, the modification of weight for zenith-distance was not so marked as for the others. The results for the solution of the zone-equations are given under the heading, "Fundamental Solution," in Table V. It will be seen that this first approximation does not indicate any material correction of the adopted normal system, B. This fact, however, has no special significance, or advantage over the original solution, except to show that B_s, as prepared for a basis to be corrected, was sufficiently precise and homogeneous for the purpose; and to demonstrate, furthermore, that in the subsequent operations the system as established by the original zone-equations has been preserved with fidelity. With comparatively few numerical errors in the subsequent operations, and especially with want of attention in drawing the curves for $\Delta\delta_s$, this might not have been the case.

Table V exhibits, after the values of $\Delta\delta_s$ and $100\ \Delta\mu'_s$, their respective probable errors, as computed from the equations. Whether these are really valid depends upon the correctness of the relative weights employed. There appears to be no mathematical method of arriving at a decision upon this point. If these probable errors may be regarded as fair approximations to the truth, then it may be said, in general, that the probable systematic error of the declinations for 1900, in the zone, $+25°$ to $-15°$, is about $\pm 0''.07$, and of $100\ \Delta\mu'_0$, $\pm 0''.19$. Further north the uncertainty is less, and further south it is greater. After the final operations to be described, the nature of the systematic error in B, which may be revealed in the future, should be such that its variation from one zone of $5°$ to an adjacent zone shall be very small. The probable errors attached to the results of fundamental solutions in Table V are largely interdependent from one zone to another; so that, for instance, if a comparatively large correction should be found for zone, $+1°$, then the true corrections for $+15°$ and $-15°$ would probably have the same sign as at $+1°$.

Secondary Revision of the System.

It may be said that the fundamental equations, in a general way, have established the position of the equator among the stars. It now remains to investigate the graduation of the sky intermediate between this equator and the poles with the aid of further evidence of observation available for the purpose.

There are many series of observed declinations which cannot be regarded as offering independent determinations, for which, nevertheless, the instrumental corrections for errors of graduation, flexure, etc., have been carefully investigated, but for which an independent determination of refraction is wanting. For some of these series the zenith-points were determined through assumed declinations of the stars. If this has been done, as at Mt. Hamilton, by means of a restricted zone of stars, or even as at Paris through a less restricted choice that is calculated to yield consistent zenith-points throughout the year, then the testimony of such observations may be of value in smoothing out sinuosities of systematic error in the observed corrections of B_s and B_s due to resultant graduation errors affecting the comparatively few catalogues upon the testimony of which the absolute normal system has been founded. Accordingly all the catalogues deemed suitable for the purpose were compared with the system, B′, which has resulted from the operation just described. It was assumed that the systematic errors of the catalogues could usually be represented by a correction of the form,

$$\Delta\varphi + a \sin z + b \cos z + k \frac{\rho}{100}$$

When $\Delta\varphi$ is omitted the last term was taken as $k\ \frac{\rho - \rho'}{100}$. $\Delta\varphi$ is not necessarily the true correction of the latitude; it simply represents a constant correction of the declinations. Only in the case of Cape 33 was an attempt made

to determine b. With the ordinary arrangement of observations, as to z.d., it is seldom possible to secure adequate discrimination between a and k. It is, therefore, impossible to attach much importance to either as standing purely for that which it is supposed to represent. The advantage of the formulas in representing the systematic corrections, as a whole, is not seriously impaired. Table III contains the result of the investigation for formulated systematic correction of each of the catalogues employed in the secondary revision of B′.

Table III. Formulated Systematic Corrections.

		$\Delta\varphi$	a	k	
		″			
Greenwich	15	−0.54	−0.48	. .	
Königsberg	21	. .	+0.05	. .	
Dorpat	24	. .	. .	+0.476	
Åbo	29	. .	−0.16	+0.230	
Greenwich	30	−0.07	−0.38	+0.313	+Corr'n, Bradley's to Bessel's refr.
St. Helena	32	−0.55	. .	−0.130	+Corr'n, Young's to Bessel's refr.
Cape	33	−0.49	+0.73	−0.474	+0.58 cos z +Div. Corr.
Cambridge	34	−0.29	−0.04	+0.30	
Cape	37	. .	+1.48	−0.094	(a applies +3° to −72°)
Greenwich	38	. .	−0.04	−0.261	
Greenwich	45	+0.05	. .	+0.031	
Pulkowa	45	. .	+0.32	. .	
Santiago	51	. .	+0.70	−0.150	
Greenwich	51	−0.04	. .	+0.016	
Paris	53	+0.13	−0.56	0.00	
Washington	56	+0.19	−0.61	−0.400	
Greenwich	57	+0.21	−0.17	−0.057	
Cape	59	. .	. .	−0.125	
Paris	60	−0.05	−0.75	−0.567	
Melbourne	62	. .	+0.07	−0.400	
Greenwich	64	. .	. .	−0.084	
Cape	65	. .	. .	−0.068	
Leiden	67	. .	−0.14	. .	
Melbourne	68	. .	. .	−0.346	+Redn. to South refr.
Pulkowa	69	. .	+0.10	−0.162	
Greenwich	72	. .	. .	+0.493	$-0''.60\,(\sin^2 z' - \sin^2 z)$
Harvard	75	. .	. .	−0.202	
Cordoba	76	. .	. .	−0.389	
Paris	76	−0.21	−0.46	−0.376	
Cape	76	. .	. .	−0.109	
Melbourne	77	. .	. .	−0.426	+Redn. to South refr.
Washington	77	−0.10	−0.81	−0.568	
Greenwich	82	. .	−0.06	−0.246	
Cape	83	. .	. .	−0.221	
Pulkowa	84	. .	+0.18	+0.055	
Radcliffe	85	−0.42	−0.90	−0.431	
Strassburg	86	. .	. .	+0.005	
Cape	89	. .	. .	−0.030	
Madison	90	. .	. .	−0.190	
Greenwich	94	. .	−0.25	−0.304	
Mt. Hamilton	95	. .	. .	−0.111	
Wien-Ottakr.	97	−0.37	−0.81	−0.250	
Albany	98	. .	. .	. .	

In the majority of instances the formulas of correction for the several catalogues do not appear to require special comment. The application of these corrections to the corresponding values of n (Table IV) results in the values of n'', final corrections to B given by each catalogue of observation. Inspection of the values of n'' for most of the better catalogues indicates that the formulas, in general, represent the discordances from B remarkably well. This must be regarded as very satisfactory for at least two reasons.

1. This could scarcely have been the case with so many catalogues, derived from observations in the two hemispheres, unless the final system, B, is a very fair approximation to a true normal system.

2. The mystery of systematic errors is very largely removed by this showing of the sources from which the greater part of them may have arisen.

In many instances, however, if we compute k from the normal equations, assuming values of a materially different from those contained in Table III, we shall still find that the combined systematic correction down to p.d., 100°, remains substantially unchanged. We should not, therefore, attach too much significance, in those cases, to the relative distribution of the correction between a and k. Some remarks upon individual catalogues may be of service.

Cape 33. This will receive attention later, in connection with the extension of the system southward.

Cape 37. The sine term, a, seems to apply only between +3° and −72° of declination, and to be due to the peculiar treatment of observations by reflection. This catalogue is in need of revision.

Melb. 68 *and* 77. See previous remarks in regard to these catalogues.

Greenwich 72. The formula for flexure adopted in the reductions for this series was $a \sin z \cos^2 z$. If we assume that the formula should have been $a \sin z$, and put $a = -0''.60$ the resulting corrections to the declinations should be,

$$-0''.60\,(\sin^2 z' - \sin^2 z),$$

z' representing the zenith-distance of the pole.

Wien-Ottakring. The formula for this series was derived from a very careful discussion in which the declinations below pole were treated separately from those above. The value of a, as distinguished from k, is entitled to a fair degree of confidence. That the flexure adopted by Grossmann in the reduction of this series can be in error so much as 0″.81 seems scarcely credible. Yet, if the flexure determined by Herz for the same instrument (as quoted by Grossmann, p. 52) had been adopted we should have had as the correction of Grossmann's present declination, $-0''.58\,(\sin z' - \sin z)$, and the discordance from B would have been reduced to a small quantity, so far as this term is concerned. Somewhere between +30° and +45° there seems to be a very large alteration in the systematic

corrections required in order to bring GROSSMANN's declinations into harmony with B. The adopted correction at +50° is −0″.03, and at +25,° −1″.01; at −45°, −0″.19, and +30°, −0″.85. Between declinations, +31° 47′ and +44° 52′, GROSSMANN has only one star, so that it is impossible to analyze the nature of this comparatively abrupt alteration in the difference, B−W.-Ott. The combined testimony of all the recent catalogues is wholly against the hypothesis that there exists a material anomaly in B in this vicinity. If the discordance should be attributed to some defect in the Vienna observations, or in the reduction of them, the most natural source of suspicion would seem to be as to the adopted division correction, though the hypothesis of some looseness in the fastening of the objective, or ocular, may not be wholly excluded.

With the formulated systematic corrections contained in Table III, the zone-equations were revised and solved anew. The results for each zone are contained in the table for B−B_s (*A.J.* 531, p. 21). By way of illustration, and in order, at the same time, to test the system, B, the process is here repeated. The necessary data are exhibited in Table IV. n'' represents the corrections to B which would have been found if the declinations of the individual catalogues had been first corrected for the effect of the formulas in Table III. Following the values of n'' in each column are the weights, p'', which were adopted in the solution of the zone-equations. These are compounded of the weights, p, which are due to the casual error of n, and of a certain maximum weight, p''_0 (Table IV), assigned to each catalogue from an estimate of the probable outstanding errors due to the unavoidable imperfection of the formulas of correction (especially because they do not take account of errors of graduation). The general aim was that the unit of weight should correspond to a probable error of ±0″.1. This probable error turned out to be ±0″.08, in the mean, so that the weights, p''_0, were too small on the whole.

The results of the solution of these zone-equations are given under the head of "Secondary Solution" in Table V.

It will be noted that the formulas in Table III are based, not upon B_f but upon B_s and B_r. The results in Table V indicate, therefore, that nothing can be gained by further approximations under the principles adopted. We must either await further observations, or adopt radical improvements of method, before a further gain in systematic accuracy can be anticipated. The questions, what authorities afford independent evidence upon the true system of star-declinations, and what relative weights should be assigned to them, are matters of individual judgment upon which the most competent critics will differ. But this difference of judgment must be somewhat radical before any very material modifications of the present results can be obtained.

TABLE V. RESULTS FROM ZONE-EQUATIONS.

FUNDAMENTAL SOLUTION.

Zone.	Mean Ep.	$\Delta\delta_s$ 1875	p.e.	100 $\Delta\mu'_s$	p.e.
°		″	″	″	″
+78	1865	−0.017	±0.014	−0.01	±0.06
60	1865	−0.030	.027	−0.04	.12
45	1865	−0.021	.034	−0.06	.15
29	1866	−0.038	.037	−0.05	.18
15	1865	−0.029	.037	−0.19	.17
+ 1	1867	−0.025	.039	−0.19	.19
−15	1867	+0.011	.046	+0.02	.23
−30	1872	+0.054	±0.071	+0.44	±0.46

SECONDARY SOLUTION.

Zone.	Mean Ep.	$\Delta\delta_s$ 1875	p.e.	100 $\Delta\mu'_s$	p.e.
°		″	″	″	″
+78	1866	+0.017	±0.013	−0.01	±0.056
60	1864	+0.015	.014	+0.01	.058
45	1866	+0.015	.012	+0.02	.054
29	1868	+0.007	.010	+0.03	.045
15	1868	−0.002	.012	−0.02	.054
+ 1	1868	+0.002	.010	−0.05	.049
−15	1869	−0.003	.008	+0.03	.040
30	1877	+0.005	.024	−0.13	.141
45	1872	−0.003	.032	+0.14	.193
60	1872	−0.023	.031	+0.06	.189
−81	1873	−0.053	±0.029	+0.07	±0.190

Extension of System to the Southern Hemisphere.

Owing to the scarcity of reliable determinations of declination from observations of the Southern hemisphere during the first half of the nineteenth century, the attempt to secure systematic accuracy in the proper-motions of the far Southern stars is one of very great difficulty. Previous to the first observations made with the Cape Transit circle, there were no measurements of star-declination at observatories in the Southern hemisphere which appear to be entitled to weight as absolutely independent determinations. HENDERSON's work at the Cape is of the first quality, so far as the skill and judgment of the observer is concerned, but his observations cover only one year, and he was handicapped by the remarkable defects of the Jones mural circle. Nevertheless, his observations appeared to offer the only practicable hope for deriving important independent evidence as to Southern declinations previous to 1860. The best plan of procedure seemed to be in accepting the system B as absolute for stars as far southward as −24°, and to employ these with other means in an attempt to determine the instrumental errors of the Jones circle. The solution of the zone observations was carried out definitively, in the first instance, only to −24°. Subsequently, and after first approximate correction of Cape 33, the zone-corrections at −30° and −34° were ascertained with close approximation to the final result. In connection with other corrections an attempt was made to determine the systematic errors of graduation. For this purpose the mean correction for the arc, 32°.5 to 37°.5, was called C_1; that of 57°.5 to 62°.5, C_6; 27°.5 to 32°.5, C_{12}, etc. Circle readings increase toward the north, and north

TABLE IV. MATERIALS FOR ZONE-EQUATIONS, — FORMATION OF NORMAL SYSTEM.

Catalogue		+78°					+60°					+45°						+29°				
	p'_0	p	n	n′	n″	p″	p	n	n′	n″	p″	δ	p	n	n′	n″	p″	p	n	n′	n″	p″
Grw. 16	–	2	+.20	. .	–.13	0.7	4	+.56	. .	–.10	0.8	46°	4	+.61	. .	+.02	0.8	5	+ .81	. .	+.09	0.8
Kön. 21	2	4	+.04	+.04	+.05	0.8	4	–.20	–.20	–.18	0.8	47	5	+.09	+.09	+.13	0.8	1	+ .02	+.02	+.07	0.5
Dpt. 24	3	6	+.13	+.13	+.07	0.5	7	+.34	+.34	+.18	0.6	45	9	+.37	+.37	+.13	0.6	6	+ .28	+.28	–.04	0.5
Åbo. 29	3	4	+.04	+.04	–.02	1.5	8	+.08	+.08	–.08	1.5	46	12	+.12	+.12	–.11	1.5	8	+ .32	+.32	.00	1.5
Grw. 30	1	2	–.10	–.17	–.09	0.4	3	+.21	–.05	–.07	0.5	45	4	+.60	–.03	+.04	0.5	4	+1.10	+.28	+.21	0.5
St. H. 32	0.5	.	.	.	.	.	.	.	.	.	.	46	.	–1.20	+.25	–.02	.	1	–1.15	+.18	–.43	0.2
Cape 33	0.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	– .19	–.47	+.14	0.4
Camb. 34	0.5	1	+.01	–.07	–.07	0.3	2	+.36	+.11	+11	0.3	46	1	+.69	+.36	+.36	0.3	2	+ .44	–.01	–.01	0.3
Cape 37	0.5	.	.	.	.	.	.	.	.	.	.	41	1	+.50	–.19	+.21	.	2	+ .18	–.26	–.01	0.4
Grw. 38	1	6	+.05	+.05	+.07	1.0	9	+.03	.00	+.11	1.5	45	8	–.04	–.08	+.07	1.5	6	– .06	–.11	+.08	1.0
Grw. 45	1	6	–.04	–.04	+.02	1.0	8	–.16	–.19	–.11	1.5	45	8	+.10	+.06	+.15	1.5	6	– .17	–.22	–.13	1.0
Paris 45	–	7	+.02	. .	+.02	1.0	6	+.17	. .	+.17	1.0	44	8	–.11	. .	–.11	1.5	7	– .02	. .	–.02	1.0
Pulk. 47	7	12	–.04	–.04	+.02	3.0	11	–.18	–.18	–.02	3.0	45	22	–.27	–.27	–.03	3.0	18	– .36	–.36	–.04	3.0
Stgo. 51	–	.	.	.	.	.	.	.	.	.	.	40	.	+.02	. .	+.70	.	1	– .59	. .	+.27	0.2
Grw. 51	–	5	+.15	. .	+.10	1.0	4	+.18	. .	+.13	1.0	45	6	+.14	. .	+.09	1.0	9	+ .02	. .	–.04	1.5
Par.(L)53	1	2	–.32	–.32	.	.	3	–.34	–.34	.	.	46	2	–.18	–.18	.	.	2	+ .18	+.18	.	.
Wash. 56	–	3	–.49	. .	–.10	1.0	2	–.37	. .	–.04	1.0	45	3	–.27	. .	–.04	1.0	4	– .04	. .	+.09	1.5
Grw. 57	3	10	–.14	–.14	+.13	1.5	7	–.30	–.29	–.06	1.5	45	8	–.18	–.16	+.02	1.5	13	– .19	–.17	–.03	1.5
Cape 59	3	.	.	.	.	.	.	.	.	.	.	40	.	–.22	–.81	–.59	.	5	+ .17	–.22	–.09	0.8
Paris 60	–	5	+.08	. .	+.26	1.5	6	–.09	. .	–.06	1.5	45	8	–.07	. .	–.15	1.5	12	+ .17	. .	–.01	1.5
Melb. 62	–	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	– .29	. .	+.13	0.3
Grw. 64	2	11	–.01	+.02	+.01	1.5	6	+.05	+.11	+.08	1.5	44	8	–.09	.00	–.05	1.5	11	– .10	+.02	–.04	1.5
Cape 65	–	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	– .01	. .	–.15	0.8
Leid. 67	8	3	–.10	–.10	–.13	1.0	10	+.18	+.18	+.12	1.5	45	15	+.16	+.16	+.06	1.5	9	+ .13	+.13	–.01	1.5
Melb. 68	2	.	.	.	.	.	.	.	.	.	.	40	.	–.77	–.22	–.49	.	4	– .13	+.05	–.11	0.6
Pulk. 69	10	19	.00	.00	+.03	3.0	15	–.05	–.05	+.05	3.0	45	30	–.23	–.23	–.08	4.0	26	– .24	–.24	–.03	3.0
Grw. 72	–	12	+.10	. .	–.05	1.0	9	+.40	. .	+.07	1.0	45	10	+.50	. .	+.09	1.0	15	+ .51	. .	–.01	1.0
Harv. 75	–	8	–.08	.	–.06	0.6	6	+.06	. .	+.13	0.6	45	9	–.10	. .	.00	0.6	9	– .23	. .	–.10	0.6
Cord. 76	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	5	+ .87	+.10	+.09	1.5
Paris 76	–	3	–.02	. .	–.12	1.0	1	+.20	. .	+.04	0.7	44	5	+.31	. .	+.08	1.5	12	+ .31	. .	+.03	1.5
Cape 76	–	.	.	.	.	.	.	.	.	.	.	41	.	–.42	. .	–.75	.	2	+ .28	. .	+.06	0.7
Melb. 77	1	.	.	.	.	.	.	.	.	.	.	40	.	–1.13	–.41	–1.13	.	5	+ .03	+.31	+.16	0.4
Wash. 77	3	23	–.04	–.01	+.10	2.0	11	–.18	–.12	–.12	1.5	45	20	–.05	+.03	–.09	2.0	24	+ .31	+.41	+.13	2.0
Grw. 82	3	16	–.07	–.04	–.04	2.0	12	–.20	–.12	–.13	1.5	44	12	–.04	+.08	+.05	1.5	19	– .06	+.09	+.05	2.0
Cape 83	3	.	.	.	.	.	.	.	.	.	.	41	2	+.91	+.21	+.21	.	9	+ .45	+.01	+.01	1.5
Pulk. 84	10	26	.00	.00	+.01	3.0	23	–.06	–.06	+.01	3.0	45	41	–.05	–.05	+.06	4.0	36	– .18	–.18	–.03	4.0
Radcl. 85	–	7	+.03	. .	–.09	0.8	5	+.51	. .	+.18	0.8	44	4	+.48	. .	–.02	0.8	8	+ .74	. .	+.08	0.8
Strass. 86	4	4	.00	.00	.00	2.0	11	+.06	+.06	+.06	3.0	46	15	+.09	+.09	+.09	3.0	9	+ .02	+.02	+.02	3.0
Cape 89	3	.	.	.	.	.	.	.	.	.	.	41	3	+.19	+.13	+.10	.	14	– .01	–.04	–.07	2.0
Madn. 90	–	20	+.11	. .	+.15	1.0	11	+.14	. .	+.23	1.0	45	19	+.01	. .	+.13	1.0	21	– .32	. .	–.17	1.0
Grw. 94	3	23	+.10	+.12	+.10	2.0	20	.00	+.04	–.01	2.0	45	26	+.10	+.16	+.07	2.0	31	+ .06	+.13	+.02	2.0
Mt. H. 95	–	13	–.05	. .	–.02	1.0	.	.	.	.	.	40	3	–.07	. .	+.01	0.8	3	+ .25	. .	+.35	0.8
W-Ott. 97	(2)	8	–.05	–.05	–.13	.	12	–.06	–.06	–.14	.	48	15	+.05	+.05	–.27	.	4	+ .98	. .	+.40	.
Alb. 98	–	6	–.07	. .	–.07	2.0	1	–.31	. .	–.31	0.7	45	6	–.05	. .	–.05	2.0	10	+ .02	. .	+.02	2.0

Catalogue	–45°					–60°				–81°			
	δ	p	n	n″	p″	p	n	n″	p″	p	n	n″	p″
St. H. 32	45°	2	–.50	+.13	0.2	2	–.14	+.15	0.2	1	–.71	–.46	.
Cape 33	45	3	–.12	–.22	0.4	3	+.25	–.21	0.4	4	+.06	–.01	0.4
Cape 37	44	8	+.38	+.06	0.5	6	+.75	+.04	0.5	5	+.19	–.04	0.5
Stgo. 51	47	1	–.30	+.01	0.2	2	–.23	–.07	0.3	1	+.07	+.08	0.2
Cape 59	45	7	+.06	–.03	0.8	6	+.02	–.05	0.8	8	–.10	–.13	0.8
Melb. 62	43	1	–.19	+.03	0.2	1	+.40	+.50	0.2	4	+.25	+.25	0.3
Cape 65	45	4	–.16	–.21	0.8	6	–.06	–.10	0.8	5	–.23	–.25	0.8
Melb. 68	45	7	+.63	+.39	0.6	6	+.46	+.28	0.6	10	+.15	+.07	0.7

(Continued on opposite page.)

TABLE IV. MATERIALS FOR ZONE-EQUATIONS, — FORMATION OF NORMAL SYSTEM.

Catalogue	p''_0	+15° p	n	n'	n''	p''	+1° p	n	n'	n''	p''
Grw. 16	1.0	10	+.72	. .	−.11	1.0	8	+.80	. .	−.11	0.8
Kön. 21	1.0	2	+.11	+.11	+.17	0.7	1	−.09	−.09	−.02	0.5
Dpt. 24	0.6	5	+.24	+.24	−.19	0.5	4	+.73	+.73	+.12	0.5
Abo. 29	2.0	8	+.44	+.44	+.03	1.5	9	+.57	+.57	+.05	1.5
Grw. 30	0.6	6	+1.50	+.42	+.30	0.5	5	+1.47	−.02	−.15	0.5
St. H. 32	0.2	1	−1.22	−.10	−.13	0.2	2	−.89	+.11	−.15	0.2
Cape 33	0.5	2	+.06	−.24	+.02	0.4	2	+.12	−.12	+.04	0.4
Camb. 34	0.4	3	+.22	−.35	−.35	0.4	3	+.32	−.43	−.43	0.4
Cape 37	0.5	3	+.18	−.17	+.04	0.4	3	−.25	−.54	+.13	0.4
Grw. 38	1.5	8	−.14	−.20	+.04	1.5	7	−.28	−.36	−.03	1.0
Grw. 45	1.5	10	+.01	−.05	+.05	1.5	8	−.11	−.19	−.08	1.5
Paris 45	1.5	12	+.03	. .	+.03	1.5	13	+.15	. .	+.15	1.5
Pulk. 47	4.0	21	−.31	−.31	+.08	3.0	10	−.40	−.40	+.04	3.0
Stgo. 51	0.3	2	−.86	. .	+.02	0.3	1	−.89	. .	−.10	0.2
Grw. 51	1.5	12	−.15	. .	−.21	1.5	11	+.11	. .	+.04	1.5
Par.(L) 53	–	3	+.18	+.18	. .	.	3	+.42	+.42	. .	.
Wash. 56	2.0	6	+.06	. .	+.10	1.5	6	+.19	. .	+.18	1.5
Grw. 57	2.0	21	−.27	−.24	−.13	2.0	19	−.03	+.01	+.09	2.0
Cape 59	1.0	11	+.28	−.03	+.09	1.0	10	+.31	+.05	+.15	1.0
Paris 60	2.0	16	+.28	. .	+.03	2.0	15	+.21	. .	−.02	2.0
Melb. 62	0.3	3	−.34	. .	+.23	0.3	4	−.85	. .	−.29	0.3
Grw. 64	2.0	19	−.04	+.11	+.03	2.0	14	−.01	+.18	+.09	1.5
Cape 65	1.0	15	+.17	. .	+.07	1.0	14	+.22	. .	+.13	1.0
Leid. 67	1.5	12	+.38	+.38	+.21	1.5	8	+.11	+.11	−.08	1.0
Melb. 68	0.7	7	+.40	+.42	+.29	0.6	7	+.16	+.10	−.02	0.6
Pulk. 69	4.0	29	−.35	−.35	−.08	3.0	17	−.34	−.34	+.02	3.0
Grw. 72	1.0	17	+.60	. .	−.11	1.0	14	+1.09	. .	+.09	1.0
Harv. 75	0.7	11	−.27	. .	−.10	0.7	8	−.09	. .	+.13	0.6
Cord. 76	3.0	19	+.60	−.04	−.01	3.0	19	+.47	−.08	−.05	3.0
Paris 76	2.0	16	+.25	. .	−.07	2.0	16	+.22	. .	−.08	2.0
Cape 76	1.0	5	+.16	. .	−.01	0.8	7	+.09	. .	−.05	0.8
Melb. 77	0.4	11	+.43	+.53	+.19	0.4	10	+.27	+.27	.00	0.4
Wash. 77	2.0	33	+.26	+.38	−.02	2.0	27	+.24	+.39	−.10	2.0
Grw. 82	2.0	27	−.28	−.09	−.14	2.0	22	−.11	+.14	+.09	2.0
Cape 83	2.0	18	+.30	−.04	−.04	2.0	22	+.28	.00	.00	2.0
Pulk. 84	4.0	41	−.25	−.25	−.08	4.0	25	−.24	−.24	−.06	3.0
Radcl. 85	1.0	11	+.75	. .	−.02	1.0	10	+.74	. .	−.07	1.0
Strass. 86	4.0	16	+.07	+.07	+.07	3.0	20	+.02	+.02	+.01	3.0
Cape 89	2.0	26	+.03	+.01	−.02	2.0	27	+.04	+.02	.00	2.0
Madn. 90	1.0	27	−.33	. .	−.15	1.0	20	−.36	. .	−.14	1.0
Grw. 94	2.0	39	+.01	+.11	−.03	2.0	38	−.08	+.05	−.08	2.0
Mt. H. 95	1.0	11	+.06	. .	+.17	1.0	19	+.09	. .	+.23	1.0
W-Ott. 97	–	5	+1.05	. .	+.33	.	6	+.79	. .	−.03	.
Alb. 98	3.0	19	+.10	. .	+.10	3.0	10	.00	. .	.00	2.0

Catalogue	−15° p	n	n'	n''	p''	−30° δ	p	n	n'	n''	p''
Grw. 16	9	+.98	. .	.00	0.8	25°	1	+1.13	. .	+.12	.
Kön. 21	1	−.14	−.14	−.07	0.5	.	.	. .	. .	. .	.
Dpt. 24	2	+.94	+.94	−.16	0.2	.	.	. .	. .	. .	.
Abo. 29	5	+.70	+.70	−.11	0.5	.	.	. .	. .	. .	.
Grw. 30	6	+2.08	−.12	−.15	0.5	25	1	+2.64	+.31	+.01	.
St. H. 32	4	−.60	+.12	−.09	0.2	30	4	+.08	+.79	+.20	0.2
Cape 33	3	−.16	−.37	+.01	0.4	31	2	−.69	−.86	−.09	0.4
Camb. 34	3	+1.13	−.03	−.03	0.3	25	.	+2.12	+.21	+.21	.
Cape 37	6	−.38	−.62	.00	0.5	31	11	−.09	−.30	−.09	0.5
Grw. 38	7	−.43	−.55	−.03	1.0	25	1	−.61	−.80	.00	.
Grw. 45	8	+.01	−.11	+.02	1.5	25	1	+.07	−.12	+.05	.
Paris 45	13	+.07	. .	+.07	1.5	26	2	−.39	. .	−.39	.
Pulk. 47	3	−.50	−.50	−.03	1.0	.	.	. .	. .	. .	.
Stgo. 51	2	−.98	. .	−.33	0.3	29	1	−.66	. .	−.17	0.2
Grw. 51	10	+.09	. .	+.02	1.5	25	1	+.07	. .	−.03	.
Par.(L) 53	1	+.27	+.27	. .	.	26	.	−.50	. .	. .	.
Wash. 56	6	+.03	. .	+.05	1.5	28	3	−.25	. .	−.04	0.5
Grw. 57	17	−.12	−.06	+.02	2.0	25	2	−.33	−.23	−.07	.
Cape 59	10	+.21	−.01	+.08	1.0	30	13	−.03	−.21	−.14	1.0
Paris 60	15	+.05	. .	.00	1.5	26	3	−.56	. .	−.05	.
Melb. 62	3	−.55	. .	−.08	0.3	30	3	−.40	. .	−.04	0.3
Grw. 64	15	−.14	+.16	+.01	1.5	25	1	−.63	−.16	−.39	.
Cape 65	18	+.11	. .	+.04	1.0	29	8	−.01	. .	−.07	1.0
Leid. 67	4	−.02	−.02	−.23	1.0	.	.	. .	. .	. .	.
Melb. 68	5	+.38	+.26	+.16	0.6	30	6	+.41	+.24	+.16	0.6
Pulk. 69	8	−.48	−.48	−.06	1.0	.	.	. .	. .	. .	.
Grw. 72	13	+1.43	. .	+.07	1.0	25	1	+2.22	. .	+.07	.
Harv. 75	8	−.31	. .	−.02	0.6	27	2	−.33	. .	+.09	0.4
Cord. 76	19	+.43	−.03	−.01	3.0	29	13	+.27	−.13	−.11	2.0
Paris 76	15	+.16	. .	−.01	1.5	26	2	−.32	. .	−.14	.
Cape 76	8	+.03	. .	−.09	0.8	30	11	+.22	. .	+.12	1.0
Melb. 77	11	+.60	+.52	+.30	0.4	29	6	+.72	+.60	+.41	0.4
Wash. 77	25	+.31	+.50	.00	2.0	29	13	+.13	+.39	+.13	1.5
Grw. 82	21	−.42	−.14	−.08	2.0	25	2	−.58	+.04	+.05	.
Cape 83	29	+.25	+.02	+.02	2.0	28	14	+.28	+.08	+.08	1.5
Pulk. 84	13	−.11	−.11	+.03	1.5	.	.	. .	. .	. .	.
Radcl. 85	10	+.51	. .	−.20	1.0	25	1	+.50	. .	+.27	.
Strass. 86	29	+.02	+.02	+.01	4.0	26	5	+.48	+.48	+.46	.
Cape 89	26	+.10	+.09	+.07	2.0	30	18	−.03	−.04	−.06	2.0
Madn. 90	6	−.29	. .	−.01	1.0	.	.	. .	. .	. .	.
Grw. 94	35	−.20	−.01	−.06	2.0	25	3	−.52	−.21	−.06	.
Mt. H. 95	11	−.20	. .	−.03	1.0	30	10	−.43	. .	−.19	1.0
W-Ott. 97	6	+.53	. .	−.28	.	28	1	+.42	. .	−.14	.
Alb. 98	14	+.07	. .	+.07	2.0	29	24	−.04	. .	−.04	3.0

Catalogue	−45° δ	p	n	n''	p''	−60° p	n	n''	p''	−81° p	n	n''	p''
Cord. 76	45°	7	+.18	−.14	2.0	5	+.07	−.18	2.0	23	+.02	−.08	3.0
Cape 76	45	9	+.17	+.09	1.0	10	−.19	−.25	1.0	10	−.25	−.28	1.0
Melb. 77	46	1	+.72	+.43	0.3	3	+.47	+.25	0.4	9	+.08	−.01	0.4
Washn. 77	40	1	−.72	+.03	.	.	.	.	.	.	.	.	.
Cape 83	46	11	+.16	−.01	1.5	9	+.29	+.16	1.5	10	+.07	+.02	2.0
Cape 89	44	18	+.01	−.01	2.0	15	−.03	−.05	2.0	13	.00	.00	2.0
Mt. H. 95	40	1	−1.18	−.78	.	.	.	.	.	.	.	.	.
Alb. 98	40	1	−.37	−.37	.	.	.	.	.	.	.	.	.

z.d.'s are regarded as positive. Then the corrections to declinations are assumed to be:

$$\Delta\varphi + a \sin z + b \cos z + k\,\frac{\rho}{100} + C_n = \Delta\delta$$

To reconcile the constant difference between direct and reflected z.d.'s a constant, Δz, is introduced. The northern declinations furnish 16 equations; D − R (see Mem. R.A.S., X, p. 78), 12 equations; and declinations below pole *minus* declinations above pole (Mem. R.A.S., X, p. 73), six equations. The weights are adopted in general accordance with HENDERSON's data wherein 0″.22 is the minimum p.e. of repeated observations on a given object, and the casual error is taken from HENDERSON's table (p. 59), but not smaller than ±0″.56 in any case. Adopted p.e. of unit of weight is ±0″.30.

In the comparison of observations above and below pole (and in two other instances) the grouping did not permit of the direct determination of a single C independent of its next neighbor. In such cases the proportional ratio with which each C was involved was expressed in the equation. Each division correction was represented in at least four of the 34 equations. The normal equations were formed with all rigor, and solved by approximations, — the convergence being rapid and requiring only three repetitions for a precise result. The values of the unknowns resulted as follows:

$\Delta\varphi = -0''.49$	$C_4 = -0''.04$
$\Delta z = +0.16$	$C_5 = -0.58$
$a = +0.73$	$C_6 = -0.40$
$b = +0.58$	$C_7 = -0.19$
$k = -0.474$	$C_8 = -0.25$
	$C_9 = -0.35$
$C_1 = +0.30$	$C_{10} = -0.48$
$C_2 = +0.65$	$C_{11} = +0.59$
$C_3 = +0.20$	$C_{12} = +0.52$

The leap between C_{10} and C_{11} is especially notable and is fully confirmed by the trend of the individual comparisons which make up the means wherever those graduations occur. The following table exhibits computed and observed values of the discordances upon which the investigation is founded.

This comparison seems to leave nothing to be desired. Only one discrepancy is relatively large (the third in "lower — upper") and this is within admissible limits relatively to its weight. The remarkable feature of this investigation is that, while the declinations north of the Cape zenith are completely reconciled to those of B, the discordance, D − R, which gave HENDERSON so much trouble is accounted for in a manner which seems to be very satisfactory. The observations above and below pole are also satisfactorily accordant. These facts seem to warrant confidence in the determination of the correction for errors of graduation. On the other hand, a small but decided discordance of the latitude obtained in this discussion from that of the modern results for the Cape transit circle subtracts somewhat from the complete satisfaction which might otherwise be felt.

RESULTS OF DISCUSSION OF HENDERSON'S OBSERVATIONS, C − O.

$\Delta\delta$, B — Cape 33.

p	δ	(O)	(C)
	°	″	″
2.2	+45.9	−1.32	−1.26
2.2	38.8	−0.05	−0.08
5.4	30.2	+0.17	+0.14
4.6	26.1	+0.31	+0.48
6.4	21.4	+0.62	+0.63
6.9	15.0	−0.47	−0.36
8.6	10.8	−0.06	−0.20
7.9	5.9	+0.02	−0.06
5.9	+ 1.2	−0.13	+0.01
5.5	− 3.6	−0.19	−0.19
7.5	8.8	−0.42	−0.37
12.2	13.6	+0.15	+0.16
9.7	19.0	+0.36	+0.39
7.0	23.8	+0.79	+0.81
5.0	29.5	+0.38	+0.42
6.5	−33.8	+0.70	+0.61

D − R (Δz).

p	z	(O)	(C)
	°	″	″
0.7	−40.8	+0.89	+0.92
1.5	−34.9	+1.81	+1.29
0.7	−29.6	+1.27	+1.19
6.6	−25.0	+0.90	+1.07
2.2	−15.1	+0.40	+0.21
2.2	+ 5.5	−1.17	−1.35
1.5	+16.0	+0.06	−0.57
1.5	+24.6	−0.46	−0.15
0.7	+32.1	−0.89	−0.30
1.5	+40.4	−1.08	−0.97
1.5	+44.5	−1.48	−1.18
2.2	+50.6	−1.65	−1.63

$\Delta\delta$, Lower — Upper.

p	δ	(O)	(C)
	°	″	″
3.0	−89.0	+0.08	−0.03
3.2	−84.1	+0.49	+0.50
0.9	−78.3	+0.73	−0.44
0.4	−74.7	−0.65	−0.75
1.4	−68.1	−1.38	−0.94
1.2	−63.9	−0.39	−0.05

The latitude of the Jones mural circle given by this discussion is −33° 56′ 3″.25 + $\Delta\varphi$ − Δz = −33° 56′ 3″.90. For the Cape transit circle we have for the seconds of latitude, and their values when the refraction-corrections of Table II are taken into account:

Cape 59 (p. 10, Int.)	3″.55	−0″.10	3″.65
Cape 83 (p. xlvii, Int.)	3.54	. .	3.54
Cape 89 (p. xxiv, Int.)	3.45	+0.03	3.48

In 1886-1891 GILL also determined the latitude by the TALCOTT method. The declinations of his northern stars were determined with the Pulkowa vertical circle. The southern element of his pairs consisted of close circumpolar stars, observed equally at both culminations. Thus, errors in the assumed declinations of the southern stars were eliminated. The seconds of this result are, 3″.65 (Cape 85, Int. p. xlvii). Assuming that the Pulkowa declinations in question require the systematic correction to B which has been found for Pulkowa 84, the seconds of latitude reduced to B would be 3″.53. Accordingly, we may assume that the mean latitude of the Cape transit circle is 3″.54, and this is numerically smaller than the latitude deduced in this discussion of HENDERSON's observations by 0″.36. For so short a period as one year, the mean latitude applicable to HENDERSON's observations, uncorrected for variation of latitude, may have been sensibly different from the true mean latitude; but this difference can scarcely have been greater than 0″.1.

This discordance in latitude may point to a numerically smaller value of k than was reached in the present discussion; but a more probable explanation is indicated in connection with the term $b \cos z$. If this term had been omitted in the equations we should have had, approximately: $\Delta\varphi = -0''.08$; $a = +0''.78$; $k = -0.50$; with small changes in the correction for errors of graduation. The seconds of latitude would then have been: $3''.49$, in excellent agreement with the modern results; supposing that there is no such thing as secular variation of latitude. The northern declinations would not have been so accordant with B as in the actual solution; but they would not have been altered at any point within 70° of the zenith by so much as $0''.2$; and ordinarily by less than half that amount.

Adopting the result of this discussion of Cape 33, the zone equations, south of −30°, were first solved, omitting St. Helena 32 and Cape 37. Then the corrections for these catalogues and for Melb. 62, were formulated as in Table III. The effect of this process is to assign some weight in the formation of the southern system to these authorities through the medium of their relations to northern stars, and to make the essential weight of Cape 33 in the formation of the system to be about one-half greater than that which is nominally assigned to it in Table IV.

It would seem to be extremely desirable that the zenith-distances observed by McClear for the Cape Catalogue of 1840 should be discussed anew on principles somewhat similar to those which have been adopted in the foregoing discussion of Cape 33. This would afford a much needed check upon the conclusions derived from Henderson's observations. However, we shall soon be in a position to determine whether this treatment of Henderson's observations has resulted in improvement of them. If errors have been introduced thereby, these will be reflected in some degree in the values of proper-motion, and the effects ought to become very sensible in the predicted declinations of B within five, or ten, years from the present time.

OBSERVATIONS OF COMET *d* 1902 (*GIACOBINI*),

MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY, UNIVERSITY OF VIRGINIA,

BY T. McN. SIMPSON, JR.

1903 Charl. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Apr. 21 9 37 50	1	10 , 4	+0 45.66	+0 50.0	7 20 39.42	+32 5 59.9	9.688	0.464	+0.99	−6.7
27 9 35 5	2	d12 , 8	+0 21.08	−6 7.9	7 29 8.57	+32 44 56.5	9.702	0.482	+0.90	−6.5
28 10 27 6	2	8 , 8	+1 52.27	−0 0.7	7 30 39.74	+32 51 3.6	9.734	0.589	+0.88	−6.5
29 9 51 49	3	8 , 6	+2 6.34	−6 18.6	7 32 6.35	+32 56 46.2	9.719	0.525	+0.87	−6.5

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority
	h m s	° ′ ″	
1	7 19 52.77	+32 5 16.6	Leiden, A.G. 3128
2	7 28 46.59	+32 51 10.8	" " 3192
3	7 29 59.14	+33 3 11.1	" " 3199

NOTES.

d refers to direct micrometrical measurement. April 27 — Comet very faint; observation difficult. These observations have been corrected for refraction.

I wish to call attention to a mistake in my observations of this comet published in *A.J.*, Nos. 537–8. Log $p\Delta$ throughout should be increased by 1 in the characteristic. The same correction should be applied to Mr. McCallie's observations published in *A.J.*, No. 534, for the reduction of which I am responsible.

Charlottesville, Va.

OBSERVATIONS OF COMET *a* 1903 (*GIACOBINI*),

MADE WITH THE 11-INCH EQUATORIAL AT THE SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS.,

BY MARY E. BYRD.

1903 Greenw. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Feb. 20 11 48 49	1	16 , 11	−0 12.97	−1 48.5	23 43 44.86	+12 3 48.7	9.639	0.740	−0.21	+0.9
22 12 6 38	2	12 , 10	−0 25.67	−6 54.5	23 47 32.72	+12 49 29.4	9.646	0.748	−0.19	+0.8
Mar. 2 11 48 24	3	8 , 8	+3 10.04	−1 46.4	0 3 35.29	+15 45 37.4	9.652	0.737	−0.19	+0.1
11 48 24	4	8 , –	+3 12.60	. . . .	0 3 35.44	. . . .	9.652	. .	−0.19	. .
12 12 10 1	5	10 , 7	−1 0.57	−6 32.2	0 21 58.47	+17 14 47.2	9.657	0.760	−0.12	−0.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	23 43 58.04	+12 5 36.3	Leipzig I, A.G. 9451	4	0 0 23.03	+15 48 46.9	Berlin A, A.G. 9778
2	23 47 58.58	+12 56 23.1	Leipzig I, A.G. 9469	5	0 22 59.16	+17 21 20.2	Berlin A, A.G. 109
3	0 0 25.44	+15 47 23.7	Berlin A, A.G. 9779				

OBSERVATIONS OF SUNSPOTS,

MADE AT BOSTON UNIVERSITY OBSERVATORY.

By C. Q. JONES and L. R. TUCKER, Students in Astronomy.

W.M.T. 1902–1903			Groups N	Groups S	Spts. in Gps. N	Spts. in Gps. S	Totals Grps.	Totals Spts.	Def.
	d	h							
Oct.	6	4	1	0	12	0	1	12	G
	6	22	2	0	15	0	2	15	G
	7	4	2	0	17	0	2	17	G
	8	1	2	0	14	0	2	14	G
	9	1	2	0	21	0	2	21	G
	10	0	2	0	4	0	2	4	F
	10	22	1	0	1	0	1	1	P
	13	0	1	0	3	0	1	3	G
	14	3	1	0	1	0	1	1	F
	15	2	1	0	3	0	1	3	G
	15	22	1	0	1	0	1	1	F
	19	23	1	0	3	0	1	3	G
	22	1	0	1	0	14	1	14	F
	23	4	0	1	0	13	1	13	G
	24	1	0	2	0	24	2	24	G
	28	4	0	2	0	8	2	8	F
	30	1	0	1	0	1	1	1	F
	31	0	0	1	0	1	1	1	G
Nov.	14	3	2	1	8	1	3	9	G
	15	0	2	1	6	1	3	7	P
	19	22	1	0	30	0	1	30	G
	20	2	1	0	25	0	1	25	G
	20	23	1	0	25	0	1	25	P
	21	2	1	0	18	0	1	18	F
	22	1	1	0	17	0	1	17	G
Jan.	5	0	0	1	0	2	1	2	F
	5	23	0	1	0	2	1	2	P
	8	1	0	1	0	1	1	1	P
Feb.	5	2	0	1	0	1	1	1	G
	6	1	0	1	0	1	1	1	F
	8	23	0	1	0	6	1	6	G
	10	3	0	1	3	10	2	13	G
	12	1	1	0	2	0	1	2	E

W.M.T. 1903			Groups N	Groups S	Spts. in Gps. N	Spts. in Gps. S	Totals Grps.	Totals Spts.	Def.
	d	h							
Feb.	13	1	1	0	2	0	1	2	P
	19	1	0	1	0	4	1	4	P
	20	0	0	1	0	5	1	5	P
	24	1	1	2	1	4	3	5	F
	24	23	1	2	1	6	3	7	G
	25	1	1	2	1	7	3	8	G
	26	2	1	1	1	9	2	10	G
	27	1	1	1	1	6	2	7	P
Mar.	2	2	1	0	2	0	1	2	G
	4	2	1	0	3	0	1	3	F
	13	2	0	1	0	5	1	5	G
	13	23	0	1	0	4	1	4	G
	25	2	1	0	6	0	1	6	F
	26	3	1	1	4	1	2	5	G
	27	0	1	1	3	6	2	9	G
Apr.	1	1	1	1	5	10	2	15	F
	2	1	1	2	2	6	3	8	F
	3	1	0	2	0	4	2	4	G
	6	1	0	1	0	2	1	2	P
	8	22	0	1	0	5	1	5	G
	9	2	0	1	0	6	1	6	G
	9	22	0	1	0	5	1	5	G
	12	23	0	1	0	3	1	3	P
	20	5	0	1	0	1	1	1	
	22	2	0	1	0	1	1	1	P
	23	0	0	1	0	1	1	1	F
	27	1	3	1	27	4	4	31	G
	28	2	2	1	19	4	3	23	G
	29	1	2	1	28	2	3	30	E
	29	23	2	1	68	9	3	77	G
May	1	0	2	2	32	3	4	35	G
	1	22	2	0	7	0	2	7	P
Totals,			54	50	442	209	104	651	

For explanations see *A.J.* 466.
Observations were made, and no spots seen, as follows: November, 8 days; December, 10 days; January, 9 days; February, 1 day; March, 7 days; May, 4 days. 23 different groups we observed, containing 223 different spots; 13 groups, with 151 spots, were in north latitude, while 10 groups, with 72 spots, were south.

PROJECTION ON *MARS*.

A dispatch *via* Harvard College Observatory, May 27, states that a large projection on *Mars* was found by SLIPHER at Lowell Observatory, Flagstaff, Ariz., May 26^d 8^h 35^m (Mountain Standard Time), in position angle 200°, lasting 35 minutes.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Captain C. M. Chester, U.S.N., Superintendent.]

1903 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(9) *Metis*.					
Apr. 5 9h 55m 5s	1	27 , 8	−0m 30.65s	+ 2′ 41.7″	12h 48m 45.85s	+ 3° 12′ 6.4″	n9.362	0.713	+2.29s	−12.8″
8 9 11 56	2	17 , 6	−0 47.55	−10 29.3	12 45 55.39	+ 3 24 20.0	n9.444	0.713	+2.30	−12.8
9 11 45 2	2	28 , 6	−1 50.31	− 6 13.0	12 44 52.63	+ 3 28 36.3	8.261	0.705	+2.30	−12.8
10 9 54 28	3	29 , 6	−0 37.86	− 9 16.9	12 44 0.99	+ 3 32 5.1	n9.274	0.708	+2.30	−12.8
17 9 22 13	4	29 , 6	−1 33.19	+ 1 7.5	12 37 48.57	+ 3 53 53.1	n9.268	0.704	+2.29	−12.6
					(230) *Athamantis*.					
Apr. 8 11 21 30	5	30 , 6	−1 38.16	+ 6 7.7	13 13 7.63	−18 11 39.3	n8.988	0.864	+2.64	−11.4
8 11 21 30	6	30 , 6	−3 28.89	+ 2 31.3	13 13 7.60	−18 11 41.2	n8.988	0.864	+2.64	−11.4
10 10 39 4	7	30 , 6	−1 38.97	− 6 53.7	13 11 20.97	−17 55 46.5	n9.213	0.858	+2.64	−11.7
17 10 34 15	8	30 , 6	+1 44.32	+ 0 9.9	13 5 7.06	−16 55 21.1	n9.019	0.858	+2.64	−13.0
18 9 37 7	9	30 , 10	−0 2.77	− 3 41.5	13 4 17.03	−16 46 44.5	n9.317	0.848	+2.64	−13.0
					(60) *Echo*.					
Apr. 18 10 45 11	10	28 , 6	+1 22.18	− 1 26.6	14 3 8.37	−10 27 36.3	n9.266	0.816	+2.62	− 9.1
21 12 12 52	11	29 , 6	+2 25.08	− 7 13.5	14 0 14.85	−10 7 18.1	8.261	0.821	+2.59	− 9.4
27 10 13 2	12	29 , 6	−1 47.52	− 0 13.5	13 54 44.21	− 9 28 40.1	n9.210	0.812	+2.61	− 9.4
29 11 15 39	13	30 , 6	+2 6.09	+ 1 12.6	13 52 53.36	− 9 15 45.3	n8.282	0.815	+2.61	− 9.8
May 2 9 13 30	14	30 , 8	−0 6.69	+ 5 0.1	13 50 35.70	− 9 0 12.7	n9.357	0.804	+2.62	− 9.9
					(83) *Beatrix*.					
Apr. 28 11 19 46	15	29 , 6	−0 55.58	+ 1 5.8	14 16 23.41	−15 17 4.1	n8.830	0.851	+2.75	− 8.1
29 12 10 51	16	30 , 6	+1 28.91	+ 3 18.4	14 15 20.58	−15 14 50.0	8.684	0.851	+2.75	− 8.4
May 2 11 28 8	16	30 , 6	−1 29.23	+ 9 46.9	14 12 22.46	−15 8 21.5	n7.980	0.851	+2.77	− 8.4
6 9 34 36	17	24 , 5	+0 33.94	+ 9 35.6	14 8 33.99	−14 59 42.9	n9.297	0.840	+2.78	− 9.2
11 9 39 3	18	35 , 7	−1 59.47	− 3 51.5	14 4 0.31	−14 49 12.3	n9.154	0.844	+2.78	− 9.2
					(16) *Psyche*.					
Apr. 28 12 53 48	19	29 , 6	−0 48.84	+ 4 6.3	14 44 33.96	−11 32 58.8	8.830	0.829	+2.68	− 6.2
May 4 10 44 39	20	30 , 6	+0 47.58	+ 3 21.2	14 39 52.77	−11 9 2.6	n9.133	0.824	+2.73	− 6.8
4 10 44 57	21	30 , 6	+1 31.24	+ 3 21.5	14 39 52.49	−11 9 2.1	n9.131	0.824	+2.72	− 6.8
7 10 26 10	22	30 , 6	+1 23.08	− 0 19.7	14 37 30.41	−10 57 19.3	n9.159	0.822	+2.74	− 6.9
8 10 26 52	22	30 , 6	+0 35.77	+ 3 33.1	14 36 43.11	−10 53 26.5	n9.126	0.822	+2.75	− 6.9
					(12) *Victoria*.					
May 7 12 14 6	23	30 , 6	+2 18.42	− 1 39.2	15 12 18.48	−19 53 4.3	7.423	0.875	+2.95	− 4.1
8 10 55 43	24	30 , 6	+0 10.53	− 5 46.2	15 11 26.01	−19 44 18.5	n9.174	0.868	+2.96	− 4.4
9 9 38 57	24	30 , 6	−0 42.72	+ 3 4.2	15 10 32.77	−19 35 28.2	n9.457	0.847	+2.97	− 4.5
11 10 26 42	25	30 , 6	−1 42.79	− 5 8.3	15 8 36.99	−19 16 12.4	n9.254	0.863	+2.97	− 4.4
11 10 51 16	26	30 , 6	+0 45.27	+ 0 55.0	15 8 35.72	−19 16 5.8	n9.114	0.867	+2.98	− 4.6

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	12h 49m 14.21s	+ 3° 9′ 37.5″	Albany, A.G. 4600	14	13h 50m 39.77s	− 9° 5′ 2.9″	Radcliffe 1890, 3610
2	12 46 40.64	+ 3 35 2.1	" " 4586	15	14 17 16.24	−15 18 1.8	Wash.A.G.Z.53,113,209
3	12 44 36.55	+ 3 41 34.8	" " 4575	16	14 13 48.92	−15 18 0.0	" " 47,119,209,231
4	12 39 19.47	+ 3 52 58.2	" " 4555	17	14 7 57.27	−15 9 9.3	" A.G.Z. 50, 114
5	13 14 43.15	−18 17 35.6	Wash.A.G.Z.51,116,204	18	14 5 57.00	−14 45 11.6	" " 45, 115
6	13 16 33.85	−18 14 1.1	" " 44,116	19	14 45 20.12	−11 36 58.9	Radcliffe 1890, 3835
7	13 12 57.30	−17 48 41.1	" " 47,117	20	14 39 2.46	−11 12 17.0	½(Schj.5220+Mün.I,10482)
8	13 3 20.10	−16 55 18.0	" " 44,116,204	21	14 38 18.53	−11 12 16.8	Mün. I, 10469
9	13 4 17.16	−16 42 50.0	" " 44,110	22	14 36 4.59	−10 56 52.7	Yarnall, 6156
10	14 1 43.57	−10 26 0.6	½(Mü.I, 9840+Mü.II, 5181)	23	15 9 57.11	−19 51 21.0	Cincinnati 1885, 2585
11	13 57 47.18	− 9 59 55.2	Wien, A.G.Z., 56, 250	24	15 11 12.52	−19 38 27.9	" " 2588
12	13 56 29.12	− 9 28 17.2	" " 134, 253	25	15 10 16.81	−19 10 59.7	" " 2587
13	13 50 44.66	− 9 16 48.1	Radcliffe 1890, 3611	26	15 7 47.47	−19 16 56.2	Radcliffe 1890, 3927

NOTES ON VARIABLE STARS, — No. 38,

By HENRY M. PARKHURST.

2689 Z *Puppis.* In the Supplementary Catalogue, *A.N.*, No. 514, No. 2690 should be 2689, the variable discovered by Perry, and announced in *A.J.*, No. 428. The four observed maxima appear to vary at least 30 days from the average period of 255 days, while both observed minima have occurred within about 60 days from the average times of maxima.

2690 *X Puppis.* The observations of 2690 show 5 maxima, with an average deviation of 14 days from the average period of 155 days. Perry's observations in April, 1899, *A.J.* 468, nearly correspond with a sixth maximum. The observed minimum of 1903 is consistent with a period of only one-half of this.

Subtangent Process. An illustration of the subtangent method (*A.J.* 400 and 456) occurs in obtaining the maximum for 976, which was lost in the twilight just before it was reached. The 8 observations given below, by smoothing, were reduced to the three following:

A 6155.36 $8^m.35$;
B 6166.81 8.09;
C 6175.60 7.92;

Making the first tangent from

AB 11.45 = −.26; 1 day = $-^m.0227$

Making the second tangent from

BC 8.79 = −.17; 1 day = $-^m.0193$

The mean of observations A, B, is 6161.08 $8^m.22$;
The mean of observations B, C, is 6171.20 $8^m.00$.
The maximum is assumed to occur at the time $C + t$.
From the first tangent, we obtain for the time $14.52 + t$, the vertex, $8.22 - (.329 + .0227\,t)$.
From the second tangent, we obtain for the time $4.40 + t$, the vertex $8.00 - (.085 + .0193\,t)$.

The subtangents are included within the parentheses. The subtangent being trisected at the vertex, the vertex is at the distance of two-thirds of the subtangent from the respective magnitudes. Multiplying the observed magnitudes by 1½, and then taking the differences, we have the equation,

$$.33 - .244 - .0034\,t = 0$$

whence $t = +25.3$; and hence the maximum is at 6201, 3 days earlier than from the elements, and 8 days after my last possible observation. From the last subtangent, making $t = 0$, the deduced maximum, $7^m.91$ is derived.

Results of Observations.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	W	Mag.	Factors			Remarks
				1902-3							d	
103	*T Andromedae*	Max.	6084	Nov. 30	65	–	E	–	–	–	–	
114	*S Ceti*	Max.	6062	Nov. 8	34	+10	2	–	–	–	–	From light-curve
434	*S Piscium*	Max.	5947	July 16	33	–	E	–	–	–	–	
466	*U Piscium*	Max.	6012	Sept. 19	48	–	E	–	–	–	–	
715	*S Arietis*	Min.	6098	Dec. 14	39	–	E	–	–	–	–	
845	*R Ceti*	Min.	6151	Feb. 5	79	–	E	–	–	–	–	
893	*U Ceti*	Max.	6124	Jan. 9	28	0	4	7.4	–	–	–	
976	*T Arietis*	Max.	6201	Mar. 27	35	− 3	9	7.91	–	–	–	Derived by the subtangent process
1113	*U Arietis*	Max.	6123	Jan. 8	9	−43	4	7.7	–	–	–	Elements, *A.J.* 403
1166	*X Ceti*	Max.	6102	Dec. 18	12	–	E	–	–	–	–	Elements, *A.J.* 438
1577	*R Tauri*	Max.	6179	Mar. 5	46	−33	9	7.34	0.65	1.34	19	
1582	*S Tauri*	Max.	6174	Feb. 28	42	−52	9	9.60	2.64	1.69	28	
1717	*V Tauri*	Max.	6136	Jan. 21	64	+30	3	9.7	–	–	–	
1761	*R Orionis*	Max.	6171	Feb. 25	46	+25	3	11.2	–	–	–	Unsatisfactory
1805	*V Orionis*	Min.	5382	Dec. 28	14	–	E	–	–	–	–	1900
"	"	Min.	6180	Mar. 6	17	–	E	–	–	–	–	1903. Reappeared shortly after
1944	*S Orionis*	Max.	6072	Nov. 18	29	–	E	–	–	–	–	Probably later
2013	*U Aurigae*	Max.	5408	Jan. 23	9	− 5	5	–	–	–	–	1901
"	"	Max. A	6210	Apr. 5	11	−17	7	7.70	0.32	0.31	5	
"	"	Max. B	6226	Apr. 21	11	− 1	3	7.8	–	–	–	Subtangent approximation
2080*a*	*Z Tauri*	Max.	6138	Jan. 23	–	–	1	10.5	–	–	–	Probably earlier
2100	*U Orionis*	Max.	6231	Apr. 26	17	−21	9	5.69	0.24	0.19	4	
2266	*V Monocerotis*	Max.	6179	Mar. 5	22	+22	9	7.58	1.33	1.11	9	Obsns. unsatisfactory
2387	*— Geminorum*	–	–	–	–	–	–	–	–	–	–	Turner's Nova
2404	*X Geminorum*	Min.	–	–	–	–	–	10.3	–	–	–	Slight changes
2445	*W Monocerotis*	Min.	6202	Mar. 28	–	–	7	10.8	–	–	–	An apparent minimum
2475	*X Monocerotis*	Max.	6191	Mar. 17	–	–	5	8.4	–	–	–	Possibly earlier
2689	*Z Puppis*	Max.	6107	Dec. 23	–	–	–	8.2	–	–	–	Period 255
2690	*X Puppis*	Max.	5121	Apr. 11	–	–	7	8.0	–	–	–	1900. Period 155 ?
"	"	Min.	6136	Jan. 21	–	–	8	8.7	–	–	–	
"	"	Min.	6171	Feb. 25	14	−18	7	8.34	–	–	–	

Individual Observations.

Including Observations by Arthur C. Perry.

103 *T Andromedae*

(Cont. from 498. Comp. Stars 346)

Julian	Calendar	Mag.
	1902	
6092	Dec. 8	10]
6103	19	11]
6115	31	12]
	1903	
6133	Jan. 18	12]

114 *S Ceti.*

(Continued from 498.)

Julian	Calendar	Mag.
	1902	
6092	Dec. 8	8.3
6107	23	8.8
	1903	
6133	Jan. 18	9.8

434 *S Piscium.*

(Continued from 400.)

Julian	Calendar	Mag.
	1902	
6111	Dec. 27	12]
6115	31	12]
	1903	
6133	Jan. 18	12]

466 *U Piscium.*

(Continued from 498.)

Julian	Calendar	Mag.
	1899	
4984.6	Nov. 25	11.4P
	1902	
6111	Dec. 27	11.8]
	1903	
6133	Jan. 18	11.8]

715 *S Arietis.*

(Continued from 498.)

Julian	Calendar	Mag.
	1902	
6115	Dec. 31	11]
	1903	
6133	Jan. 18	11]

845 *R Ceti.*

(Continued from 498.)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1	12]
6133	18	12]
6189	Mar. 15	12]

893 *U Ceti.*

(Cont. from 498. Comp. Stars 346)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1	8.0
6124	9	7.4
6133	18	8.0
6145	30	8.7

976 *T Arietis.*

(Cont. from 498. Comp. Stars 403)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1	8.9
6133	18	8.3
6145	30	9.0
6168	Feb. 22	8.0
6171	25	7.8
6177	Mar. 3	8.0
6189	15	7.7
6193	19	7.8

1113 *U Arietis.*

(Cont. from 498. Comp Stars 314)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1	8.0
6133	18	7.7
6145	30	8.5

1166 *X Ceti.*

(Cont. from 498. Comp. Stars 468)

Julian	Calendar	Mag.
	1903	
6133	Jan. 18	10.1
6136	21	10.4
6145	30	11.0

1577 *R Tauri.*

(Continued from 498.)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1 to	
6136	21	11]
	3 dates	
6145	Jan. 30	10.3
6168	Feb. 22	8.4
6171	25	7.5
6174	28	7.1
6177	Mar. 3	7.3
6180	6	7.5
6189	15	7.4
6201	27	7.6
6217	Apr. 12	7.8

1582 *S Tauri.*

(Continued from 498.)

Julian	Calendar	Mag.
	1903	
6133	Jan. 18	9.9
6136	21	9.9
6145	30	9.9
6168	Feb. 22	9.6
6171	25	9.7
6174	28	9.5
6177	Mar. 3	9.5
6189	15	10.0
6201	27	10.1]

1717 *V Tauri.*

(Cont. from 513. Comp. Stars 513)

Julian	Calendar	Mag.
	1903	
6133	Jan. 18	9.9
6136	21	9.7
6145	30	10.3

1761 *R Orionis.*

(Continued from 438.)

Julian	Calendar	Mag.
	1899	
4984.6	Nov. 25	10.3
	1900	
5347.5	Nov. 23	10.0
5376.5	Dec. 22	10.1
5378.5	24	10.25_2
	1901	
5471.5	Mar. 27	11.5]
	1903	
6138	Jan. 23	11.9
6168	Feb. 22	11.2
6171	25	11.2
6174	28	11.2
6177	Mar. 3	11.4]
6189	15	11.4]
6201	27	11.4]

1805 *V Orionis.*

(Cont. from 468. Comp. Stars 319)

Julian	Calendar	Mag.
	1900	
5376.5	Dec. 22	11]
	1903	
6169	Feb. 23	10.2]
6205	Mar. 31	10.3
6217	Apr. 12	10.2

1944 *S Orionis.*

(Continued from 403)

Julian	Calendar	Mag.
	1903	
6169.5	Feb. 23	7.4
6172.5	26	8.2
6175.5	Mar. 1	8.51_2
6176.5	2	7.83_2
6177.5	3	8.38_2
6180.5	6	9.08_2
6186.5	12	9.58_2

2013 *U Aurigae.*

(Continued from 468.)

Julian	Calendar	Mag.
	1900	
5376.5	Dec. 22	9.5
5378.5	24	8.94_2
5383.5	29	8.24_2
	1901	
5471.5	Mar. 27	10.46_2
	1903	
6206	Apr. 1	7.55_2
6208	3	8.39_3
6209	4	7.65_2
6210	5	7.76_2
6215	10	8.11_3
6217	12	7.95_2
6222	17	8.86_2
6223	18	8.43_2
6224	19	6.85_2
6225	20	8.13_2
6226	21	8.16_2
6229	24	8.77_2

2080a *Z Tauri.*

Julian	Calendar	Mag.
	1903	
6138	Jan. 23	10.5
6168	Feb. 22	11.0
6171	25	11.4
6226	Apr. 21	12]

2100 *U Orionis.*

(Continued from 438)

Julian	Calendar	Mag.
	1903	
6138	Jan. 23	10.4]
6168	Feb. 22	10.1
6171	25	10.0
6177	Mar. 3	10.5
6203	29	7.4
6217	Apr. 12	5.7
6222	17	5.7
6223	18	5.36_2
6224	19	7.99_2
6225	20	7.61_2
6226	21	6.48_2
6229	24	5.89_2
6230	25	5.70_2

2100 *U Orionis.*—Cont.

Julian	Calendar	Mag.
	1903	
6231	Apr. 26	5.82_2
6232	27	5.40_2
6233	28	5.82_2
6234	29	6.48_2
6235	30	6.31_2
6236	May 1	6.84_2
6237	2	6.41_2

2266 *V Monocerotis.*

(Cont. from 468. Comp. Stars 403)

Julian	Calendar	Mag.
	1903	
6169.5	Feb. 23	7.6
6172.5	26	8.0
6175.5	Mar. 1	7.4
6176.5	2	7.54_2
6177.5	3	7.92_2
6178.5	4	7.29_2
6180.5	6	7.49_2
6186.5	12	7.92_2
6201.5	27	7.95_2

2387 *Nova Geminorum.*

Julian	Calendar	Mag.
	1903	
6205	Mar. 31	7.4
6206	Apr. 1	7.54_2
6208	3	7.88_2
6209	4	7.80_2
6210	5	8.51_2
6215	10	8.47_2
6217	12	8.83_2
6222	17	8.07_2
6223	18	8.74_2
6224	19	7.62_2
6225	20	8.01_2
6226	21	6.60_2
6229	24	9.23_2
6230	25	9.39_4
6232	27	9.54_2
6233	28	9.95_2
6235	30	9.13_4
6236	May 1	9.20_2
6237	2	9.99_2

2404 *X Geminorum.*

(Continued from 441.)

Julian	Calendar	Mag.
	1903	
6205	Mar. 31	10.3
6206	Apr. 1	10.47_2
6222	17	10.55_2
6225	20	10.40_2
6226	21	9.10_2
6233	28	10.66_2

2445 *W Monocerotis.*

Julian	Calendar	Mag.
	1903	
6188	Mar. 14	10.4
6189	15	10.02_2
6192	18	10.08_2
6200	26	10.94_2
6205	31	10.91_2
6206	Apr. 1	10.65_2
6222	17	9.9

2475 *X Monocerotis.*

Julian	Calendar	Mag.
	1903	
6189	Mar. 15	8.84_2
6191	17	8.1
6192	18	8.62_2
6193	19	8.38_2
6200	26	9.08_2
6205	31	9.32_2
6206	Apr. 1	9.23_2
6222	17	8.8

2689 *Z Puppis.*

(Cont. from 513. Comp. Stars 403)

Julian	Calendar	Mag.
	1899	
4975.7	Nov. 16	12P
	1902	
5832.5	Mar. 23	10.4P
5835.5	26	10.4P
5853.5	Apr. 13	10.2P
5867.5	27	10.2P
5871.5	May 1	9.9P
6106.7	Dec. 22	8.3P
6107	23	8.2
6114	30	8.2
6114.6	30	8.4P
	1903	
6121.6	Jan. 6	8.7
6130.6	15	9.1
6132.6	17	9.9
6145.6	30	9.7
6160.6	Feb. 14	10.2
6168.6	22	9.9P
6174.6	28	10.5P
6178.6	Mar. 4	9.9]
6201.6	27	9.9
6208.5	Apr. 3	9.9]P
6222.6	17	9.9
6224.5	19	10.4P

2690 *X Puppis.*

(Cont. from 468. Comp. Stars 403)

Julian	Calendar	Mag.
	1900	
5056.6	Feb. 5	8.5P
5081.6	Mar. 2	8.5P
5102.5	23	8.1P
5105.5	26	8.3P
5123.5	Apr. 13	8.0P
5137.5	27	8.3P
5141.5	May 1	8.7P
	1902	
6106.7	Dec. 22	8.2P
6107	23	8.0
6114	30	8.2
6114.6	30	8.0P
	1903	
6121	Jan. 6	8.5
6130.6	15	8.9
6132	17	8.7
6145	30	8.7
6160	Feb. 14	8.2
6168	22	8.3
6187	Mar. 13	8.05_2
6201	27	9.15_2
6222	Apr. 17	8.3
6224.5	19	8.7P

OBSERVATIONS OF THE COMPANIONS OF *SIRIUS* AND *PROCYON*,

MADE WITH THE 40-INCH REFRACTOR,

By E. E. BARNARD.

Sirius.

Sirius is favorably placed for observation during the worst season for observation here, and as this star is a strong atmospheric test, it has been impossible to get a good series of measures until the latter part of the past winter.

A few scattering isolated measures were obtained with difficulty in the endeavor to get a complete set of two or more nights closely following each other. These are perhaps of little value, but it may be well to print them with the above caution.

			°	″	
1	1899.903	Nov. 25	148.88	4.60	Good
2	1901.885	Nov. 19	133.13	5.34	
3	1902.153	Feb. 25	125.71	5.85	Believed to be good
4	1902.917	Dec. 1	121.90	6.06	
5	1903.054	Jan. 20	123.26	6.18	Angle uncertain

Nos. 1, 3 and 4 were made with the star in the center of field, the others by occulting the bright star by the edge of the field, which makes it specially difficult to determine the angle.

The following measures of the companion were made under fair conditions with the star in the center of the field, and are believed to be very good.

		°	″
1903.147	Feb. 23	119.74	6.28
.150	24	118.53	6.05
.166	Mar. 2	118.95	5.93
1903.154		119.07	6.09

The increasing distance of the companion now makes it an easy object with the great telescope if the seeing is good.

Procyon.

The small companion of *Procyon* is much more difficult than that of *Sirius*, though it is easier now to observe, since its distance has increased. The same remarks apply to this star that are referred to *Sirius*. It requires a very good night to see and measure it.

The following two measures were made with difficulty.

		°	″	
1901.882	Nov. 18	340.53	4.93	Single dist. very diff'lt
.885	19	346.46	5.20	Good
1901.883		343.49	5.06	

One of these angles is bad.

The following measures are good, and were made with the star in the center of the field.

		°	″
1903.147	Feb. 23	350.47	5.17
.150	24	351.11	5.23
.166	Mar. 2	351.50	5.07
1903.154		351.03	5.16

My previous measures, which were printed in *A.J.* 435, 462, 482, are

	°	″
1898.213	326.0	4.83
1899.073	330.6	4.91
1900.055	336.0	5.09

It will be seen that the direct angular motion is about 5° a year. The distance seems not to have materially increased in the past two or three years.

Here are two measures of the old distant companion.

		°	″
1901.882	Nov. 18	345.43	60.76
1902.153	Feb. 25	346.29	61.17

In all the measures a magnifying power of 700 diameters has been used.

Yerkes Observatory, Williams Bay, Wis., 1903 *April* 15.

EPHEMERIS OF FAYE'S COMET,

By F. E. SEAGRAVE.

Greenw. Midnight 1903	α h m s	δ ° ′ ″	log r	log Δ
July 1	4 57 52	+18 40 23	0.223590	0.406606
5	5 9 37	+18 41 47	0.225444	0.405785
9	5 21 17	+18 39 58	0.227522	0.404999
13	5 32 50	+18 34 59	0.229814	0.404236
17	5 44 15	+18 26 56	0.232313	0.403489

This ephemeris is based upon elements by Stromgren, published in a recent number of the *Astr. Nachrichten.* The comet will be in perihelion on June 3, but will be very unfavorably situated for observations, as it will rise only about fifty minutes before the sun. It should be seen towards the middle of July.

CONTENTS.

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AN EXAMPLE IN PERIODIC ORBITS, THE SECOND-ORDER PERTURBATIONS OF *JUPITER* AND *SATURN* INDEPENDENT OF THE ECCENTRICITIES AND OF THE MUTUAL INCLINATION,

By JAMES PARK McCALLIE.

INTRODUCTORY.

A periodic solution is a particular integral in the problem of three bodies. It is possible only under certain restricting conditions which do not exist in nature. Yet such solutions are of great beauty and interest, and often possess real value in assisting us to obtain a more general solution of the differential equations of motion; as, for example, the periodic orbit used as an intermediary by Dr. G. W. HILL in his "*Researches in the Lunar Theory.*"[1]

Analytically a solution is said to be periodic when the coordinates, referred to axes rotating with a uniform angular velocity, may be expressed in series periodic with respect to the time, while geometrically a periodic orbit is one in which a body, referred to the same rotating axes, returns periodically to the same position with reference to the other two bodies.

LAGRANGE in a very elegant manner discovered the first periodic solutions in the problem of three bodies, which however are without much practical value, since to obtain them he assumed the mutual distances as always being in a constant ratio to each other. These are the straight line and equilateral triangle solutions.

The next periodic solutions were obtained by G. W. HILL.[1] By neglecting the lunar inclination and the solar parallax and eccentricity he finds a particular integral of the equations for the moon's motion about the earth under the influence of the disturbing force of the sun. The curve corresponding to this particular integral is closed when referred to rotating axes, and is what is known as the *variational* orbit of the moon. This is used by Dr. HILL as an intermediary instead of the ellipse or modified ellipse of other lunar theorists.

POINCARÉ has shown that there are an infinite number of such solutions that are really distinct. Since POINCARÉ's wholly analytical treatment, the subject of periodic orbits has attracted many astronomers and mathematicians, and a number of memoirs, both analytical and numerical, have been produced. The whole field of periodic orbits is recognized as a fertile one, though by no means easy of entrance.

[1] *The American Journal of Mathematics*, Vol. 1.

Of the memoirs on the subject may be mentioned one by C. V. L. CHARLIER,[1] in which he obtains analytically some of the results found by DARWIN in his extensive numerical work on periodic orbits.[2] In the majority of memoirs one mass is assumed infinitesimal, but periodic orbits exist, whether the mass be infinitesimal or not. It is in the case where none of the masses are infinitesimal that I have selected the following numerical example in periodic orbits. The case is purely an ideal one, but it was in the hope that the results might be of some interest to astronomers that the work was undertaken.

The suggestion of the problem is due to Dr. G. W. HILL, and I desire to express my great indebtedness to him, and also my appreciation to Prof. ORMOND STONE for his encouragement and helpful suggestions, and to Mr. T. McN. SIMPSON, Jr., for checking some of the numerical work.

Example. If two masses, small relatively to a third mass, revolve around the latter in coplanar orbits, having no proper eccentricities, they will have symmetrical conjunctions and oppositions, *i.e.*, their conjunctions and oppositions will be symmetrically placed with regard to their mutually perturbed orbits, which will cut the line of syzygies perpendicularly. Let us take the time of such a symmetrical conjunction as the origin of time, and the longitude of this conjunction as the origin of longitudes. The differential equations of the two bodies will then have particular integrals, or periodic solutions, as is shown by HILL and POINCARÉ. Assume the masses of the two planets to be, for the inner, the mass of *Jupiter*, and for the outer, the mass of *Saturn*, with periods also respec-

[1] *Meddelanden fran Lunds Astronomiska Observatorium*, No. 18.
[2] *Acta Mathematica*, Vol. 21.

tively equal to those of *Jupiter* and *Saturn*, while the mass of the largest body is the mass of the sun. The problem in hand is to find the expressions for the coordinates of the two small bodies as far as the terms proportional to the squares and products of the masses. These terms have been found before, as for instance in HILL's "*New Theory of Jupiter and Saturn*," but they are there mixed up with terms involving the eccentricities, etc., and it is the present purpose to determine them entirely separate from such influences, and in the light of a periodic solution. It may be of some interest to know just how large these terms of the second order are.

Coordinates. I shall refer to the three bodies in question as the *Sun*, *Jupiter*, and *Saturn*. That the latter two may have the same expression for their perturbative functions it is necessary only to use symmetrical differential-equations as explained in TISSERAND, Vol. I, Chap. IV. *Jupiter* is referred to the center of the *Sun* as origin, while *Saturn* is referred to the center of mass of *Jupiter* and the *Sun*. Allowing the subscripts 0, 1, 2 to refer to the *Sun*, *Jupiter*, and *Saturn* respectively, and denoting the masses severally by $m_i\,(i=0,1,2)$, we have for the heliocentric coordinates of *Jupiter* and *Saturn*

$$\xi_1 = x_1 \quad , \quad \xi_2 = x_2 + \kappa_1 x_1$$
$$\eta_1 = y_1 \quad , \quad \eta_2 = y_2 + \kappa_1 y_1$$

where

$$\kappa_i = \frac{m_i}{\mu_i} \quad , \quad \mu_i = m_0 + m_1 + \ldots + m_i$$

If $\mathrm{r}_i,\, v_i\,(i=1,2)$ represent the radii vectores and true longitudes of *Jupiter* and *Saturn* respectively, then

$$x_1 = \mathrm{r}_1 \cos v_1 \ , \ y_1 = \mathrm{r}_1 \sin v_1 \ ; \ x_2 = \mathrm{r}_2 \cos v_2 \ , \ y_2 = \mathrm{r}_2 \sin v_2$$

PERTURBATIVE FUNCTION.

The potential of the system is

$$U = \frac{m_0 m_1}{\Delta_{0,1}} + \frac{m_0 m_2}{\Delta_{0,2}} + \frac{m_1 m_2}{\Delta_{1,2}}$$
$$= \frac{m_0 m_1}{\mathrm{r}_1} + \frac{m_0 m_2}{\mathrm{r}_2} + m_1 m_2 F$$

where

$$m_1 m_2 F = m_0 m_2 \left[\frac{1}{\Delta_{0,2}} - \frac{1}{\mathrm{r}_2}\right] + \frac{m_1 m_2}{\Delta_{1,2}}$$
$$= m_0 m_2 \left[\left\{\mathrm{r}_2^2 + \kappa_1^2 \mathrm{r}_1^2 + 2\kappa_1 \mathrm{r}_1 \mathrm{r}_2 \cos(v_2 - v_1)\right\}^{-\frac{1}{2}} - \frac{1}{\mathrm{r}_2}\right] + \frac{m_1 m_2}{\Delta_{1,2}}$$

If we put

$$\frac{\mu_1}{m_0}\mathrm{r}_2 = r_2 \ , \ \mathrm{r}_1 = r_1 \quad , \quad v_2 - v_1 = \theta \ , \ \frac{\mu_1}{\mu_0} = \mathrm{m}$$

F has the approximate expression

$$F = \frac{\mathrm{m}}{r_2}\left[\left\{1 - 2\frac{r_1}{r_2}\cos\theta + \left(\frac{r_1}{r_2}\right)^2\right\}^{-\frac{1}{2}} - \frac{r_1}{r_2}\cos\theta + \tfrac{1}{4}\frac{m_1}{m_0}\left(\frac{r_1}{r_2}\right)^2\left\{1 + 3\cos 2\theta\right\}\right]$$
$$= F_0 + F_1$$

in which F_1 is the part having as a factor the small mass m_1. Since the planets have no proper eccentricities, and lie in the same plane, the perturbations will depend on the single argument $v_2 - v_1$, or the elongation. Hence it is sufficient to put in the function F, as a *first approximation*,

$$r_1 = a_1 \ , \ r_2 = a_2 \ , \ v_2 = l_2 = n_2 t, \ v_1 = l_1 = n_1 t \ , \ \theta_0 = (n_2 - n_1) t$$

Then F may be written separately in its two parts,

$$\left.\begin{aligned} F_0 &= \mathrm{m}\left[\tfrac{1}{2}\sum_{-\infty}^{+\infty}{}_i A^i \cos i\,\theta_0 - \frac{a_1}{a_2^2}\cos\theta_0 + \tfrac{1}{2}A^0\right] \\ F_1 &= \tfrac{1}{4}\,\mathrm{m}\, m_1 \frac{a_1^2}{a_2^3}\left[1 + 3\cos 2\theta_0\right] \end{aligned}\right\} \qquad (1)$$

where

$$\tfrac{1}{2}\sum_{-\infty}^{+\infty}{}_i A^i \cos i\,\theta_0 = \frac{1}{a_2}\left[1 - 2\alpha\cos\theta_0 + \alpha^2\right]^{-\frac{1}{2}}$$

In F_0 the value of A^i for $i = 0$ has been taken from under the sign Σ, and so hereafter.

DIFFERENTIAL EQUATIONS OF MOTION.

1. *For Jupiter.* The equations for *Jupiter* in rectangular coordinates are

$$\mu_0 \kappa_1 \frac{d^2 x_1}{dt^2} = \frac{\partial U}{\partial x_1} \quad , \quad \mu_0 \kappa_1 \frac{d^2 y_1}{dt^2} = \frac{\partial U}{\partial y_1}$$

These equations expressed in the polar coordinates r_1, v_1 after the manner of DEPONTECOULANT's equations in the "*Lunar Theory*,"[1] are

$$\left.\begin{aligned} \tfrac{1}{2}\frac{d^2}{dt^2}(r_1^2) - \frac{\mu_1}{r_1} + \frac{\mu_1}{a_1} &= \mathrm{m}_2\left[r_1\frac{\partial F}{\partial r_1} + 2\int d'F + 2\mathrm{m} g_1\right] \\ \frac{dv_1}{dt} &= \frac{1}{r_1^2}\left[h_1 + \mathrm{m}_2\int\frac{\partial F}{\partial v_1}\,dt\right] \end{aligned}\right\} \qquad (2)$$

In these equations the new expressions introduced have the following significance:

$$d'F = \left\{\frac{\partial F}{\partial r_1}\frac{dr_1}{dt} + \frac{\partial F}{\partial v_1}\frac{dv_1}{dt}\right\}dt \quad , \quad \mathrm{m}_2 = \mathrm{m}\, m_2$$

$$-\frac{\mu_1}{2\mathrm{m}_2 a_1} + \mathrm{m}\, g_1 = \text{constant of integration attached to } \int d'F$$
$$h_1 = \text{constant of integration.}$$

2. *For Saturn.* The equations for *Saturn* formed in the same way are (3)

$$\left.\begin{aligned} \tfrac{1}{2}\frac{d^2}{dt^2}(r_2^2) - \frac{\mu_2 \mathrm{m}^2}{r_2} + \frac{\mu_2 \mathrm{m}^2}{a_2} &= \frac{\mu_2 \mathrm{m}\, m_1}{m_0}\left[r_2\frac{\partial F}{\partial r_2} + 2\int d''F + 2\mathrm{m} g_2\right] \\ \frac{dv_2}{dt} &= \frac{1}{r_2^2}\left(h_2 + \frac{\mu_2 \mathrm{m}\, m_1}{m_0}\int\frac{\partial F}{\partial v_2}\,dt\right) \end{aligned}\right\}$$

where the corresponding terms have an exactly similar meaning to those employed in *Jupiter's* equations.

3. *Equations Connecting Constants.* The above equations for *Jupiter*, or *Saturn*, are of the second and first

[1] See BROWN's *Lunar Theory*, pp. 16, 17.

order respectively, and are sufficient to determine three arbitrary constants, besides h_i and a_i ($i = 1$ or 2). But since the orbits have no inclinations or nodes there are only four constants, e_i, π_i, n_i, ϵ_i ($i = 1$ or 2), to be determined for each body, and therefore we must have another equation connecting the constants. Three of the constants are immediately determined by the special conditions of the problem. For since the orbits of the planets have no eccentricity other than that caused by their mutual perturbations, and hence their perihelia are indeterminate, we have

$$e_i \sin \pi_i = 0$$
$$e_i \cos \pi_i = 0$$

By reason of the way in which we have chosen our origins of longitudes and of time, ϵ_1 and ϵ_2 are zero. Hence the only two independent constants are n_1 and n_2. The equations above referred to are, for *Jupiter* and *Saturn*, respectively,[1]

$$(4)\quad \left\{ \begin{aligned} &\frac{1}{r_1}\frac{d^2 r_1}{dt^2} - \left(\frac{dv_1}{dt}\right)^2 + \frac{\mu_1}{r_1^3} = \frac{\mathrm{m}_2}{r_1}\frac{\partial F}{\partial r_1} \\ &\frac{1}{r_2}\frac{d^2 r_2}{dt^2} - \left(\frac{dv_2}{dt}\right)^2 + \frac{\mu_2 \mathrm{m}^2}{r_2^3} = \frac{\mu_2 \mathrm{m} m_1}{m_0}\frac{1}{r_2}\frac{\partial F}{\partial r_2} \end{aligned} \right.$$

Units Employed. Let us take m_0, the mass of the *Sun*, as our unit of mass, and let the mean distance of the earth from the *Sun* be the unit of length. Then that k, the *Gaussian Constant*, may also be unity, the unit of time must be 58.13245 mean solar days. Hence we may put

$$\mu_1 = 1 + m_1 = n_1^2 a_1^3 \quad , \quad \mu_2 \mathrm{m}^2 = (1 + m_1 + m_2)(1 + m_1)^2 = n_2^2 a_2^3$$

$$\alpha = \frac{a_1}{a_2} = [(1 + m_1 + m_2)(1 + m_1)]^{-\frac{1}{3}} \left(\frac{n_2}{n_1}\right)^{\frac{2}{3}}$$

The values of m_1, m_2, n_1, n_2 are taken from p. 558 of Hill's "*New Theory of Jupiter and Saturn*," and are

$$m_1 = \tfrac{1}{1047.879} \quad , \quad n_1 = 109256''.62552$$
$$m_2 = \tfrac{1}{3501.6} \quad , \quad n_2 = 43996''.21506$$

The above mean motions are for a sidereal year. Taking as our values for the mass and mean motion (in a sidereal year) of the earth,

$$m' = \tfrac{1}{327000} \quad , \quad n' = 1295977''.41516$$

from the equation $a' = (1 + m')^{\frac{1}{3}} n'^{-\frac{2}{3}}$ we obtain the numerical value of a', which, used as the unit of distance, gives

$$\log a_1 = 0.716237409 \qquad \log \nu = \log \frac{n_2}{n_1} = 9.604967534$$
$$\log a_2 = 0.979909852 \qquad \log \alpha = 9.736327557$$

Integration of Equations of Motion. In order to solve equations (3) and (4) it seems best to put

$$r_1^2 = a_1^2(1 + u_1 + \delta u_1) \quad , \quad \frac{dv_1}{dt} = n_1 + z_1 + \delta z_1$$

$$r_2^2 = a_2^2(1 + u_2 + \delta u_2) \quad , \quad \frac{dv_2}{dt} = n_2 + z_2 + \delta z_2$$

[1] Brown's *Lunar Theory*, pp. 16, 17.

where u_1, z_1, u_2, z_2 represent perturbations of the first order with respect to the masses and δu_1, δz_1, δu_2, δz_2 are of the second order.

1. *First Order Terms for Jupiter.* The radius-vector equation for *Jupiter* becomes, to terms of the first order,

$$\frac{d^2 u_1}{dt^2} + n_1^2 u_1 = 2 n_1^2 a_1 \mathrm{m}_2 \left[a_1 \frac{\partial F_0}{\partial a_1} + 2 n_1 \int \frac{\partial F_0}{\partial l_1} dt + 2 \mathrm{m}\, g_1 \right] \qquad (5)$$

This linear differential equation of the second order may be solved by indeterminate coefficients. Since its right member is a cosine function of the elongation, θ_0, only, we put

$$u_1 = 2 \sum_{-\infty}^{+\infty} \mathrm{a}_i \cos i\, \theta_0$$

Substituting this value of u_1 in the above equation, and equating coefficients of the same argument on either side we have

$$\mathrm{a}_0 = \mathrm{m}_2 \left[\tfrac{1}{2} a_1^2 \frac{\partial A^0}{\partial a_1} + 2 a_1 g_1 \right] .$$

$$\mathrm{a}_{-1} = \mathrm{a}_1 = \frac{\mathrm{m}_2}{\nu(1-\nu)(2-\nu)} \left[\frac{1-\nu}{2} a_1^2 \frac{\partial A^1}{\partial a_1} + a_1 A^1 - \frac{3-\nu}{2} \alpha^2 \right]$$

$$\mathrm{a}_i = \frac{\mathrm{m}_2}{(1-\nu)\{1 - i^2(1-\nu)^2\}} \left[\frac{1-\nu}{2} a_1^2 \frac{\partial A^i}{\partial a_1} + a_1 A^i \right] (i = \pm 2, \ldots)$$

To the same order the longitude equation is

$$n_1 + z_1 - \frac{h'_1}{a_1^2} = \left[h'_1 + \delta_1 h_1 + \mathrm{m}_2 \int \frac{\partial F_0}{\partial l_1} dt \right] \frac{1 - u_1}{a_1^2} - \frac{h'_1}{n_1^2}$$

where h_1 has been replaced by $h'_1 + \delta_1 h_1$. In the circular orbit $h_1 = h'_1 = n_1 a_1^2$. Hence $\delta_1 h_1$ is a small constant of the order of the masses. Then

$$z_1 = \frac{\delta_1 h_1}{a_1^2} - n_1 u_1 + \frac{\mathrm{m}_2}{\mathrm{m}} n_1^2 a_1 \int \frac{\partial F_0}{\partial l_1} dt \qquad (6)$$

Putting

$$\delta_1 v_1 = \int z_1\, dt = \sum_{-\infty}^{+\infty} \alpha_i \sin i\, \theta_0$$

we find

$$-\alpha_{-1} = \alpha_1 = \frac{\mathrm{m}_2}{2\nu(1-\nu)^2(2-\nu)} \left[2(1-\nu) a_1^2 \frac{\partial A^1}{\partial a_1} + (\nu^2 - 2\nu + 4) a_1 A^1 - (\nu^2 - 4\nu + 6)\alpha^2 \right]$$

$$\alpha_i = \frac{\mathrm{m}_2}{2(1-\nu)\, i\, \{1 - i^2(1-\nu)^2\}} \left[2(1-\nu) a_1^2 \frac{\partial A^i}{\partial a_1} + \{3 + i^2(1-\nu)^2\} a_1 A^i \right] (i = \pm 2, \ldots)$$

The constant term of z_1 is $\dfrac{\delta_1 h_1}{a_1^2} - 2 n_1 \mathrm{a}_0$; but we shall define n_1 as the mean motion of *Jupiter* in disturbed as well as in undisturbed orbit, and it will be obtained directly from observation. Hence

$$\frac{\delta_1 h_1}{a_1^2} - 2 n_1 \mathrm{a}_0 = 0$$

Since the arbitraries e_1 and π_1 of the general solution of the problem are zero in this case, and a_1 is not independent of n_1, all the arbitrary constants have now been fixed, for ϵ_1 is zero by the conditions laid down. Hence g_1 is not independent of the other arbitraries, and we find it by means of the first of equations (4). This equation will also enable us to verify the preceding work, inasmuch as the coefficients of $\cos i\,\theta_0$ on each side of the equation should be identical. To terms of the first order the equation is

$$\frac{d^2u_1}{dt^2} - 4n_1 z_1 - 3n_1^2 u_1 = 2\,\frac{\mathrm{m}_2}{\mathrm{m}}\,n_1^2 a_1^2 \frac{\partial F_0}{\partial a_1}$$

Substituting in this the above values of u_1 and z_1 we find

$$g_1 = -\tfrac{1}{3}\,a_1 \frac{\partial A^0}{\partial a_1}\ , \ \text{or}\ \ \mathrm{a}_0 = -\tfrac{1}{6}\,\mathrm{m}_2\,a_1^2\,\frac{\partial A^0}{\partial a_1}$$

2. *First Order Terms for Saturn.* The radius-vector equation for *Saturn* is

$$(7)\quad \frac{d^2u_2}{dt^2} + n_2^2 u_2 = 2\frac{m_1}{\mathrm{m}}\,n_2^2 a_2\left[a_2\frac{\partial F_0}{\partial a_2} + 2\int\frac{\partial F_0}{\partial l_2}\,dt + 2\mathrm{m}\,g_2\right]$$

Let
$$u_2 = 2\sum_{-\infty}^{+\infty}\mathrm{b}_i\cos i\,\theta_0$$

In forming $a_2\partial F_0/\partial a_2$ we make use of the relation

$$a_2\frac{\partial F_0}{\partial a_2} + a_1\frac{\partial F_0}{\partial a_1} = -F_0$$

Then

$$\mathrm{b}_0 = \frac{m_1}{2}\left[4a_2 g_2 - a_1 a_2\frac{\partial A^0}{\partial a_1} - a_2 A^0\right]$$

$$\mathrm{b}_{-1} = \mathrm{b}_1 = \frac{m_1\nu^2}{(1-\nu)(1-2\nu)}\left[\frac{1-\nu}{2}\,a_1 a_2\frac{\partial A^1}{\partial a_1} + \frac{1+\nu}{2}\,a_2 A^1 - \alpha\right]$$

$$\mathrm{b}_i = \frac{m_1\nu^2}{(1-\nu)\{i^2(1-\nu)^2-\nu^2\}}\left[\frac{1-\nu}{2}\,a_1 a_2\frac{\partial A^i}{\partial a_1} + \frac{1+\nu}{2}\,a_2 A^i\right]$$

The differential equation for longitude of *Saturn* is, to terms of first order,

$$(8)\qquad z_2 = \frac{\delta_1 h_2}{a_2^2} - n_2 u_2 + \frac{m_1}{\mathrm{m}}\,n_2^2 a_2\int\frac{\partial F_0}{\partial l_2}\,dt$$

Putting

$$\delta_1 v_2 = \int z_2\,dt = \sum_{-\infty}^{+\infty}\beta_i\sin i\,\theta_0$$

we find

$$-\beta_{-1} = \beta_1 = \frac{m_1\nu^2}{2(1-\nu)^2(1-2\nu)}\left[2\nu(1-\nu)a_1 a_2\frac{\partial A^1}{\partial a_1} + (1+2\nu^2)\,a_2 A^1 - (1+2\nu)\alpha\right]$$

$$\beta_i = \frac{m_1\nu^2}{2(1-\nu)^2 i\{i^2(1-\nu)^2-\nu^2\}}\left[2\nu(1-\nu)\,a_1 a_2\frac{\partial A^i}{\partial a_1} + \{2\nu+\nu^2+i^2(1-\nu)^2\}a_2 A^i\right]$$

and since the constant term in $\delta_1 v_2$ is zero

$$\frac{\delta_1 h_2}{a_2^2} - 2n_2\mathrm{b}_0 = 0$$

The equation determining the constant term in u_2 is to terms of first order,

$$\frac{d^2u_2}{dt^2} - 4n_2 z_2 - 3n_2^2 u_2 = 2\,\frac{m_1}{\mathrm{m}}\,n_2^2 a_2^2\,\frac{\partial F_0}{\partial a_2}$$

This gives

$$\mathrm{b}_0 = \tfrac{1}{2}m_1 a_2 g_2 = \tfrac{1}{6}\,m_1\left[a_1 a_2\frac{\partial A^0}{\partial a_1} + a_2 A^0\right]$$

Differential Equations Including Second Order Terms.

Having now solved the differential equations as far as the terms proportional to the masses we are prepared to push our approximation still further and include all terms proportional to the squares and products of the masses. It is well known that the form of the solution remains unchanged in all the successive approximations of including the squares, cubes, and higher powers of the masses, and hence our differential equations preserve the same form, and are solved precisely in the same way as before.

1. *For Jupiter.* — *a*) *Radius Vector Equation.* When we extend the radius vector equation to terms of the second order, and omit all terms of the first order, we get

$$\frac{d^2}{dt^2}\delta u_1 + n_1^2\delta u_1 = \tfrac{3}{2}\,n_1^2 u_1^2 + 2\,\frac{\mathrm{m}_2}{\mathrm{m}}\,n_1^2 a_1\left[\delta\left(r_1\frac{\partial F}{\partial r_1}\right) + 2\delta\int d'F + a_1\frac{\partial F_1}{\partial a_1} + 2\int d'F_1 + 2\mathrm{m}\,\delta g_1\right]$$

In this equation F_1 has the value given above, and $\mathrm{m}\,\delta g_1$ is the constant of integration attached to $\delta\int d'F$, and is of the second order. We shall proceed to express fully the right member of this equation.

Since

$$r_1 = a_1(1+\tfrac{1}{2}u_1+\tfrac{1}{2}\delta u_1 - \tfrac{1}{8}u_1^2)$$

we have

$$\frac{\delta_1 r_1}{a_1} = \tfrac{1}{2}u_1\ \ ;\ \ \text{also}\ \ \frac{\delta_1 r_2}{a_2} = \tfrac{1}{2}\,u_2$$

Also since F_0 is a function of $l_2 - l_1$ we have

$$\frac{\partial F_0}{\partial l_2} = -\frac{\partial F_0}{\partial l_1}$$

With these relations and that given above, with reference to $a_2\partial F_0/\partial a_2$ we may easily express $\delta(r_1\partial F/\partial r_1)$. We also have

$$\delta\int d'F = \int\delta\left[\frac{\partial F}{\partial r_1}\frac{dr_1}{dt} + \frac{\partial F}{\partial v_1}\frac{dv_1}{dt}\right]dt$$

in which

$$\delta\left(\frac{dr_1}{dt}\right) = \frac{d}{dt}\,(\delta_1 r_1) = \frac{a_1}{2}\,\frac{du_1}{dt}$$

$$\delta\left(\frac{dv_1}{dt}\right) = \frac{d}{dt}\,(\delta_1 v_1) = z_1$$

Then the above differential equation in its expanded form is

(9)

$$\frac{d^2}{dt^2}(\delta u_1)+n_1^2\delta u_1 = n_1^2\Bigg[\tfrac{3}{4}u_1^2+2\frac{\mathrm{m}_2}{\mathrm{m}}a_1\Bigg\{\left(a_1\frac{\partial F_0}{\partial a_1}+a_1^2\frac{\partial^2 F_0}{\partial a_1^2}\right)\frac{u_1}{2}$$

$$-\left(2a_1\frac{\partial F_0}{\partial a_1}+a_1^2\frac{\partial^2 F_0}{\partial a_1^2}\right)\frac{u_2}{2}+a_1\frac{\partial^2 F_0}{\partial l_1\partial a_1}(\delta_1 v_1-\delta_1 v_2)+a_1\frac{\partial F_1}{\partial a_1}$$

$$+2\int d'F_1+2\mathrm{m}\,\delta g_1$$

$$+2n_1\int\Bigg[\frac{a_1\partial^2 F_0}{\partial a_1\partial l_1}\left(\frac{u_1}{2}-\frac{u_2}{2}\right)-\frac{\partial F_0}{\partial l_1}\frac{u_2}{2}+\frac{\partial^2 F_0}{\partial l_1^2}(\delta_1 v_1-\delta_1 v_2)$$

$$+\frac{1}{n_1}\left(\frac{a_1}{2}\frac{\partial F_0}{\partial a_1}\frac{du_1}{dt}+\frac{\partial F_0}{\partial l_1}z_1\right)\Bigg]dt\Bigg\}\Bigg]$$

We see immediately that the right member is composed of products of series, either cosine by cosine, sine by sine, or, underneath the integral sign, cosine by sine. In every case we get, after multiplication, and integration of the last mentioned products, a cosine series. If now we attach the factor $2\mathrm{m}_2 a_1/\mathrm{m}$ above to F_0, every coefficient in each factor is of the order of the masses. We shall designate the coefficients of cosine series by Roman letters, of sine series by Greek letters. In each series the subscript i has every integral value from $-\infty$ to $+\infty$ including zero. For cosine series we may put $\mathrm{a}_i = \mathrm{a}_{-i}$, for sine series $\alpha_i = -\alpha_{-i}$. Hence we may write

$$\Sigma_i \mathrm{a}_i \cos i\,\theta_0 \times \Sigma_j \mathrm{b}_j \cos j\,\theta_0 = \Sigma_i\Sigma_j \mathrm{a}_i\mathrm{b}_j\cos(i+j)\,\theta_0$$
$$\Sigma_i \mathrm{a}_i \cos i\,\theta_0 \times \Sigma_j \alpha_j \sin j\,\theta_0 = \Sigma_i\Sigma_j \mathrm{a}_i\alpha_j\sin(i+j)\,\theta_0$$
$$\Sigma_i \alpha_i \sin i\,\theta_0 \times \Sigma_j \beta_j \sin j\,\theta_0 = -\Sigma_i\Sigma_j \alpha_i\beta_j\cos(i+j)\,\theta_0$$

whence the equation for δu_1 becomes

$$\frac{d^2}{dt^2}(\delta u_1)+n_1^2\,\delta u_1 = n_1^2\left[\Sigma_i\Sigma_j S_{i+j}\cos(i+j)\,\theta_0+2\sigma\mathrm{m}\,\delta g_1\right]$$

the solution of which gives

$$\delta u_1 = \Sigma_i\Sigma_j\frac{S_{i+j}}{1-(i+j)^2(1-\nu)^2}\cos(i+j)\,\theta_0+2\sigma\,\mathrm{m}\,\delta g_1$$

where $\sigma = 2\mathrm{m}_2 a_1/\mathrm{m}$ and

$$S_{i+j} = 3\,\mathrm{a}_i\mathrm{a}_j+\mathrm{e}_i\mathrm{a}_j-\mathrm{f}_i\mathrm{b}_j-\epsilon_i\gamma_j+\mathrm{k}_{i+j}+\mathrm{l}_{i+j}$$
$$+\frac{2}{(i+j)(1-\nu)}\left[\epsilon_i\mathrm{c}_j-\zeta_i\mathrm{b}_j+\mathrm{g}_i\gamma_j+\mathrm{h}_i\delta_j+\zeta_i\mathrm{d}_j\right]$$

These letters express in order the coefficients of the various factors just as they occur in the right-hand member of the expanded equation (9) given above.

b) *Longitude Equation.* To terms of the second order this equation becomes, when we put $h_1 = h'_1+\delta_1 h_1+\delta_2 h_1$ and omit terms of the first order,

(10)

$$\delta z_1 = -u_1 z_1 - n_1\delta u_1+\frac{\delta_2 h_1}{a_1^2}+\tfrac{1}{2}\sigma n_1\int\Bigg\{a_1\frac{\partial^2 F_0}{\partial l_1\partial a_1}\left(\frac{u_1}{2}-\frac{u_2}{2}\right)$$

$$-\frac{\partial F_0}{\partial l_1}\frac{u_2}{2}+\frac{\partial^2 F_0}{\partial l_1^2}(\delta_1 v_1-\delta_1 v_2)+\frac{\partial F_1}{\partial l_1}\Bigg\}dt$$

$$= n_1\Sigma_i\Sigma_j P_{i+j}\cos(i+j)\,\theta_0$$

where

$$P_{i+j} = -2\mathrm{a}_i\mathrm{d}_j-\frac{1}{1-(i+j)^2(1-\nu)^2}S_{i+j}$$

$$+\frac{1}{2(i+j)(1-\nu)}\left[\epsilon_i\mathrm{c}_j-\zeta_i\mathrm{b}_j+\mathrm{g}_i\gamma_j\right]+\tfrac{1}{2}\mathrm{l}_{i+j}-2\sigma\mathrm{m}\delta g_1+\frac{\delta_2 h_1}{n_1 a_1^2}$$

where the whole constant part is included in P_0, which for reasons given above must be equated to zero.

Then

$$\delta_2 v_1 = \int\delta z_1\,dt = \Sigma_i\Sigma_j\frac{P_{i+j}}{(i+j)(\nu-1)}\sin(i+j)\,\theta_0$$

c) *Equation Determining Constant Part of* δu_1. The first of equations (4) extended to terms of the second order is sufficient to determine the constant δg_1 which occurs in δu_1, and at the same time to verify our equations for δu and δz_1. For on summing the coefficients of like cosines we shall find that they vanish identically, and only a constant term is left. If we let

$$\dot u_1 = \frac{du_1}{dt}\quad,\quad \ddot u_1 = \frac{d^2 u_1}{dt^2}$$

this equation is (11)

$$\frac{\delta u_1}{n_1^2}-3\delta u_1-4\frac{\delta z_1}{n_1}-\frac{u_1\ddot u_1}{n_1^2}-\tfrac{1}{2}\frac{\dot u_1^2}{n_1^2}-2\frac{z_1^2}{n_1^2}+\tfrac{15}{4}u_1^2$$

$$= \tfrac{1}{2}\sigma\left[-a_1\frac{\partial F_0}{\partial a_1}u_1+2a_1\delta\left(\frac{\partial F}{\partial r_1}\right)+2a_1\frac{\partial F_1}{\partial a_1}\right]$$

Substituting in this equation the expressions for δu_1, $\delta\ddot u_1$, δz_1, and making use of equations (5) and (6) we arrive at the equation

$$\tfrac{3}{2}u_1^2-\tfrac{1}{2}\frac{\dot u_1^2}{n_1^2}-2\frac{z_1^2}{n_1^2}+2\sigma\mathrm{m}\,\delta g_1-4\frac{\delta_2 h_1}{n_1 a_1^2}$$

$$+2\sigma\int\left\{a_1\frac{\partial F_0}{\partial a_1}\frac{\dot u_1}{2}+\frac{\partial F_0}{\partial l_1}z_1\right\}dt = 0$$

We shall find a different expression for the last term. Equation (6) is

$$\frac{z_1}{n_1}+u_1 = \frac{\delta_1 h_1}{n_1 a_1^2}+\frac{\sigma}{2}n_1\int\frac{\partial F_0}{\partial l_1}dt$$

By means of this relation and its derivative we find

$$2\sigma\int z_1\frac{\partial F_0}{\partial l_1}dt = 2\frac{z_1^2}{n_1^2}+4\frac{\delta_1 h_1}{n_1 a_1^2}u_1-2u_1^2$$

$$+\int 2\sigma n_1\dot u_1\left\{\int\frac{\partial F_0}{\partial l_1}dt\right\}dt$$

By adding

$$2\sigma\int a_1\frac{\partial F_0}{\partial a_1}\frac{\dot u_1}{2}dt$$

to each member of this equation, the integral in the right member becomes

$$\sigma\int\left\{a_1\frac{\partial F_0}{\partial a_1}+2n_1\int\frac{\partial F_0}{\partial l_1}dt\right\}\dot u_1\,dt$$

which by means of equation (5) may be completely integrated. Hence we obtain

$$2\sigma\int\left\{a_1\frac{\partial F_0}{\partial a_1}\frac{\dot u_1}{2}+\frac{\partial F_0}{\partial l_1}z_1\right\}dt = \left[2\frac{z_1^2}{n_1^2}-\tfrac{3}{2}u_1^2+\tfrac{1}{2}\frac{\dot u_1^2}{n_1^2}\right]_0$$

where $[\ ]_{-0}$ means that the constant term is absent. The equation under consideration then gives for the constant in δu_1

$$\sigma \mathrm{m} \delta g_1 = 2\frac{\delta_2 h_1}{n_1 a_1^2} + \left[\frac{z_1^2}{n_1^2} - \tfrac{3}{4} u_1^2 + \tfrac{1}{4}\frac{\dot{u}_1^2}{n_1^2}\right]_0$$

Here $[\ ]_0$ means that only the constant part is present. Thus it is seen that all periodic terms identically vanish, and the equations for δu_1 and δz_1 are verified. We shall use this same equation to verify the numerical work.

Since the constant term in δz_1 is zero we have

$$\frac{\delta_2 h_1}{n_1 a_1^2} = 2\sigma \mathrm{m}\, \delta g_1 + 2\,[\mathrm{a}_i \mathrm{d}_j]_0 + S_0$$

Hence

$$-3\sigma \mathrm{m}\, \delta g_1 = 4\,[\mathrm{a}_i \mathrm{d}_j]_0 + 2\, S_0 + \left[\frac{z_1^2}{n_1^2} - \tfrac{3}{4} u_1^2 + \tfrac{1}{4}\frac{\dot{u}_1^2}{n_1^2}\right]_0$$

2. *For Saturn.* — *a*) *Radius Vector Equation.* The equations for *Saturn*, being formed in a manner exactly similar to that pursued in forming *Jupiter's* equations, may simply be written down. The first is

$$(12)\qquad \frac{d^2}{dt^2}(\delta u_2) + n_2^2\, \delta u_2$$

$$= n_2^2\Bigg[\tfrac{3}{2} u_2^2 - 2\frac{m_1}{\mathrm{m}} a_2 \Bigg\{ \left(2a_1 \frac{\partial F_0}{\partial a_1} + a_1^2 \frac{\partial^2 F_0}{\partial a_1^2}\right)\left(\frac{u_1}{2} - \frac{u_2}{2}\right)$$

$$- \left(F_0 + a_1 \frac{\partial F_0}{\partial a_1}\right)\frac{u_2}{2} + \left(\frac{\partial F_0}{\partial l_1} + a_1 \frac{\partial^2 F_0}{\partial l_1 \partial a_1}\right)(\delta_1 v_1 - \delta_1 v_2)$$

$$- a_2 \frac{\partial F_1}{\partial a_2} - 2\int d'' F_1 - 2\mathrm{m}\delta g_2$$

$$+ 2n_2 \int\Bigg[a_1 \frac{\partial^2 F_0}{\partial l_1 \partial a_1}\left(\frac{u_1}{2} - \frac{u_2}{2}\right) - \frac{\partial F_0}{\partial l_1}\frac{u_2}{2} + \frac{\partial^2 F_0}{\partial l_1^2}(\delta_1 v_1 - \delta_1 v_2)$$

$$+ \left(F_0 + a_1 \frac{\partial F_0}{\partial a_1}\right)\frac{\dot{u}_2}{2n_2} + \frac{\partial F_0}{\partial l_1}\frac{z_2}{n_2}\Bigg] dt \Bigg\}\Bigg]$$

We see that, as in *Jupiter's* radius vector equation, the right member is composed of the products of series, all of which result in cosine series. Many of the individual series are the same as those entering *Jupiter's* equation, except for the constant factor $2m_1 a_2/\mathrm{m}$. Denoting this constant by ω, we can put

$$\omega = \frac{\omega}{\sigma}\cdot \sigma$$

and we can then use the same letters as before to denote the same coefficients here. New letters will be used where we have new coefficients, and arranging them in exactly the order in which they occur above, we may write the equation for δu_2,

$$\frac{d^2}{dt^2}(\delta u_2) + n_2^2 \delta u_2 = n_2^2[\Sigma_i \Sigma_j\, R_{i+j} \cos(i+j)\,\theta_0 + 2\omega \mathrm{m}\, \delta g_2]$$

the solution of which is

$$\delta u_2 = \Sigma_i \Sigma_j \frac{\nu^2}{\nu^2 - (i+j)^2(1-\nu)^2} R_{i+j} \cos(i+j)\,\theta_0 + 2\omega \mathrm{m}\, \delta g_2$$

where

$$R_{i+j} = 3\mathrm{b}_i \mathrm{b}_j - \frac{\omega}{\sigma}\Big[\mathrm{f}_i \mathrm{c}_j - \mathrm{q}_i \mathrm{b}_j - \theta_i \gamma_j - \mathrm{m}_{i+j} - 2\mathrm{n}_{i+j}$$

$$+ \frac{2\nu}{(i+j)(1-\nu)}\Big\{\epsilon_i \mathrm{c}_j - \zeta_i \mathrm{b}_j + \mathrm{g}_i \gamma_j + \mathrm{q}_i \eta_j + \zeta_i \mathrm{p}_j\Big\}\Big]$$

b) *Longitude Equation.* To terms of the second order this is (13)

$$\delta z_2 = -u_2 z_2 - n_2 \delta u_2 + \frac{\delta_2 h_2}{a_2^2}$$

$$+ \tfrac{1}{2}\omega \int d'' F_0 - \tfrac{1}{2} n_2 \omega \int \Big\{ a_1 \frac{\partial^2 F_0}{\partial l_1 \partial a_1}\left(\frac{u_1}{2} - \frac{u_2}{2}\right)$$

$$- \frac{\partial F_0}{\partial l_1}\frac{u_2}{2} + \frac{\partial^2 F_0}{\partial l_1^2}(\delta_1 v_1 - \delta_1 v_2)\Big\} dt$$

$$= n_2\, \Sigma_i \Sigma_j\, K_{i+j} \cos(i+j)\,\theta_0$$

where

$$K_{i+j} = -2\mathrm{b}_i \mathrm{p}_j - \frac{\nu^2}{\nu^2 - (i+j)^2(1-\nu)^2} R_{i+j} - 2\omega \mathrm{m}\, \delta g_2 + \frac{\delta_2 h_2}{n_2 a_2^2}$$

$$+ \tfrac{1}{2}\frac{\omega}{\sigma} n_{i+j} - \tfrac{1}{2}\frac{\omega}{\sigma}\frac{\nu}{(i+j)(1-\nu)}[\epsilon_i \mathrm{c}_j - \zeta_i \mathrm{b}_j + \mathrm{g}_i \gamma_j]$$

Then

$$\delta_2 v_2 = \int \delta z_2 dt = \Sigma_i \Sigma_j \frac{\nu}{(i+j)(\nu-1)} K_{i+j} \sin(i+j)\,\theta_0$$

c) *Equation Determining Constant Part of* δu_2. The second of equations (4), expressed to terms of the second order, is

$$\frac{\delta \ddot{u}_2}{n_2^2} - 3\,\delta u_2 - 4\frac{\delta z_2}{n_2} - \frac{u_2 \ddot{u}_2}{n_2^2} - \tfrac{1}{2}\frac{\dot{u}_2^2}{n_2^2} - 2\frac{z_2^2}{n_2^2} + \tfrac{15}{4} u_2^2 \qquad (14)$$

$$= -\omega\left[A\left(\frac{u_1}{2} - \frac{u_2}{2}\right) - 3B\frac{u_2}{2} + \frac{\partial B}{\partial l_1}(\delta_1 v_1 - \delta_1 v_2) - a_2 \frac{\partial F_1}{\partial a_2}\right]$$

where

$$A = 2a_1 \frac{\partial F_0}{\partial a_1} + a_1^2 \frac{\partial^2 F_0}{\partial a_1^2}\ , \qquad B = F_0 + a_1 \frac{\partial F_0}{\partial a_1}$$

From this equation we get, exactly as in the equation for *Jupiter*,

$$-3\omega \mathrm{m}\, \delta g_2 = 4\,[\mathrm{b}_i \mathrm{p}_j]_0 + 2R_0 + \left[\frac{z_2^2}{n_2^2} - \tfrac{3}{4} u_2^2 + \tfrac{1}{4}\frac{\dot{u}_2^2}{n_2^2}\right]_0$$

which is exactly similar to the expression for δg_1.

Reference of Coordinates of *Saturn* to Center of *Sun*.

Let r_2', v_2' be the polar coordinates of *Saturn* referred to the center of the *Sun* as origin. Then in the triangle of *Sun*, *Saturn*, mass-center of *Sun* and *Jupiter*, the angles are respectively $v_2' - v_1$, φ, and $\pi - (v_2 - v_1)$, and the sides opposite r_2, $\kappa_1 \mathrm{r}_1$, and r_2'. If we put

$$v_2 - v_1 = l_2 - l_1 + \delta_1 v_2 - \delta_1 v_1 + \ldots = \theta_0 + \theta_1 + \ldots,$$

we get $\quad \mathrm{r}_2' = [\mathrm{r}_2^2 + \kappa_1^2 \mathrm{r}_1^2 + 2\kappa_1 \mathrm{r}_1 \mathrm{r}_2 \cos(v_2 - v_1)]^{\frac{1}{2}}$

or, approximately

$$\mathrm{r}_2' = \mathrm{r}_2 + \kappa_1 \mathrm{r}_1 \cos(\theta_0 + \theta_1) + \tfrac{1}{4}\kappa_1^2 \frac{\mathrm{r}_1^2}{\mathrm{r}_2}[1 - \cos 2(\theta_0 + \theta_1)]$$

$$= \mathrm{a}_2\Big[1 + \tfrac{1}{2} u_2 + \tfrac{1}{2}\delta u_2 - \tfrac{1}{8} u_2^2 + \tfrac{1}{4}\kappa_1^2 \frac{\mathrm{a}_1^2}{\mathrm{a}_2^2}(1 - \cos 2\theta_0)$$

$$+ \kappa_1 \frac{\mathrm{a}_1}{\mathrm{a}_2}\Big\{(1 + \tfrac{1}{2} u_1)\cos\theta_0 - \theta_1 \sin\theta_0\Big\}\Big]$$

since $\theta_1 = \delta_1 v_2 - \delta_1 v_1$ is a very small angle. As far as to terms of the first order

$$r_2' = a_2\left[1+\tfrac{1}{2}u_2+\kappa_1\frac{a_1}{a_2}\cos\theta_0\right]$$

so that to terms of this order r_2' differs from r_2 only in the term of argument θ_0. It is also seen that r_2' has the same mean value as has r_2. When terms of the second order are included this ceases to be true.

In the same triangle as mentioned above we have

$$v_2' = v_2 - \varphi$$

and $$\frac{\sin\varphi}{\sin(v_2-v_1)} = \frac{\kappa_1 r_1}{r_2'} = \tfrac{1}{3000}\,, \qquad \text{approximately.}$$

Hence

$$\sin\varphi = \varphi = \frac{\kappa_1 r_1}{r_2'}\sin(v_2-v_1)$$

and therefore

$$v_2' = v_2 - \kappa_1\frac{a_1}{a_2}\left[1+\frac{u_1}{2}-\frac{u_2}{2}-\kappa_1\frac{a_1}{a_2}\cos\theta_0\right][\sin\theta_0+\theta_1\cos\theta_0]$$

$$= v_2 - \kappa_1\frac{a_1}{a_2}\Big[\sin\theta_0+\theta_1\cos\theta_0 + \left(\frac{u_1}{2}-\frac{u_2}{2}\right)\sin\theta_0 - \tfrac{1}{2}\kappa_1\frac{a_1}{a_2}\sin 2\theta_0\Big]$$

It is seen that v_2' has the same mean rate of increase, n_2, as has v_2, being as much less than the latter in the first and second quadrants as greater in the third and fourth.

Computation of First-Order Terms.

It is necessary first to obtain the values of the functions A^i entering into the perturbative function. Let

$$[1-2\alpha\cos\theta_0+\alpha^2]^{-\frac{1}{2}} = \tfrac{1}{2}\sum_{-\infty}^{+\infty} b^i\cos i\,\theta_0$$

Hence if we compute $b^i,\ \alpha\dfrac{db^i}{d\alpha},\ \alpha^2\dfrac{d^2b^i}{d\alpha^2}$ we can obtain from them $A^i,\ a_1\dfrac{\partial A^i}{\partial a_1},\ a_1^2\dfrac{\partial^2 A^i}{\partial a_1^2}$ by well known relations. These quantities may be computed in several ways, all well known, and it is unnecessary here to reproduce the formulas. By glancing at the perturbations under consideration as given by DePontécoulant, "*Théorie Analytique du Système du Monde,*" it is seen that several coefficients are quite large; for instance, 196″ is the coefficient of $\sin 2\theta_0$ in $\delta_1 v_1$. For this and similar terms nine-place logarithms are necessary, but only a few terms demand so many figures. In general seven-place logarithms suffice for terms of the first order, while five-, and for one or two terms, six-place logarithms, will give the same accuracy for the second-order terms. The b^i and their derivatives have been computed for $\log\alpha = 9.736327557$, and the computations were checked twice, and in some cases, three times by recomputation.

The values found for $b^i,\ \alpha\dfrac{db^i}{d\alpha},\ \alpha^2\dfrac{d^2b^i}{d\alpha^2}$ are

i	b^i	$\alpha\dfrac{db^i}{d\alpha}$	$\alpha^2\dfrac{d^2b^i}{d\alpha^2}$
0	0.338438916	9.643539018	9.930590
1	9.792423038	9.907211461	9.878787
2	9.410262287	9.779191774	0.018692
3	9.07072475	9.59673039	0.020155
4	8.7510906	9.3914979	9.948196
5	8.4430357	9.173599	9.833742
6	8.1425680	8.947617	9.691950
7	7.847463	8.715983	9.53118
8	7.556353	8.480187	9.35651
9	7.268330	8.24120	9.17124
10	6.98277	7.99967	8.97758
11	6.69922	7.75609	8.7772
12	6.4174	7.5105	8.5716

From these data we immediately compute the first-order terms of *Jupiter* given below. The coefficients are expressed in abstract numbers for $\delta_1 r_1/a_1$, in seconds of arc for $\delta_1 v_1$.

$$\frac{\delta_1 r_1}{a_1} = \left\{\begin{array}{ll}
-0.00001\,14252 & \\
+0.00012\,45421 & \cos\theta_0\\
-0.00053\,33873 & \cos 2\theta_0\\
-0.00005\,55968 & \cos 3\theta_0\\
-0.00001\,43934 & \cos 4\theta_0\\
-0.00000\,47600 & \cos 5\theta_0\\
-0.00000\,17772 & \cos 6\theta_0\\
-0.00000\,07141 & \cos 7\theta_0\\
-0.00000\,03016 & \cos 8\theta_0\\
-0.00000\,01320 & \cos 9\theta_0\\
-0.00000\,00593 & \cos 10\theta_0\\
-0.00000\,00273 & \cos 11\theta_0\\
-0.00000\,00127 & \cos 12\theta_0
\end{array}\right\}$$

$$\delta_1 v_1 = \left\{\begin{array}{rl}
+\ 79.24829 & \sin\theta_0\\
-195.77043 & \sin 2\theta_0\\
-\ 16.33180 & \sin 3\theta_0\\
-\ 3.75436 & \sin 4\theta_0\\
-\ 1.15702 & \sin 5\theta_0\\
-\ 0.41297 & \sin 6\theta_0\\
-\ 0.16100 & \sin 7\theta_0\\
-\ 0.06656 & \sin 8\theta_0\\
-\ 0.02868 & \sin 9\theta_0\\
-\ 0.01275 & \sin 10\theta_0\\
-\ 0.00581 & \sin 11\theta_0\\
-\ 0.00269 & \sin 12\theta_0
\end{array}\right\}$$

The corresponding values for *Saturn* are

$$\frac{\delta_1 r_2}{a_2} = \left\{\begin{array}{ll} +0.00041\,67147 & \\ +0.00034\,91670 & \cos\theta_0 \\ +0.00014\,74618 & \cos 2\theta_0 \\ +0.00003\,40816 & \cos 3\theta_0 \\ +0.00001\,05662 & \cos 4\theta_0 \\ +0.00000\,37863 & \cos 5\theta_0 \\ +0.00000\,14794 & \cos 6\theta_0 \\ +0.00000\,06123 & \cos 7\theta_0 \\ +0.00000\,02639 & \cos 8\theta_0 \\ +0.00000\,01173 & \cos 9\theta_0 \\ +0.00000\,00534 & \cos 10\theta_0 \\ +0.00000\,00247 & \cos 11\theta_0 \\ +0.00000\,00116 & \cos 12\theta_0 \end{array}\right\}$$

$$\delta_1 v_2 = \left\{\begin{array}{rl} +103''.82924 & \sin\theta_0 \\ +\ 32.01024 & \sin 2\theta_0 \\ +\ 6.66903 & \sin 3\theta_0 \\ +\ 1.99553 & \sin 4\theta_0 \\ +\ 0.70687 & \sin 5\theta_0 \\ +\ 0.27562 & \sin 6\theta_0 \\ +\ 0.11428 & \sin 7\theta_0 \\ +\ 0.04944 & \sin 8\theta_0 \\ +\ 0.02206 & \sin 9\theta_0 \\ +\ 0.01008 & \sin 10\theta_0 \\ +\ 0.00469 & \sin 11\theta_0 \\ +\ 0.00222 & \sin 12\theta_0 \end{array}\right\}$$

We have shown that in order to reduce $\delta_1 r_2 / a_2$ and $\delta_1 v_2$ to $\delta_1 r_2' / a_2$ and $\delta_1 v_2'$ respectively, it is necessary to change the coefficient of argument θ_0 only, adding $\kappa_1 a_1 / a_2$ in the first case, and subtracting it in the second. This amounts to

$$\text{Red. to } \frac{\delta_1 r_2'}{a_2} = +0.00052\,00157 \qquad \text{Red. to } \delta_1 v_2' = -107''.26093$$

Computation of Second-Order Terms.

With the values obtained for A^i, $a_1 \dfrac{\partial A^i}{\partial a_1}$, $a_1^2 \dfrac{\partial^2 A^i}{\partial a_1^2}$ were computed the coefficients $a_i, \ldots, q_i$ and $\alpha_i, \ldots, \theta_i$. In order then to find the numerical values of δu_1, δz_1, δu_2, δz_2 it was necessary to multiply together series having the above as coefficients. This multiplication was performed by the method of special values as set forth in Hansen's "*Auseinandersetzung,*" pp. 159–164, or in Tisserand's "*Mécanique Céleste,*" Tome IV. The semi-circumference was divided into twelve equal parts, and to θ_0 were given the thirteen equidistant values 0°, 15°, 30°, ... , 180°. It is important in these computations to take advantage of any checks that may present themselves. When no checks were available the computations were repeated. After all the products had been computed equation (11), determining the constant part of the radius-vector, was employed as a partial verification of the work.

1. *Computation of* δu_1 *and* $\delta z_1 / n_1$. The numerical values of the coefficients entering into δu_1 and δz_1 are tabulated below in terms of their logarithms. It will be denoted whether the series (which is a product of two other series) is a cosine or a sine series, and by the numbers $i+j$ at the left what is the multiple of the argument θ_0 whose coefficient is opposite. By multiplying by two each of the coefficients $a_i, \ldots, \alpha_i, \ldots$, except when $i = 0$, we may regard $i+j$ as always positive.

$i+j$	cosine $a_i a_j$	cosine $e_i a_j$	cosine $f_i b_j$	cosine $-\varepsilon_i \gamma_j$
0	3.18127	3.13068n	3.39200	3.37675n
1	2.588179n	3.260253n	3.681298	3.506168n
2	2.322029	3.261497n	3.741587	3.331918n
3	2.80822n	3.10171n	3.70378	2.97340n
4	3.13372	3.21001n	3.61656	3.07604
5	2.44947	3.15498n	3.48959	3.22051
6	1.9421	3.05398n	3.33996	3.19793
7	1.501	2.9239n	3.17452	3.10977
8	1.098	2.7740n	2.99747	2.98594
9	0.718	2.6055n	2.8098	2.8358
10	0.35	2.4241n	2.6133	2.6738
11	9.95	2.1817n	2.3949	2.5558
12	9.7	2.0110n	2.1544	2.3660

$i+j$	sine $\varepsilon_i c_j$	sine $\zeta_i b_j$	sine $g_i \gamma_j$	sine $h_i d_j$
1	3.228778n	2.624453	3.264226n	1.597713
2	3.502550n	2.976050	3.256116n	2.420552n
3	3.54078n	2.95902	3.04924n	2.19131n
4	3.59282n	2.84670	2.96747	2.73957n
5	3.52926n	2.67205	3.15861	2.64359n
6	3.42139n	2.4729	3.15214	2.48659n
7	3.28530n	2.2601	3.07239	2.3024n
8	3.12894n	2.0379	2.9536	2.1014n
9	2.9597n	1.812	2.8096	1.8935n
10	2.7762n	1.577	2.7506	1.680n
11	2.5576n	1.33	2.5291	1.472n
12	2.3456n	1.06	2.3477	1.249n

$i+j$	sine $\zeta_i d_j$	cosine $a_i d_j$	cosine k_{i+j}	cosine l_{i+j}
0	. . . .	3.50648n	2.64482	. . . .
1	2.349305	2.796439	. . . .	. . . .
2	2.340550	2.514531n	3.121940	3.044708
3	2.04846	3.10381	. . . .	. . . .
4	2.95404	3.46114n	. . . .	. . . .
5	2.85854	2.81880n	. . . .	. . . .
6	2.69805	2.35268n	. . . .	. . . .
7	2.5099	1.9517n	. . . .	. . . .
8	2.3051	1.5856n	. . . .	. . . .
9	2.0929	1.245n	. . . .	. . . .
10	1.872	0.926n	. . . .	. . . .
11	1.613	0.573n	. . . .	. . . .
12	1.395	0.28n	. . . .	. . . .

In order to find the constant δg_1 which enters into δu_1, and at the same time verify the preceding calculations, it is necessary to compute the additional products in equation (11), namely,

$$\frac{\dot{u}_1^2}{4n_1^2},\ \frac{z_1^2}{n_1^2},\ \frac{u_1 \ddot{u}_1}{4n_1^2},\ \text{and } \sigma a_1 \frac{\partial F_0}{\partial a_1} \frac{u_1}{2}$$

the numerical values of which are tabulated below. The same nomenclature is used as before, and the tabulation is in the same order in which the terms are here written.

$i+j$	cosine $-\delta_i\delta_j$	cosine d_id_j	cosine a_io_j	cosine a_ir_j
0	3.32522	3.83271	3.32522*n*	2.55100*n*
1	2.30343	2.96510*n*	2.12056*n*	2.97100*n*
2	2.12794	2.66130	2.45334*n*	2.89591*n*
3	2.73772	3.39792*n*	2.68638	2.88204*n*
4	3.28384*n*	3.78892	3.29248*n*	2.76675*n*
5	2.77321*n*	3.18525	2.82099*n*	2.82824*n*
6	2.39858*n*	2.75328	2.49363*n*	2.80472*n*
7	2.0656*n*	2.38169	2.20678*n*	2.72459*n*
8	1.7530*n*	2.0415	1.9372*n*	2.60879*n*
9	1.451*n*	1.7187	1.673*n*	2.4579*n*
10	1.158*n*	1.407	1.412*n*	2.301*n*
11	0.886*n*	1.087	1.147*n*	2.068*n*
12	0.60*n*	0.80	0.89*n*	1.911*n*

It was found that the last three or four coefficients (except the twelfth) obtained by the method of special values did not satisfy the checks, whereas the same coefficients computed by direct multiplication of series did. Hence all these coefficients were thus recomputed. In this way were obtained the twelfth coefficients in the sine series, which are not given by the method of special values. In the cosine series the twelfth coefficient is the same for both ways of computing.

From the above data we get for δu_1 and $\delta z_1 / n_1$,

$i+j$	cosine δu_1	cosine $\delta z_1 / n_1$
0	3.78191	. . . .
1	4.559802*n*	4.501489
2	4.554736	4.569894
3	3.84237	4.04265*n*
4	2.84305	3.63491
5	2.6374	2.6812
6	2.2726	1.713
7	1.909	1.125*n*
8	1.555	1.232*n*
9	1.211	1.076*n*
10	0.86	0.78*n*
11	0.09	0.46
12	9.4	0.45

We have

$$\frac{\delta_2 r_1}{a_1} = \tfrac{1}{2}\left[\delta u_1 - \tfrac{1}{4}u_1^2\right] \quad , \quad \delta_2 v_1 = \int \delta z_1 \, dt$$

The numerical values of these quantities are here given.

$$\frac{\delta_2 r_1}{a_1} = \left\{\begin{array}{ll} +0.00000\,02267 & \\ -0.00000\,17952 & \cos\theta_0 \\ +0.00000\,17830 & \cos 2\theta_0 \\ +0.00000\,03800 & \cos 3\theta_0 \\ -0.00000\,00332 & \cos 4\theta_0 \\ +0.00000\,00076 & \cos 5\theta_0 \\ +0.00000\,00050 & \cos 6\theta_0 \\ +0.00000\,00025 & \cos 7\theta_0 \\ +0.00000\,00012 & \cos 8\theta_0 \\ +0.00000\,00006 & \cos 9\theta_0 \\ +0.00000\,00003 & \cos 10\theta_0 \end{array}\right\} \qquad \delta_2 v_1 = \left\{\begin{array}{ll} -1\overset{\prime\prime}{.}09575 & \sin\theta_0 \\ +0.64134 & \sin 2\theta_0 \\ +0.12699 & \sin 3\theta_0 \\ -0.03725 & \sin 4\theta_0 \\ -0.00332 & \sin 5\theta_0 \\ -0.00030 & \sin 6\theta_0 \\ +0.00007 & \sin 7\theta_0 \\ +0.00007 & \sin 8\theta_0 \\ +0.00005 & \sin 9\theta_0 \\ +0.00002 & \sin 10\theta_0 \\ -0.00001 & \sin 11\theta_0 \\ -0.00001 & \sin 12\theta_0 \end{array}\right\}$$

These values of $\delta_2 r_1 / a_1$ and $\delta_2 v_1$ constitute the solution of the problem for *Jupiter's* coordinates, but, that the expressions for $\frac{r_1}{a_1}$ and v_1 may be complete, we add the first- and second-order terms, thus forming the tables

$$\frac{r_1}{a_1} = \left\{\begin{array}{ll} 1-0.00001\,11985 & \\ +0.00012\,27470 & \cos\theta_0 \\ -0.00053\,16043 & \cos 2\theta_0 \\ -0.00005\,52168 & \cos 3\theta_0 \\ -0.00001\,44266 & \cos 4\theta_0 \\ -0.00000\,47524 & \cos 5\theta_0 \\ -0.00000\,17722 & \cos 6\theta_0 \\ -0.00000\,07116 & \cos 7\theta_0 \\ -0.00000\,03004 & \cos 8\theta_0 \\ -0.00000\,01314 & \cos 9\theta_0 \\ -0.00000\,00591 & \cos 10\theta_0 \\ -0.00000\,00272 & \cos 11\theta_0 \\ -0.00000\,00127 & \cos 12\theta_0 \end{array}\right\} \qquad v_1 = n_1 t + \left\{\begin{array}{ll} +\ 78\overset{\prime\prime}{.}15254 & \sin\theta_0 \\ -195.12909 & \sin 2\theta_0 \\ -\ 16.20481 & \sin 3\theta_0 \\ -\ 3.79161 & \sin 4\theta_0 \\ -\ 1.16033 & \sin 5\theta_0 \\ -\ 0.41327 & \sin 6\theta_0 \\ -\ 0.16093 & \sin 7\theta_0 \\ -\ 0.06649 & \sin 8\theta_0 \\ -\ 0.02863 & \sin 9\theta_0 \\ -\ 0.01273 & \sin 10\theta_0 \\ -\ 0.00582 & \sin 11\theta_0 \\ -\ 0.00270 & \sin 12\theta_0 \end{array}\right\}$$

2. *Computation of* δu_2 *and* $\delta z_2 / n_2$. Several of the series entering into δu_2 and $\delta z_2 / n_2$ have already been computed as they enter also into δu_1 and $\delta z_1 / n_1$. The remaining coefficients, in terms of their logarithms, are tabulated below.

$i+j$	cosine b_ib_j	cosine f_ic_j	cosine q_ib_j	cosine $-\theta_i\gamma_j$
0	3.39116	3.63496n	3.32535	3.52548n
1	3.541489	3.849053n	3.415321	3.622736n
2	3.29549	3.89548n	3.36101	3.42446n
3	2.92530	3.81759n	3.19688	3.01129n
4	2.52038	3.79771n	3.02462	3.29467
5	2.09649	3.68966n	2.81428	3.36085
6	1.69460	3.55232n	2.59122	3.30380
7	1.3130	3.39590n	2.36141	3.19518
8	0.948	3.22480n	2.12730	3.05590
9	0.595	3.0439n	1.8916	2.89760
10	0.25	2.8505n	1.647	2.7281
11	9.91	2.6250n	1.395	2.5953
12	9.60	2.4080n	1.119	2.4079

$i+j$	sine $q_i\eta_j$	sine ζ_ip_j	cosine m_i+_j	cosine n_i+_j
0	. . .	. . . .	2.82091n	. . .
1	3.169687	2.848613n	. . .	. . .
2	3.19792	2.79111n	3.29803n	2.64967n
3	3.09005	3.00085n	. . .	. . .
4	3.05805	3.00635n	. . .	. . .
5	2.91879	2.88634n	. . .	. . .
6	2.74412	2.72391n	. . .	. . .
7	2.55063	2.53883n	. . .	. . .
8	2.34727	2.3401n	. . .	. . .
9	2.1352	2.1324n	. . .	. . .
10	1.9187	1.9153n	. . .	. . .
11	1.7057	1.684n	. . .	. . .
12	1.503	1.428n	. . .	. . .

There is one additional product needed for $\delta z_2 / n_2$ and three for the numerical expression of equation (14). These are, respectively,

$$\frac{u_2 z_2}{2n_2}, \quad \frac{\dot{u}_2^2}{4n_2^2}, \quad \frac{z_2^2}{n_2^2}, \quad \frac{u_2 \ddot{u}_2}{4n_2^2}$$

The coefficients of these products are given below in the same order in which they occur here. Then δu_2 and $\delta z_2 / n_2$ have the values given.

$i+j$	cosine b_ip_j	cosine $-\gamma_j\gamma_j$	cosine p_ip_j	cosine s_ib_j
0	3.22296n	3.38700	3.59894	3.38700n
1	3.669376n	3.48381	3.62334	3.83723n
2	3.56529n	2.35849n	3.62027	3.92742n
3	3.32728n	3.25359n	3.60254	3.81581n
4	3.01265n	3.18388n	3.37820	3.63415n
5	2.66516n	2.94751n	3.08329	3.40628n
6	2.33235n	2.68587n	2.78977	3.16908n
7	2.00986n	2.41787n	2.50202	2.92815n
8	1.7018n	2.1475n	2.2185	2.6852n
9	1.3983n	1.8775n	1.9386	2.4422n
10	1.106n	1.6090n	1.6602	2.192n
11	0.80n	1.3485n	1.378	1.937n
12	0.46n	1.0995n	1.080	1.655n

$i+j$	cosine δu_2	cosine $\delta z_2/n_2$
0	4.20364n	. . .
1	5.005758n	5.074954
2	4.04903n	4.36740
3	3.59051n	4.07674
4	3.08096n	3.71344
5	2.65093n	3.37455
6	2.25643n	3.0540
7	1.8850n	2.7447
8	1.5294n	2.4442
9	1.1877n	2.150
10	0.835n	1.852
11	0.13n	1.34
12	9.43n	0.89

As in the case of r_1 and v_1 we have

$$\frac{\delta_1 r_2}{a_2} = \tfrac{1}{2}\left[\delta u_2 - \tfrac{1}{4}u_2^2\right] \quad , \quad \delta_1 v_2 = \int \delta z_2\, dt$$

In order to determine $\frac{\delta_2 r_2'}{a_2}$ and $\delta_2 v_2'$, the second-order perturbations of *Saturn's* coordinates when referred to the center of the sun, we must apply to the former the following reductions respectively: —

$$+\tfrac{1}{4}\kappa_1^2\frac{a_1^2}{a_2^2}(1-\cos 2\theta_0) + \kappa_1\frac{a_1}{a_2}\left[\frac{u_1}{2}\cos\theta_0 + (\delta_1 v_1 - \delta_1 v_2)\sin\theta_0\right]$$

and

$$+\tfrac{1}{2}\kappa_1^2\frac{a_1^2}{a_2^2}\sin 2\theta_0 - \kappa_1\frac{a_1}{a_2}\left[\left(\frac{u_1}{2}-\frac{u_2}{2}\right)\sin\theta_0 - (\delta_1 v_1 - \delta_1 v_2)\cos\theta_0\right]$$

In order to show the amount of these reductions the values of $\frac{\delta_2 r_2}{a_2}$ and $\delta_2 v_2$ are placed beside them below. In $\frac{\delta_2 r_2}{a_2}$ and its reduction the numbers are expressed in units of the tenth decimal.

$i+j$	cosine $\frac{\delta_2 r_2}{a_2}$	Reduction	sine $\delta_2 v_2$ ″	Reduction ″
0	− 9222	+ 690	. . . .	. . . .
1	−52407	−4318	−1.65252	−0.04982
2	− 6585	− 477	−0.16202	+0.02275
3	− 2368	+1374	−0.05531	−0.02554
4	− 768	+ 109	−0.01797	−0.00211
5	− 286	+ 22	−0.00659	−0.00051
6	− 115	+ 6	−0.00262	−0.00017
7	− 49	+ 2	−0.00110	−0.00007
8	− 21	0	−0.00048	−0.00003
9	− 10	0	−0.00022	−0.00001
10	− 4	0	−0.00010	0.00000
11	− 1	0	−0.00003	0.00000
12	0	0	−0.00001	0.00000

We can now form the tables for r_2' / a_2 and v_2'.

$$\frac{r_2'}{a_2} = \left\{ \begin{array}{ll} 1+0.00041\,58615 & \\ +0.00086\,35112 & \cos\theta_0 \\ +0.00014\,67556 & \cos 2\theta_0 \\ +0.00003\,39822 & \cos 3\theta_0 \\ +0.00001\,05003 & \cos 4\theta_0 \\ +0.00000\,37599 & \cos 5\theta_0 \\ +0.00000\,14685 & \cos 6\theta_0 \\ +0.00000\,06076 & \cos 7\theta_0 \\ +0.00000\,02618 & \cos 8\theta_0 \\ +0.00000\,01163 & \cos 9\theta_0 \\ +0.00000\,00530 & \cos 10\theta_0 \\ +0.00000\,00246 & \cos 11\theta_0 \\ +0.00000\,00116 & \cos 12\theta_0 \end{array} \right\}$$

$$v_2' = n_2 t + \left\{ \begin{array}{ll} -\ 5''.13402 & \sin\theta_0 \\ +31.87097 & \sin 2\theta_0 \\ +\ 6.58817 & \sin 3\theta_0 \\ +\ 1.97545 & \sin 4\theta_0 \\ +\ 0.69977 & \sin 5\theta_0 \\ +\ 0.27283 & \sin 6\theta_0 \\ +\ 0.11311 & \sin 7\theta_0 \\ +\ 0.04893 & \sin 8\theta_0 \\ +\ 0.02183 & \sin 9\theta_0 \\ +\ 0.00998 & \sin 10\theta_0 \\ +\ 0.00466 & \sin 11\theta_0 \\ +\ 0.00221 & \sin 12\theta_0 \end{array} \right\}$$

Thus we get

$$\frac{r_1}{a_1} = 1 + \sum_1^\infty A_i \cos i\,(l_2 - l_1)$$

$$v_1 = l_1 + \sum_1^\infty B_i \sin i\,(l_2 - l_1)$$

$$\frac{r_2'}{a_2} = 1 + \sum_1^\infty A_i' \cos i\,(l_2 - l_1)$$

$$v_2' = l_2 + \sum_1^\infty B_i' \sin i\,(l_2 - l_1)$$

Leander McCormick Observatory, 1903 *May* 15.

If we refer the coordinates to axes intersecting in the *Sun*, and rotating in the direction of motion with the uniform velocity n_1, it is evident that we may write

$$v_1 - l_1 = w_1 = \sum_1^\infty B_i \sin ikt$$

$$v_2' - l_1 = w_2 = kt + \sum_1^\infty B_i' \sin ikt$$

where $k = n_2 - n_1$. Then all the coordinates are periodic with respect to the time.

WHITE SPOT ON *SATURN*,

By E. E. BARNARD.

I always make a habit of looking at *Saturn* frequently during the time the planet is visible each year. In all the observations I have ever made of it, I had never seen any marking that could be used for determining the rotation period. On June 15th, at $15^h\ 0^m$, while examining the planet with the 40-inch, I noticed a decided bright spot half way from the central meridian of the planet and the following limb. The sky immediately clouded over, and no opportunity occurred to see the spot again until this morning (a.m. of June 24).

On this occasion the spot was very noticeable, and was preceded by a smaller white spot, which was separated from the main spot by a small dusky patch extending north and south. This dusky patch was estimated to be in transit at $2^h\ 59^m$.

At $15^h\ 3^m$ the following measures were made:

Center of spot fol. p. limb	11''.7 (2)
Center of spot pre. f. limb	6.1 (2)

At $15^h\ 7^m$ its center was

Dist. from S. limb	11''.8 (2)
Dist. from N. limb	5.7 (2)

When on the central meridian the east and west diameter was 2''.6, from one setting of the wires. The spot was then slightly elongated, and very luminous, though not definitely defined.

In transit the position was

From S. limb	11''.51 (2)
From N. limb	6.00 (2)

This would make it 2''.75 north of the center of the disc.

The rotational motion of the spot was rapid.

The following careful records were made:

$15^h\ 37^m$ the large white spot is now in transit.
15 42 I think it is now in transit.
15 50 it is certainly past transit.
15 55 it is noticeably past transit.

I think the second observation is perhaps the best, and have adopted it for the time of transit, viz.:

June 23 $15^h\ 42^m$ Central St'd Time ($6^h\ 0^m$ slow of G.M.T.),

At $16^h\ 0^m$ the spot was strikingly distinct, and a little elongated. At this time no other spot was visible, so it will be easy to definitely identify it for determination of the rotation period. The belt south of the spot was very heavy, and was extended out to the south.

The spot was observed again last night, with the 12-inch and a power of about 300 diameters.

It was decidedly conspicuous, and when in transit was somewhat elongated.

The following observations of its transit were made:

June 24 12h 48m not quite in transit.
12 52 " " "
12 54 " " "
12 55 transit, or not quite.
12 58 transit.
13 0 transit.
13 2 past transit.
June 24 13h 4m perhaps past.
13 7 certainly past transit.
13 11 noticeably past.
13 13 conspicuously past.

At transit there was no other spot visible. The central transit was perhaps close to

June 24d 12h 58m Central Standard Time.

Yerkes Observatory, Williams Bay, Wis., 1903 *June* 25.

COMET *c* 1903.

[From RITCHIE'S Circular of July 6, No. 135.]

A message received from Dr. KREUTZ, via Harvard College Observatory, on June 22, announced the discovery of a comet with nucleus by BORRELLY, of Marseilles, on June 21. Subsequent positions were received from Lick Observatory and from Professor PAYNE, which are given below.

From observations of June 22, 23 and 24, Mr. BELLAMY has computed the following approximate

ELEMENTS.

T = 1903 August 21.707 Greenw. M.T.

$$\left.\begin{array}{l}\pi = 67^\circ 55' \\ \Omega = 292\ 37 \\ i = 92\ 11\end{array}\right\} 1903.0$$

q = 0.2375

EPHEMERIS.

Gr. Midnight	α	δ	Br.
1903 July 3	21h 32m 47s	+ 8° 51′	3.9
7	21 9 16	+21 42	
11	20 24 13	+43 42	
15	17 53 41	+66 24	16.8

Brightness June 21 = 1.

POSITIONS.

Greenw. M.T.	α	δ	Observer
1903 June 21.469	21h 52m 52s.	−8° 10′ –″	Borrelly
22.8742	21 51 38.8	−7 0 48	Aitken
23.8278	21 50 51.4	−6 9 26	Aitken
23.8613	21 50 50.1	−6 7 38	Wilson
24.8745	21 49 52.0	−5 8 48	Aitken

OBSERVATIONS OF THE SATELLITE OF *NEPTUNE*,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

BY W. W. DINWIDDIE.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

All of the following measures were made by double distances, five comparisons on each side of coincidence; and ten position angles. The center of *Neptune* was bisected in each case. A magnifying power of six hundred diameters was used. The measures have been corrected for refraction. An unusual amount of cloudy weather this season prevented a greater number of observations being secured. The seeing has been uniformly bad, except on Oct. 30, Nov. 1, and Dec. 5.

Washington M.T.	p	Wash. M.T.	s	Washington M.T.	p	Wash. M.T.	s
1902 Oct. 24 16h 28m 45s	101.99°	16h 28m 45s	16.28″	1903 Jan. 6 11h 8m 30s	263.97°	11h 8m 45s	16.48″
28 16 14 45	235.03	16 14 15	14.39	22 10 5 30	19.61	10 6 30	10.86
29 16 41 15	154.10	16 41 0	11.39	Feb. 20 8 2 36	52.00	8 3 0	13.57
30 16 14 30	96.33	16 15 15	16.05	22 8 30 0	269.53	8 29 40	16.55
31 16 44 0	50.96	16 43 45	13.68	25 8 37 30	88.07	8 38 0	16.88
Nov. 1 16 3 0	327.27	16 2 45	11.88	26 8 5 15	43.24	8 6 30	13.17
2 16 4 30	272.43	16 4 15	16.51	Mar. 3 7 34 45	84.92	7 35 15	16.77
21 14 43 0	205.87	14 44 45	11.74	12 7 27 58	257.50	7 28 17	15.97
Dec. 1 14 6 30	291.17	14 7 45	15.34	17 7 32 58	295.24	7 32 43	13.46
5 14 14 30	69.20	14 16 0	15.93	18 7 51 35	251.43	7 51 17	16.45
7 13 33 15	286.28	13 34 45	15.62	26 7 41 28	108.42	7 42 7	14.79

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A 4½-INCH REFRACTOR,

By A. W. QUIMBY.

1903	Time	New Grs.	Total Grs.	Total Spots	Fac Grs.	Def.
Jan. *1	1	–	–	–	–	poor
*3	11	–	–	–	–	poor
6	1	1	1	2	1	fair
7	7	–	–	–	–	poor
8	9	–	–	–	1	fair
9	10	–	–	–	1	fair
10	9	–	–	–	1	fair
12	1	–	–	–	–	fair
13	9	–	–	–	–	fair
14	11	–	–	–	–	poor
15	3	–	–	–	–	fair
16	3	–	–	–	–	fair
17	2	–	–	–	–	fair
18	8	1	1	1	1	fair
19	10	–	1	5	1	fair
20	11	–	1	3	1	poor
21	11	–	1	17	–	v. good
22	9	1	2	11	1	poor
23	8	–	2	11	1	poor
24	10	–	1	6	1	poor
26	9	–	–	–	–	poor
27	3	–	–	–	1	fair
28	11	2	2	7	1	fair
29	9	–	1	8	1	poor
30	9	–	1	8	–	poor
31	10	–	1	8	–	poor
Feb. 2	10	–	1	2	1	poor
3	3	–	1	2	1	poor
4	1	–	–	–	–	poor
5	8	1	1	2	–	fair
6	8	–	1	2	–	fair
7	9	–	–	–	–	poor
8	5	–	–	–	–	poor
9	8	2	2	3	1	fair
10	9	1	3	10	1	fair
12	8	–	3	6	1	fair
13	9	–	2	4	–	fair
14	3	–	1	2	–	poor
17	8	–	1	2	–	fair
18	8	1	2	3	–	poor
19	8	1	3	7	–	fair
20	3	–	1	8	3	fair
21	8	–	1	1	2	poor
22	8	–	1	2	–	poor
23	8	–	1	3	–	fair
24	8	2	3	7	1	fair
25	8	–	3	6	2	fair
26	8	–	2	7	2	fair
27	8	–	2	5	–	fair
28	2	–	2	3	–	poor
Mar. 1	8	–	1	1	1	poor
2	8	–	1	3	1	fair
3	8	–	1	2	2	fair
4	8	–	1	2	3	fair

1903	Time	New Grs.	Total Grs.	Total Spots	Fac Grs.	Def.
Mar. 5	8	–	–	–	1	fair
6	8	–	–	–	1	good
10	11	–	–	–	–	poor
11	5	–	–	–	–	poor
12	8	1	1	1	1	fair
13	8	–	1	4	–	fair
14	8	–	1	7	–	fair
15	8	–	–	–	1	fair
17	8	–	–	–	–	fair
18	8	–	–	–	–	fair
19	8	–	–	–	2	good
20	10	–	–	–	–	poor
21	8	–	–	–	–	poor
23	4	1	1	1	–	poor
24	8	1	2	5	1	fair
25	8	–	2	6	1	fair
26	8	1	2	3	1	fair
27	8	–	2	7	1	fair
28	8	–	2	12	1	good
29	8	1	3	24	1	fair
31	4	–	3	42	1	good
Apr. 1	8	–	3	23	1	fair
2	8	1	3	7	2	poor
3	8	–	2	3	1	poor
4	8	–	1	3	1	fair
5	9	–	1	3	1	fair
6	8	–	1	8	2	fair
7	5	–	1	1	–	poor
9	8	1	2	7	1	fair
10	8	–	2	7	1	fair
11	7	–	2	6	1	fair
13	10	–	1	2	1	poor
16	2	–	–	–	–	poor
17	8	–	–	–	–	fair
18	7	–	–	–	2	fair
19	8	–	–	–	–	fair
20	8	–	–	–	1	fair
21	7	–	–	–	1	fair
22	4	1	1	1	2	fair
23	7	–	1	1	2	fair
24	8	2	3	9	3	fair
25	11	–	2	3	1	poor
26	5	1	3	10	1	fair
27	8	1	4	24	2	good
28	8	–	4	27	2	fair
29	10	–	3	42	2	good
30	8	–	3	80	2	good
May 1	8	–	3	24	2	fair
2	6	1	3	18	2	fair
3	8	–	3	16	2	fair
4	8	–	3	10	3	fair
5	4	–	–	–	1	good
6	8	–	–	–	1	fair
7	8	–	–	–	1	fair

1903	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
May 8	8	–	–	–	1	fair
9	8	1	1	1	1	good
10	8	–	–	–	1	fair
11	7	–	–	–	1	fair
12	6	1	1	1	2	fair
13	8	–	1	1	2	fair
14	8	–	1	1	2	fair
15	8	–	–	–	2	fair
16	8	1	1	1	2	fair
17	8	–	1	2	1	fair
18	7	–	1	2	1	fair
19	8	–	1	1	1	fair
20	8	1	1	2	1	fair
21	7	1	2	10	2	fair
22	7	–	2	4	–	poor
23	10	–	1	2	–	fair
24	6	–	1	1	–	poor
25	7	–	1	1	–	poor
26	7	1	2	13	1	good
27	7	–	1	1	–	poor
28	8	–	1	2	2	fair
29	3	–	–	–	1	good
30	8	–	–	–	–	poor
31	6	–	–	–	1	fair
June 1	8	–	–	–	1	good
2	6	–	–	–	–	poor
3	8	–	–	–	–	fair
4	7	–	–	–	–	fair
5	7	–	–	–	–	poor
6	7	–	–	–	–	poor
7	6	–	–	–	–	poor
8	8	–	–	–	–	fair
9	8	–	–	–	–	fair
10	8	–	–	–	–	fair
11	8	–	–	–	–	fair
12	10	–	–	–	–	poor
13	4	1	1	1	2	fair
14	2	–	1	1	1	poor
15	8	–	1	1	2	poor
16	8	1	2	6	2	fair
17	10	–	1	1	1	poor
18	10	–	1	3	2	fair
19	8	–	1	2	1	poor
20	4	2	3	30	–	poor
21	8	–	3	50	–	good
22	8	–	3	16	1	poor
24	4	–	2	5	2	fair
25	7	–	2	3	–	poor
26	7	–	2	5	1	fair
27	7	–	1	1	1	poor
28	7	–	1	1	1	fair
29	4	–	1	1	1	poor
30	8	1	1	4	–	good

*2½-inch refractor.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Captain C. M. Chester, U.S.N., Superintendent.]

1903 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(387) *Aquitania*.					
h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 4 10 16 33	1	15 , 4	−1 21.06	− 1 37.9	9 35 39.88	+19 11 3.6	n9.451	0.527	+1.99	−15.7
4 10 29 35	2	20 , 5	−1 11.42	− 6 56.7	9 35 39.60	+19 11 8.9	n9.414	0.517	+1.99	−15.7
5 11 48 22	3	19 , 4	−1 27.76	− 2 54.9	9 34 45.83	+19 20 17.3	n8.982	0.472	+2.00	−15.7
					(402) *Chloe*.					
Feb. 21 10 0 22	4	29 , 6	+0 30.81	+ 0 0.3	9 45 8.71	+20 3 11.3	n9.319	0.483	+2.10	−15.7
22 9 14 48	5	30 , 10	−0 11.02	− 1 2.8	9 44 21.07	+20 12 52.7	n9.452	0.510	+2.11	−15.6
23 9 57 46	6	19 , 4	−2 56.36	−13 55.4	9 43 31.36	+20 23 0.8	n9.291	0.472	+2.11	−15.6
25 10 8 19	7	25 , 5	+1 6.16	−13 39.6	9 41 56.05	+20 42 15.1	n9.193	0.454	+2.12	−15.4
					(11) *Parthenope*.					
Jan. 30 11 0 6	8	24 , 5	+1 55.53	− 6 38.4	10 8 22.70	+13 43 10.4	n9.461	0.608	+1.84	−15.2
Feb. 5 10 43 55	9	30 , 6	+1 10.08	− 1 48.8	10 3 21.70	+14 21 58.9	n9.430	0.594	+1.94	−15.5
9 10 45 13	10	23 , 6	+2 22.84	+ 0 21.2	9 59 45.86	+14 48 34.8	n9.368	0.579	+2.00	−15.8
12 10 58 51	11	23 , 6	−0 16.47	+ 3 19.8	9 56 58.32	+15 8 38.0	n9.258	0.563	+2.03	−15.9
					(79) *Eurynome*.					
Feb. 19 9 56 18	12	30 , 6	−2 34.51	+ 1 24.4	10 20 31.87	+ 3 49 48.3	n9.442	0.709	+2.08	−15.7
20 9 58 41	13	29 , 10	−0 0.09	− 4 43.2	10 19 35.35	+ 3 56 25.1	n9.424	0.708	+2.09	−15.9
21 11 3 39	14	35 , 8	+0 24.96	− 4 53.1	10 18 36.27	+ 4 3 21.3	n9.147	0.701	+2.10	−16.0
22 10 13 10	15	29 , 6	+2 10.61	+ 3 29.8	10 17 41.84	+ 4 9 52.4	n9.351	0.703	+2.10	−16.1
					(18) *Melpomene*.					
Mar. 27 10 49 32	16	10 , 2	−1 39.04	− 1 2.4	10 29 34.99	+13 47 42.5	8.885	0.571	+2.06	−15.0
Apr. 1 9 46 24	17	27 , 7	−1 15.32	− 7 22.2	10 26 53.60	+14 13 32.4	n7.840	0.562	+2.02	−14.6
4 9 37 56	18	30 , 6	+0 54.62	−11 28.4	10 25 33.51	+14 26 45.8	7.427	0.558	+1.98	−14.4
5 9 8 28	18	28 , 6	+0 31.38	− 7 32.3	10 25 10.26	+14 30 42.0	n8.692	0.558	+1.97	−14.3
					(29) *Amphitrite*.					
Mar. 26 11 39 25	19	30 , 6	+0 10.07	− 1 18.8	13 1 32.64	− 9 33 25.6	n9.134	0.814	+2.38	−11.7
Apr. 4 10 35 14	20	30 , 6	+0 43.67	+10 32.0	12 53 11.06	− 9 3 40.8	n9.243	0.808	+2.44	−12.8
4 11 9 41	21	20 , 4	−3 29.94	+ 3 41.9	12 53 9.62	− 9 3 37.3	n9.036	0.811	+2.44	−12.5
5 10 45 24	22	25 , 5	+2 52.93	+ 0 51.6	12 52 13.82	− 9 0 5.4	n9.166	0.809	+2.44	−13.0
8 9 54 42	22	30 , 6	+0 5.41	+11 46.0	12 49 26.31	− 8 49 10.4	n9.330	0.804	+2.45	−12.4

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	9 36 58.95	+19 12 57.2	Berlin (A) A.G. 3903	12	10 23 4.30	+ 3 48 39.6	Albany, A.G. 4047
2	9 36 49.03	+19 18 21.3	" " " 3900	13	10 19 33.35	+ 4 1 24.2	" " 4030
3	9 36 11.59	+19 23 27.9	" " " 3895	14	10 18 9.21	+ 4 8 30.4	" " 4021
4	9 44 35.80	+20 3 26.7	Berlin (B) A.G. 3854	15	10 15 29.13	+ 4 6 38.7	" " 4007
5	9 44 29.98	+20 14 11.1	" " " 3853	16	10 31 11.97	+13 48 59.9	Bonn VI, +14°2267
6	9 46 25.61	+20 37 11.8	" " " 3860	17	10 28 6.90	+14 21 9.2	Leipzig I, A.G. 4053
7	9 40 47.77	+20 56 10.1	" " " 3840	18	10 24 36.91	+14 38 28.6	" " " 4042
8	10 6 25.33	+13 50 4.0	Leipzig I, A.G. 3962	19	13 1 20.19	− 9 31 55.1	Schjellerup 4728 [140
9	10 2 9.68	+14 24 3.2	" " " 3948	20	12 52 24.95	− 9 14 0.0	Wien, A.G. Zones 122 &
10	9 57 21.02	+14 48 29.4	" " " 3928	21	12 56 37.12	− 9 7 6.7	" " Zone 131
11	9 57 12.76	+15 5 34.1	Berlin (A) A.G. 4017	22	12 49 18.45	− 9 0 44.0	Newcomb's Fund. Catal.

OBSERVATIONS OF COMET *d* 1902 (GIACOBINI),

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By W. W. DINWIDDIE.

[Communicated by Captain C. M. Chester, U.S.N., Superintendent.]

1902–3 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Dec. 18 12 26 28	1	t15, 2	−0 48.39	+ 1 35.1	7 11 15.14	+ 0 44 20.3	n9.053	0.733	+4.65	−15.1
26 10 44 23	2	10,10	+0 3.00	+ 3 22.8	7 6 27.16	+ 2 33 14.6	n9.370	0.718	+4.81	−16.1
Jan. 17 12 12 30	3	10,10	+0 6.91	+ 0 40.9	6 50 39.24	+ 9 0 33.8	9.129	0.644	+1.94	−12.6
18 8 17 49	4	10,10	−0 21.82	+10 6.7	6 50 4.49	+ 9 16 59.6	n9.481	0.662	+1.93	−12.6
22 11 27 30	5	10,10	−0 37.19	− 3 34.9	6 47 17.18	+10 38 45.7	8.963	0.619	+1.94	−12.8
Feb. 4 9 36 5	6	t18, 8	−0 37.13	+ 0 57.8	6 40 12.42	+14 56 34.3	n8.454	0.550	+1.90	−12.3
5 8 28 12	7	t39,10	−0 49.44	− 1 40.1	6 39 49.28	+15 15 20.4	n9.163	0.555	+1.90	−12.3
22 7 55 39	8	t29, 6	+2 16.08	− 3 10.8	6 36 31.16	+20 31 5.8	n8.856	0.443	+1.80	−10.8
23 8 4 14	9	10,10	−0 5.99	+ 0 19.1	6 36 33.63	+20 48 27.3	n8.658	0.434	+1.80	−10.8
26 8 33 44	10	10,10	−0 20.89	− 1 37.6	6 36 51.07	+21 39 24.9	8.627	0.413	+1.75	−10.4
Mar. 1 9 0 18	11	10,10	−0 7.51	− 6 38.8	6 37 23.69	+22 28 40.2	9.091	0.405	+1.70	−10.0
3 10 31 39	12	t32, 6	−1 11.79	+ 5 21.4	6 37 54.55	+23 1 20.9	9.496	0.474	+1.67	− 9.9
17 9 15 19	13	10,10	−0 24.83	− 3 36.1	6 44 23.86	+26 20 56.0	9.434	0.371	+1.48	− 8.6
18 9 56 34	14	t25, 5	−0 49.08	+ 3 25.8	6 45 4.72	+26 34 15.9	9.545	0.430	+1.47	− 8.6
Apr. 17 8 34 15	15	t12, 6	+1 11.47	+ 4 4.7	7 15 14.85	+31 36 43.0	9.577	0.328	+1.04	− 7.0
18 8 45 10	16	t11, 6	−1 17.54	+ 6 41.9	7 16 34.34	+31 44 17.5	9.602	0.356	+1.04	− 7.0
27 9 33 32	17	10,10	+0 20.54	− 6 13.0	7 29 8.01	+32 44 51.3	9.696	0.494	+0.89	− 6.5

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	7 11 58.88	+ 0 43 0.3	Nicolajew, A.G. 2097	9	6 36 37.82	+20 48 19.0	Berlin B, A.G. 2535
2	7 6 19.35	+ 2 30 7.9	Albany, A.G. 2677	10	6 37 10.21	+21 41 12.9	Berlin B, A.G. 2543
				11	6 37 29.50	+22 35 29.0	Berlin B, A.G. 2545
3	6 50 30.39	+ 9 0 5.5	Leipzig II, A.G. 3353	12	6 39 4.67	+22 56 9.4	Berlin B, A.G. 2564
4	6 50 24.38	+ 9 7 5.5	Leipzig II, A.G. 3348	13	6 44 47.21	+26 24 40.7	Camb., Eng., A.G. 3532
5	6 47 52.43	+10 42 32.4	Leipzig I, A.G. 2557	14	6 45 52.33	+26 30 58.7	Camb., Eng., A.G. 3547
6	6 40 47.65	+14 55 48.8	Leipzig I, A.G. 2477	15	7 14 2.34	+31 32 45.3	Leiden, A.G. 3076
7	6 40 36.82	+15 17 12.8	Berlin A, A.G. 2369	16	7 17 50.84	+31 37 42.6	Leiden, A.G. 3113
8	6 34 13.28	+20 34 27.4	Berlin B, A.G. 2511	17	7 28 46.58	+32 51 10.8	Leiden, A.G. 3192

Comparisons in α were made by transits when marked *t*, otherwise Δα was determined by the micrometer.

OBSERVATIONS OF COMET *b* 1902 (PERRINE),

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By C. W. FREDERICK.

[Communicated by Captain C. M. Chester, U.S.N., Superintendent.]

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 5 11 0 8	1	23, 5	+1 37.17	−5 24.2	8 3 39.73	−33 23 12.4	n7.862	0.922	+2.27	−17.4
5 11 15 47	1	20, 4	+1 32.89	−4 32.7	8 3 35.45	−33 22 20.9	8.483	0.922	+2.27	−17.4
20 9 45 57	2	21, 5	+1 37.64	−2 22.2	6 54 39.38	−18 9 39.2	9.023	0.864	+1.70	−20.7
21 10 50 12	3	d10,10	−0 10.45	+6 34.7	6 52 0.15	−17 20 1.5	9.385	0.846	+1.69	−20.7
22 8 59 15	4	d10,10	−0 26.32	+0 33.5	6 49 48.93	−16 37 52.0	8.538	0.859	+1.67	−20.7
23 10 33 14	5	d10,10	+0 10.96	−7 35.9	6 47 27.49	−15 50 46.5	9.366	0.840	+1.64	−20.6
25 9 33 30	6	27, 6	+1 24.29	−3 34.5	6 43 33.66	−14 29 8.8	9.146	0.843	+1.58	−20.4
26 9 5 50	7	d10,10	+0 9.89	−5 52.0	6 41 48.59	−13 50 22.5	8.982	0.842	+1.56	−20.3
Mar. 1 7 59 51	8	28, 6	+0 44.27	+4 12.9	6 37 12.53	−12 1 37.7	n7.746	0.833	+1.47	−20.1
3 8 58 42	9	d10,10	−0 28.72	−4 55.4	6 34 34.11	−10 53 1.6	9.126	0.822	+1.43	−19.9
4 8 44 47	10	24, 6	+2 12.40	+1 46.8	6 33 25.78	−10 21 28.8	9.065	0.820	+1.39	−19.9
17 8 33 42	11	20, 6	+1 49.37	−1 53.0	6 24 51.16	− 4 54 30.4	9.314	0.778	+1.15	−18.6
18 8 39 40	12	30, 6	+0 48.12	−3 17.5	6 24 34.39	− 4 34 37.5	9.350	0.775	+1.14	−18.6
20 8 17 59	15	26, 6	−1 23.35	−2 44.6	6 24 7.97	− 3 57 13.2	9.301	0.772	+1.12	−18.5
25 7 49 29	16	d10, 5	−0 20.19	−5 41.4	6 23 39.73	− 2 32 48.8	9.266	0.761	+1.03	−17.8
26 8 23 8	18	d10,10	+0 11.91	−0 24.4	6 23 39.79	− 2 17 0.4	9.398	0.757	+1.01	−17.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	8 2 0.29	−33 17 30.8	C.G.C. 10728	10	6 31 11.99	−10 22 55.7	Camb., U.S., A.G.Zones
2	6 53 0.04	−18 6 56.3	Washington, A.G.Zones	11	6 23 0.64	− 4 52 18.8	Strassburg, A.G. Zones
3	6 52 8.91	−17 26 15.5	Washington, A G.Zones	12	6 23 45.13	− 4 31 1.4	Strassburg, A.G. Zones
4	6 50 13.58	−16 38 4.8	Washington, A.G.Zones	13	6 25 57.57	− 3 38 45.9	Strassburg, A.G. Zones
5	6 47 14.89	−15 42 50.0	Washington, A.G.Zones	14	6 25 45.21	− 3 46 56.6	Mic. Comp. with *13
6	6 42 7.79	−14 25 13.9	Washington, A.G.Zones	15	6 25 30.20	− 3 54 10.1	Mic. Comp. with *14
7	6 41 37.14	−13 44 10.2	Camb., U.S., A.G.Zones	16	6 23 58.89	− 2 26 49.6	Strassburg, A.G. Zones
8	6 36 26.79	−12 5 30.5	Camb., U S., A.G.Zones	17	6 22 41.55	− 2 9 2.6	{Strassburg, A.G. Zones} {Nicolajew, A.G. 1691}
9	6 35 1.40	−10 47 46.3	Camb., U.S., A.G.Zones	18	6 23 26.87	− 2 16 18.2	Mic. Comp. with *17.

The second observation by W. W. Dinwiddie. Comparisons in α were determined by the micrometer when marked *d*, otherwise by transits. The comet was best seen at the last observation. Continued cloudy weather prevented further observations.

SEARCHING EPHEMERIS FOR APPEARANCE IN 1903 OF COMET 1896 V.

Continued from *A.J.* 534, as Abridged from M. Ebell's Communication in *A.N.* 3881.

Boston M.T.	Assumed Per. Pass. June 6.5 α	δ	Assumed Per. Pass. June 22.5 α	δ	Br.	Assumed Per. Pass. July 8.5 α	δ
1903	h m s	° ′	h m s	° ′		h m s	° ′
July 16.5	2 32 34	+17 30.6	1 59 36	+17 33.9	2.55	1 18 42	+17 32.9
20.5			2 10 22	+17 57.4			
24.5	2 53 9	+18 2.9	2 20 51	+18 16.6	2.60	1 40 13	+18 29.4
28.5			2 31 1	+18 31.3			
Aug. 1.5	3 12 29	+18 19.7	2 40 51	+18 41.6	2.64	2 0 32	+19 4.7
5.5			2 50 19	+18 47.5			
9.5	3 30 23	+18 21.9	2 59 22	+18 49.1	2.67	2 19 19	+19 19.1
13.5			3 7 59	+18 46.5			
17.5	3 46 40	+18 9.9	3 16 7	+18 39.7	2.69	2 36 12	+19 12.0
21.5			3 23 45	+18 28.8			
25.5	4 1 6	+17 44.9	3 30 50	+18 14.0	2.70	2 50 52	+18 44.3
29.5			3 37 20	+17 55.4			
Sept. 2.5	4 13 30	+17 7.7	3 43 15	+17 33.2	2.70	3 2 55	+17 56.2
6.5			3 48 32	+17 7.4			
10.5	4 23 42	+16 19.6	3 53 10	+16 38.3	2.70	3 12 11	+16 49.6
14.5			3 57 8	+16 6.0			
18.5	4 31 30	+15 21.7	4 0 23	+15 30.8	2.69	3 18 18	+15 25.7
22.5			4 2 55	+14 52.8			
26.5	4 36 44	+14 15.4	4 4 43	+14 12.2	2.66	3 21 24	+13 47.4

Unit of brightness assumed as for 1897 Jan. 4, when last seen. At discovery in 1896 it was 11^M–12^M (Br. = 2.93.)

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ON THE FIFTH SATELLITE OF *JUPITER*,

BY E. E. BARNARD.

The large southern declination of *Jupiter*, and the consequent low altitude of the planet, even when on the meridian, have made it impossible to secure any measures of the fifth satellite from the spring of 1899, until the past summer and fall.

Though it was looked for at different times in the interval, when the conditions were favorable, and the opportunity occurred, it could not be seen.

In the latter part of July, and in August and September of the past year, the satellite was again observed, but it was at all times difficult to measure. Fortunately the elongations occurred when the planet was near the meridian; otherwise it would have been impossible to secure the measures. Even when the seeing was best the satellite was difficult from the low altitude.

The following west elongation times were observed in 1902. They are $6^h\ 0^m$ slow of G.M.T.

July	28^d	$13^h\ 59^m.5$
Aug.	25	11 28.7
Sept.	9	10 6.3

Reducing these to the *Sun* of 1892 Oct. 9, we have

Julian Day	5, 959.590750
	5, 987.490542
	6, 002.434626

The following west elongations were observed in 1892. They are $8^h\ 0^m\ 0^m$ slow of G.M.T.

Oct.	9^d	$16^h\ 3^m$	Nov.	4^d	$13^h\ 44^m$
	17	15 17		11	13 6
	23	14 48		18	12 25
	28	14 26			

These reduced to the elongation of 1892 Oct. 9 give

Julian Day	2, 381.669584	Julian Day	2, 381.670025
	.668656		.669880
	.668628		.664901
	.672942	Mean	2, 381.669231

From the three elongations of 1902, above, and the normal elongation time for 1892 Oct. 9, we get the following ~~three values~~ of the periodic time:

	$0^d.49817899$	7182 periods
	0.49817923	7238 "
	0.49817906	7268 "
Mean	0.49817909	$\pm 0^d.00000006$

The sidereal period of the satellite is, therefore,

$$0^d\ 11^h\ 57^m\ 22^s.6698 \quad \pm 0^s.0052$$

The probable error of a single determination is $\pm 0^s.007$. The mean daily motion in the orbit is, 722°.631636.

The following are comparisons of the predicted times of west elongation from the "*Connaissance des Temps*" for 1902, and the observed times. They are all in Paris mean time.

		C—O
Pred. 1902 July $28^d\ 20^h\ 11^m.4$	Obs'd July $28^d\ 20^h\ 8^m.9$	+2.5
Aug. 25 17 38.4	Aug. 25 17 38.1	+0.3
Sept. 9 16 17.4	Sept. 9 16 15.7	+1.7
	Mean	+1.5

It would seem, therefore, that the satellite was about one and a half minutes ahead of its predicted time, which rested upon the periodic time derived by Dr. COHN.

As in the previous observations of this object, a piece of smoked mica covering one-half of the field was placed in front of the field-lens of the eye-piece -- between it and the wires. This dulled the light of *Jupiter* so that the planet could be distinctly seen and measured, the eye at the same time not being blinded to the feeble light of the satellite which was visible in the unobscured part of the field. It is by this means only that this object can be referred direct to *Jupiter*. I have previously called attention to the fact that my measures of the diameters of *Jupiter* with the 36-inch of the Lick Observatory were made through similar smoked mica. These diameters are therefore specially suited for reducing these observations of the satellite to the center of the planet.

Miss E. E. DOBBIN, of the University of Chicago, has recently undertaken a new determination of the orbit of

this satellite, under the supervision of Dr. KURT LAVES. Miss DOBBIN will use all the measures obtained here with the 40-inch in the years 1898, 1899 and 1902, and will base her work on Dr. COHN's orbit, *A.N.* 3404. Until her important investigation is finished, it has seemed desirable in sending the present list of measures to the *Astronomical Journal*, to make some effort to approximately determine some of the elements of the orbit (by another method than that which she will use), in which the elongation distances alone are taken into account.

In 1894, TISSERAND, from the observations made up to December of 1893, during which time the satellite had been under observation, but little more than a year, computed certain elements of the orbit from the observed elongation distances (*C.R.*, CXIX, No. 15, for 1894 Oct. 8).

The great polar compression of *Jupiter* must produce a large motion in the orbit of this satellite. From the known compression of *Jupiter* TISSERAND computed the amount of this motion, which he found to be 882° a year in a positive direction, or a complete revolution of the orbit in about five months. With this motion, and the elongation distances measured by the writer at the Lick Observatory, he derived the mean distance, the eccentricity and the longitude of the perijove of the satellite's orbit. These elements represented the observations with great exactness.

His elements were

$$a = 47''.906 \quad , \quad e = 0.0073$$

ω_0 (the longitude of the perijove) $= -4°$ epoch 1892 Nov. 1. The daily motion of the orbit used by TISSERAND was $\omega_1 = +2°.42$.

No matter how well the orbit of this satellite may be determined, it is impossible, on account of the rapid motion of the line of apsides, to get a fair representation of the observations unless the motion of the perijove has been well determined. This very fact also militates against a satisfactory determination of the orbit from observations extending over any considerable period of time.

The values of the polar compression of *Jupiter* from the various measures of his diameters are very discordant. There is, furthermore, a characteristic difference between the compression derived from filar-micrometer and from heliometer measures amounting to about one unit in its value.

Not being satisfied with the way in which my measures with the 40-inch refractor were represented in 1898 and 1899 by TISSERAND's elements, I varied his motion of the perijove until these observations were satisfied. At the same time the measures of 1892 and 1893 were, if anything, better represented than with TISSERAND's motion (see *A.J.* 472, Vol. XX, p. 126). It was then shown that by accelerating the motion of the line of apsides to 900° a year a very close agreement was obtained between observation and theory over the entire six or seven years. The observed elongation distances up to 1902 July 28 are represented with great exactness with this combination of TISSERAND's elements and my value of the apsidal motion. But for the other measures of 1902, with the exception of Oct. 7, which is closely represented, the results are disappointing, and I believe the discordances are not warranted by the character of the observations.

A comparison of various ephemerides (computed from different elements) with the observations, shows that TISSERAND's elements with his motion of the line of apsides, give a fair representation of all the observations. The agreement between observation and calculation is remarkable, with the exception of the measures of 1899. But the observations of 1899 are, I believe, very exact, especially the elongation distance of May 1, which is combined with that of April 25, with which it is accordant, to form the normal of April 28.

In *A.N.* 3403–4, Dr. FRITZ COHN, using the observations made by HERMANN STRUVE at Pulkowa, and by the writer at the Lick Observatory up to the end of 1894, made a thorough investigation of the orbit of the satellite. He did not derive the mean distance from the elongation distances, as was done by TISSERAND, but determined it from a modification of KEPLER's third law in which he took into account the motion of the line of apsides.

Of these elements, the values with which we are concerned, are

$$\begin{aligned} a &= 48''.065 \\ e &= 0.00501 \\ \omega_0 &= 207°.2 \quad \text{Epoch 1892 Nov. 1} \end{aligned}$$

The motion of the perijove determined by him was 911°.7 a year, or $+2°.4961$ daily.

This motion differed from TISSERAND's value by 30° a year, while the longitude of the perijove was 149° less than TISSERAND's. It is evident that one or the other of these computations must be very badly out.

I have shown that with an increase of 18° annually in TISSERAND's motion his other elements closely represented the place of the satellite up to the end of the observations in 1899, while those of COHN gave badly discordant results. A comparison of the residuals (O—C) from an ephemeris of each is given in the following table:

	TISSERAND	COHN	COHN, using TISSERAND's ω_0
	$''$	$''$	$''$
1892 Sept. 12	+0.03	+0.25	−0.07
Oct. 8	−0.11	+0.27	−0.16
19	−0.11	−0.66	−0.36
26	+0.07	+0.15	−0.02
Nov. 10	+0.08	−0.22	−0.08
12	+0.02	+0.05	−0.13
1893 Sept. 27	−0.07	−0.65	−0.25
Nov. 12	+0.03	−0.29	−0.23
Dec. 10	0.00	+0.29	−0.13
1894 Dec. 3	+0.15	−0.03	+0.12
1898 Mar. 15	0.00	−0.14	+0.14
1899 May 6	+0.09	+0.30	+0.08
June 16	+0.05	−0.25	+0.18

It would seem from this comparison that TISSERAND'S results are nearer the truth. If, however, we reject COHN'S longitude of the perijove and substitute, instead, TISSERAND'S value, there is a great improvement in the results, as will be seen from the residuals in last column above.

This would seem to show that COHN'S longitude of the perijove is wrong, and this is further proved to be true later on.

In trying to get a satisfactory agreement between calculation and observation, I have computed nearly twenty ephemerides from different elements, several sets of which have been computed new, or by varying portions of them.

The results in general would almost leave one in doubt as to what the motion of the orbit really is.

Some of these comparisons may be of interest. A few of them are therefore incorporated in the following table:

TABLE OF RESIDUALS FROM VARIOUS ELEMENTS.

	1	2	3	4	5	6	7	8	9	10	11
	″	″	″	″	″	″	″	″	″	″	″
1892 Sept. 12	+0.03	+0.12	0.00	+0.25	+0.02	+0.01	−0.01	+0.01	−0.01	−0.06	−0.02
Oct. 8	−0.11	+0.02	−0.11	+0.27	−0.11	−0.01	−0.15	−0.04	−0.03	−0.13	−0.07
19	−0.11	−0.27	−0.36	−0.65	−0.11	−0.31	−0.11	−0.25	−0.23	−0.16	−0.17
26	+0.06	+0.11	−0.02	+0.15	+0.07	+0.14	+0.05	+0.14	+0.17	+0.11	+0.15
Nov. 10	+0.09	0.00	−0.12	−0.22	+0.07	+0.07	+0.07	+0.09	+0.14	+0.13	+0.15
12	+0.02	+0.05	−0.06	+0.06	+0.03	−0.05	0.00	−0.03	−0.09	−0.06	−0.03
1893 Sept. 27	−0.07	−0.23	−0.34	−0.62	−0.16	−0.21	−0.16	−0.17	−0.08	−0.24	−0.21
Nov. 12	+0.03	−0.08	−0.17	−0.29	+0.11	−0.35	+0.10	−0.01	−0.01	+0.15	+0.13
Dec. 10	0.00	+0.10	−0.02	+0.29	+0.09	+0.26	+0.07	+0.07	+0.04	+0.15	+0.16
1894 Dec. 3	+0.15	+0.13	0.00	−0.07	−0.02	+0.06	−0.04	+0.05	−0.02	−0.02	+0.03
1898 Mar. 4	−0.10	0.00	−0.13	−0.04	+0.34	+0.07	+0.18	+0.03	−0.01	+0.01	+0.03
Apr. 5	+0.32	+0.18	+0.07	−0.17	−0.12	−0.03	+0.16	−0.09	−0.06	−0.19	−0.13
1899 Apr. 28	+0.16	+0.28	+0.16	+0.38	+0.72	+0.54	+0.42	+0.58	+0.57	+0.06	+0.10
May 23	+0.07	+0.19	+0.06	+0.12	+0.42	+0.36	+0.39	+0.35	+0.30	+0.02	+0.07
June 16	+0.05	+0.01	−0.12	−0.28	−0.17	−0.10	−0.20	−0.06	−0.15	−0.19	−0.19
1902 July 28	−0.03	−0.08	−0.20	−0.32	0.00	−0.01	+0.01	+0.03	−0.08	+0.03	+0.04
Aug. 5	+0.22	+0.11	0.00	+0.13	+0.01	+0.05	+0.17	+0.02	−0.04	+0.04	+0.06
25	+0.58	+0.36	+0.27	+0.20	−0.07	+0.03	+0.22	0.00	−0.01	−0.08	−0.03
Sept. 9	+0.61	+0.40	+0.31	+0.31	−0.05	+0.18	+0.24	0.00	+0.04	−0.10	−0.05
Oct. 7	+0.07	+0.05	−0.06	+0.04	+0.04	−0.01	+0.03	0.00	+0.10	−0.03	0.00

Elements used for the ephemerides from which the above table of residuals was determined:

	a	e	ω_0	ω_1	Epoch
	″		° ′	°	
1	47.906	0.0073	− 4 –	+2.466	1892 Nov. 1
2	47.945	0.0036	−18 30	+2.466	" "
3	48.060	0.0041	−18 30	+2.466	" "
4	48.065	0.00501	207 12	+2.4961	" "
5	47.906	0.0073	− 4 –	+2.42	" "
6	47.958	0.00426	7 27	+2.4148	" "
7	47.921	0.00769	20 58	+2.42	1897 Oct. 1
8	47.932	0.00542	31 44	+2.4175	1902 Aug. 27
9	47.917	0.00578	48 54	+2.42052	" "
10	47.921	0.00768	20 57	+2.414784	1897 Oct. 1
11	47.903	0.00687	22 12	+2.414784	" "

Of these elements 4 and 5 are those of COHN and TISSERAND, respectively. No. 9 was determined by computing corrections to 8 from the residuals given by 8. No. 3 was computed from the same observations as for 2, with the semi-diameter of *Jupiter* increased by 0″.12, in an endeavor to get an agreement between theory and observation in the mean distance.

I have adopted 11 as final, so far as the present observations are concerned.

In a delicate problem of this kind, it is useless to combine observations by different observers, for some of the quantities sought will be masked by the personalities of the observers. It is therefore best to depend on the work of one observer alone, in the hope that the consistency of his measures may in the end more nearly attain to the truth.

With the exception of the measures of HERMANN STRUVE, there are almost no other observations but my own. To avoid the effects of personal equation, it has appeared best to use only my measures. These seem, at least, to have the merit of consistency.

It seems probable that by this time the motion of the orbit can be determined with great exactness from the observations of elongation alone. The measures of 1892 are sufficiently numerous and exact to determine the longitude of the perijove during that year with a small amount of error. In the last few years the low altitude of *Jupiter*, for one thing, has not permitted a sufficient number of observations of elongation distances to very exactly locate the perijove. It is probable that the observations which will be made at the coming opposition of *Jupiter* will be enough when combined with those of last year, to determine its position very closely, and this combined with the position in 1892 will give a very exact value for the motion. I have thought, however, that after all, one might be able

to determine the apsidal motion closely with the material already on hand. With this point in view the following investigations have been made.

TISSERAND has given the following formulas which were used by him, in "*Comptes Rendus*," Tome CXIX, p. 583.

For the computation of the elongation distances:

$$(1) \qquad \begin{aligned} r_0 &= a - ae \sin(l - \omega_0 - \omega_1 t) \\ r_e &= a + ae \sin(l - \omega_0 - \omega_1 t) \end{aligned}$$

For the eccentricity and the longitude of the perijove:

$$(2) \qquad ae \cos \omega_0 = x \quad , \quad ae \sin \omega_0 = y$$

For the equations of condition:

$$(3) \qquad \begin{aligned} r_0 &= a - x \sin(l - \omega_1 t) + y \cos(l - \omega_1 t) \\ r_e &= a + x \sin(l - \omega_1 t) - y \cos(l - \omega_1 t) \end{aligned}$$

In these r_0 and r_e are the west and east elongation distances, and

l = the geocentric longitude of *Jupiter*.
ω_1 = the daily motion of the perijove.
ω_0 = the longitude of the perijove.
t = the time interval from the epoch.

From the measures of 1902 I have deduced the following elongation distances. In all these the satellite was to the west of the planet, that is, preceding. These are reduced to the distance 5.20.

	Observed	Computed	O—C
	″	″	″
1902 July 28	47.94	47.96	−0.02
Aug. 5	48.07	48.05	+0.02
25	48.18	48.20	−0.02
Sept. 9	48.17	48.18	−0.01
Oct. 7	47.92	47.92	0.00

Though these measures do not cover a revolution of the orbit of the satellite, I have thought they might give some idea of the orbit, and especially of the position of the perijove. Adopting the epoch 1902 Aug. 27, and with a daily motion of $+2°.42$ the following equations of condition were formed by the aid of eq. (3):

$$\begin{aligned} 47''.94 &= a - 0.44\,x + 0.90\,y \\ 48.07 &= a - 0.09\,x + 1.00\,y \\ 48.18 &= a + 0.79\,x + 0.61\,y \\ 48.17 &= a + 0.99\,x + 0.14\,y \\ 47.92 &= a + 0.47\,x - 0.88\,y \end{aligned}$$

These give the following normal equations:

$$\begin{aligned} 240.28 &= 5.00\,a + 1.72\,x + 1.77\,y \\ 82.85 &= 1.72\,a + 2.02\,x - 0.28\,y \\ 85.18 &= 1.77\,a - 0.28\,x + 2.97\,y \end{aligned}$$

and these give:

$$x = +\ 0.221 \qquad y = +\ 0.137 \qquad a = 47''.932$$

From these

$$\omega_0 = 31°\ 44' \quad \text{Epoch 1902 August 27} \qquad e = 0.00542$$

These elements closely represent the observations used, as will be seen by the residuals (O—C) in the above table.

These also closely represent all the other observations back to 1892, with the exception of those of 1899, as will be seen by the following table of residuals:

	″		″
1892 Sept. 12	+0.01	1893 Dec. 10	+0.07
Oct. 8	−0.04	1894 Dec. 3	+0.05
19	−0.25	1898 Mar. 4	+0.03
26	+0.14	Apr. 5	−0.09
Nov. 10	+0.09	1899 Apr. 28	+0.58
12	−0.03	May 23	+0.35
1893 Sept. 27	−0.17	June 16	−0.06
Nov. 12	−0.01		

The position of the perijove as given by this last orbit ought to enable one to determine the motion of the line of apsides very closely; for, combining it with TISSERAND's position in 1892, there will be an interval of ten years, which ought to reduce any error in the motion so determined to a small quantity.

The longitude of the perijove from this orbit is $+31°.7$. If we combine this with TISSERAND's value we shall get a daily motion of the orbit of $+2°.4200$, which is exactly the value derived by TISSERAND. It is equal to $883°.9$ a year.

At the suggestion of Mr. W. S. ADAMS, who kindly supplied me with the formula for the purpose, equations of condition were formed from the residuals given by this last orbit for a determination of corrections to that orbit. In this manner the elements (No. 9) of the preceding table of elements were obtained. They show a slight improvement over the first one, but I do not feel satisfied with the results.

With the above value of the apsidal motion ($+2°.4200$) and the following observations, assuming the epoch 1897 Oct. 1, a new set of elements was computed.

	″		
1892 Sept. 12	48.11	(4)	East
Oct. 8	48.14	(4)	East
19	47.51	(4)	West
26	48.19	(3)	East
Nov. 10	47.98	(3)	East
12	47.97	(3)	West
1893 Sept. 27	47.67	(3)	East
Nov. 12	47.74	(2)	East
Dec. 10	48.12	(1)	East
1894 Dec. 3	48.17	(1)	East
1898 Mar. 15	48.12	(3)	East
1899 May 6	48.31	(3)	East
June 16	48.03	(1)	East
1902 Aug. 1	47.96	(2)	West
Sept. 1	48.17	(2)	West
Oct. 7	47.92	(1)	West

Equations of condition were formed from these observations, which by the method of least-squares gave the following normal equations:

$$\begin{aligned} 768''.11 &= +16.00\,a + 2.68\,x + 3.39\,y \\ 130.74 &= +\ 2.68\,a + 7.09\,x - 0.99\,y \\ 163.28 &= +\ 3.39\,a - 0.99\,x + 8.86\,y \end{aligned}$$

The solution of these gave

$$x = +0.3444$$
$$y = +0.1318$$
$$a = 47''.921$$

From these, by eq. (2),

$$\omega_0 = +20°\ 58' \quad \text{Epoch 1897 Oct. 1}$$
$$e = 0.00769$$

If we combine the longitudes of the perijove derived from the orbits of 1897 Oct. 1 and 1902 Aug. 27, we get a daily motion of $+2°.4194$ or $883°.7$ annually.

Reducing the two values of the longitude of the perijove to 1892 Nov. 1, using a motion of $+2°.4200$, for comparison with TISSERAND's, we have

Tisserand,	1892 Nov. 1	$\omega_0 = -4°$
Orbit of 1897 Oct. 1	"	$\omega_0 = -3°$
Orbit of 1902 Aug. 27	"	$\omega_0 = -4°$

The close agreement of the middle one of these with the others is doubtless accidental. The first and last should agree of course, for the motion was derived from them.

Several different sets of elements were computed from the above observed elongation distances. These in general gave somewhat different results, depending on the assumed epoch and motion of the orbit.

Forming equations of condition from the same set of observations with a motion of 882° a year, and the epoch 1892 Nov. 1, the following normal equations resulted:

$$768''.11 = +16.00\,a + 3.48\,x + 2.57\,y$$
$$168.19 = +\ 3.48\,a + 6.43\,x - 0.28\,y$$
$$123.45 = +\ 2.57\,a - 0.28\,x + 9.57\,y$$

These gave

$$a = 47''.958$$
$$x = +\ 0.2024$$
$$y = +\ 0.0265$$
$$\omega_0 = +7°.4 \quad \text{Epoch 1892 Nov. 1}$$
$$e = 0.00426$$

With $\omega_1 = 900°$ a year, and the epoch 1897 Oct. 1, equations of condition were formed from the preceding observations, from which resulted the following normal equations;

$$768''.11 = +16.00\,a + 0.45\,x - 5.79\,y$$
$$21.61 = +\ 0.45\,a + 10.51\,x - 0.48\,y$$
$$-278.54 = -\ 5.79\,a - 1.28\,x + 5.48\,y$$

The solution of these gave

$$a = 47''.944$$
$$x = -\ 0.0046$$
$$y = -\ 0.173$$
$$\omega_0 = -88°.5 \quad \text{Epoch 1897 Oct. 1}$$
$$e = 0.00365$$

If this value of ω_0 were carried back to 1892 Nov. 1 with the motion of 900° a year its longitude would be $-18°.5$, which is 14° from TISSERAND's value.

As will be seen later this last set of elements is badly influenced by the large motion of the perijove which was used.

After much experimenting I decided to separate some of the normal observations because the interval between them was too great for a simple mean to be taken. This gave twenty observations of elongation distance. The observations of 1898 and 1899 were also corrected for the final value of the micrometer screw.

I also decided to reject my motion of the perijove, and to adopt one nearly in accord with the value derived from a comparison with TISSERAND's elements, and my elements from the observations of 1902 alone.

These observations, all of which were made with either the 36-inch or the 40-inch refractors, are given in the following table:

Date	Observations		Computed Elong. Dist.	Residuals O—C
	"		"	"
1892 Sept. 12	48.11 (4)	E	48.129	−0.02
Oct. 8	48.14 (4)	E	48.214	−0.07
19	47.51 (4)	W	47.680	−0.17
26	48.19 (3)	E	48.041	+0.15
Nov. 10	47.98 (3)	E	47.829	+0.15
12	47.97 (3)	W	48.005	−0.03
1893 Sept. 27	47.67 (3)	E	47.885	−0.21
Nov. 12	47.74 (2)	E	47.614	+0.13
Dec. 10	48.12 (1)	E	47.958	+0.16
1894 Dec. 3	48.17 (1)	E	48.140	+0.03
1898 Mar. 4	48.08 (2)	E	48.048	+0.03
Apr. 5	48.08 (1)	E	48.210	−0.13
1899 Apr. 28	48.27 (2)	E	48.167	+0.10
May 23	48.27 (1)	E	48.197	+0.07
June 16	47.99 (1)	E	48.176	−0.19
1902 July 28	47.94 (1)	W	47.896	+0.04
Aug. 5	48.07 (1)	W	48.012	+0.06
25	48.18 (1)	W	48.213	−0.03
Sept. 9	48.17 (1)	W	48.216	−0.05
Oct. 7	47.92 (1)	W	47.924	0.00

Assuming the epoch 1897 Oct. 1, which falls near the middle of the series, and adopting a motion of the perijove of $+2.414784$ daily, which I have found by experiment to be close to the true motion, we have with formula (3) the following equations of condition:

$48''.11 = a + 0.36\,x + 0.93\,y$	$48''.08 = a + 0.07\,x + 1.00\,y$
$48.14 = a + 1.00\,x + 0.05\,y$	$48.08 = a + 1.00\,x + 0.08\,y$
$47.51 = a - 0.91\,x + 0.42\,y$	$48.27 = a + 0.52\,x + 0.86\,y$
$48.19 = a + 0.73\,x - 0.68\,y$	$48.27 = a + 1.00\,x - 0.08\,y$
$47.98 = a + 0.16\,x - 0.99\,y$	$47.99 = a + 0.56\,x + 0.83\,y$
$47.97 = a - 0.07\,x + 1.00\,y$	$47.94 = a - 0.40\,x + 0.92\,y$
$47.67 = a + 0.33\,x - 0.95\,y$	$48.07 = a - 0.05\,x + 1.00\,y$
$47.74 = a - 0.99\,x + 0.11\,y$	$48.18 = a + 0.74\,x + 0.67\,y$
$48.12 = a - 0.22\,x + 0.98\,y$	$48.17 = a + 1.00\,x + 0.08\,y$
$48.17 = a + 0.93\,x - 0.37\,y$	$47.92 = a + 0.44\,x - 0.90\,y$

NORMAL EQUATIONS.

$$960''.57 = +20.00\,a + 6.20\,x + 4.96\,y$$
$$299.62 = +\ 6.20\,a + 9.00\,x - 0.97\,y$$
$$238.68 = +\ 4.96\,a - 0.97\,x + 11.06\,y$$

Solving these normal equations we have

$$\begin{aligned} x &= +0.30447 \\ y &= +0.12432 \\ a &= 47''.9033 \\ \omega_0 &= 22^\circ\, 12' \quad \text{Epoch 1897 Oct. 1} \\ e &= 0.0068655 \end{aligned}$$

The smallness of the residuals and the nearly equal distribution of the signs, viz.: -0.87 and $+0.82$ are very satisfactory, and would seem to show that both the orbit and its motion are closely determined.

I would therefore take the following elements and motion as the finally adopted values for the orbit of the satellite from the elongation distances observed by me.

Elements.

Mean distance $= a$	$= 47''.903$ (at Δ 5.20)
Eccentricity $= e$	$= 0.006866$
Longitude of perijove $= \omega_0$	$= 22^\circ.0$ Epoch 1897 Oct. 1
Daily motion of the line of apsides $= \omega_1$	$= +\ 2^\circ.414784$
Annual " " " "	$= +82^\circ.0$
Sidereal period of the satellite	$= 0^d\ 11^h\ 22^m.6698 \pm 0^m.0052$
Mean daily motion in the orbit	$= 722^\circ.631636$
Mean distance from theory	$= 48''.066$

The above values for the mean distance of the satellite would give

From the observed elongations $= 112300$ miles
From the theoretical value $= 112670$ "

It has not been deemed necessary to give the time of the epoch closer than to the nearest day.

In the early observations of the satellite I pointed out the fact that the measured elongation distances showed the orbit to be eccentric. Though this eccentricity is small, it is clearly indicated in the observations; from a mere glance at these one can form some idea of the amount of the eccentricity.

The various determinations of the mean distance from my observations of elongation do not vary much from $47''.92$. Assuming this value we can deduce a close approximation to the eccentricity of the orbit from a simple inspection of the measures; taking the mean of all the elongation distances that fall below $47''.80$, and of all those that fall above $48''.10$ as containing the least and greatest distances of the satellite, the eccentricity will be determined by the formulas,

$$e = 1 - \frac{r}{a} \quad , \quad e = \frac{r'}{a} - 1$$

The observations give

$$r = 47''.606 \text{ (8 obs.)} \quad , \quad r' = 48''.215 \text{ (17 obs.)}$$

From the formulas the first gives $e = 0.00655$ (wt. 8)
the second gives $e = 0.00610$ (wt. 17)

The mean of these is $e = 0.00625$

This is in good accord with the adopted value.

In deriving the mean distance of the satellite from the measures, the personality of the observer enters, and no matter how consistent his measures may be there will nevertheless be a small error introduced into the mean distance. The combination of measures by different observers may lessen this uncertainty, and if there are very few observers, as in the present case, it may increase the error.

It therefore needs some method to determine this quantity in which the personality of the observer does not enter.

Dr. Cohn has shown (*A.N.* 3404, p. 322) that the semi-major axis of the orbit of the satellite can be better determined theoretically, from the periodic time, by the aid of Kepler's third law. He gives the following formula:

$$\sin^3(\Delta) = \frac{1}{\mu}\left(\frac{N}{n}\right)^2 \frac{1}{(\rho)^3}\left(1 + \frac{\delta P}{n}\right)(1 - \Sigma m)$$

where (Δ) is the semi-major axis of the satellite's orbit.

$\frac{1}{\mu}$ = the mass of *Jupiter* $= \frac{1}{1047.35}$ (Newcomb; *A.N.* 136, pp. 133–134).
$(\rho) = 5.20280$, The mean distance of *Jupiter* from the sun.
$N = 59'.14$, The mean daily motion of the earth.
$n = 43356'$, The mean daily motion of the satellite.
δP = the annual motion of the line of apsides of the orbit in minutes of arc.
$\Sigma m = 0.00017$, The sum of the masses of the four old satellites, according to Tisserand (*Méc. Cél.*, T. IV, p. 81).

n in the second parenthesis of the above formula is multiplied by 365.25.

From this formula, using $\delta P = 911^\circ.7$, his value for the motion of the perijove, Dr. Cohn gets for the semi-major axis of the orbit, $(\Delta) = 48''.065$.

(An actual computation seems to give $48''.068$ instead of the above quantity.)

If the annual motion is assumed to be 900° this formula would give $(\Delta) = 48''.067$; or, using a value of 884°, $(\Delta) = 48''.066$.

From these it will be seen that a considerable variation in δP has very little effect on the mean distance. Hence any uncertainty in the motion of the line of apsides will not materially affect the resulting mean distance.

The mean distance derived from this formula is doubtless very exact, and is to be preferred to the value derived from the elongation distances. The difference between theory and observation in this case, considering the difficulty of the object, is small. This difference is in part due to errors of observation, and in part to a small error, perhaps, in the diameter used to reduce the measures.

I have long ago called attention (*A.J.* 325) to the discordance between micrometer and heliometer measures of the diameters of *Jupiter*, and have shown that the former are uniformly about one second of arc the greater. The values for the diameters of the planets adopted in the almanacs have in the main depended on heliometer measures. In the case of *Jupiter* it would be fatal to use such

"standard values" for the reduction of my measures of this satellite, for they would make a discordance some four or five times as great as those mentioned above. No other diameters, therefore, but those determined by me should ever be used in reducing my observations of this satellite. This statement should emphasize the necessity of the greatest caution in the reduction of such observations by the indiscriminate use of any adopted value of the diameter.

The inference drawn from the preceding investigations would lead to the following conclusion.

The daily motion of the line of apsides is close to +882° a year, which is the value derived by TISSERAND from theoretical considerations.

Dr. COHN's motion of the orbit is evidently very much too large.

The longitude of the perijove for 1892 Nov. 1 was within a few degrees of 360°, which is also very near the value assigned by TISSERAND. The position assigned it by COHN must therefore be in error pretty nearly a half revolution of the orbit.

If we give preference to the theoretical determination of the mean distance, it will be very close to 48″.07 at the mean distance of *Jupiter*, while the value from my measures will be 47″.90, which is almost identical with the value derived by TISSERAND.

From the close representation of all the observations, extending through ten years, it would seem that the orbit has suffered no noticeable change in the interval.

In conclusion, I am indebted to Mr. W. S. ADAMS for valuable advice, and for a kindly interest in the subject.

NOTE: An elongation distance of the satellite observed 1903 July 21 gives a residual from elements No. 11 of —0″.07.

OBSERVATIONS OF THE FIFTH SATELLITE OF JUPITER IN 1902.

July 21.

h m s	Dist. from pr. limb ″	Dist from center ″	Comp.
13 39 43	28.18	52.69	3
13 43 17	29.83	54.33	3
13 47 40	79.90*	55.40	3
13 53 12	31.76	56.26	3

Position angle of the wires = 161°.6.

* From following limb.

July 28.

h m s	From pr. limb. ″	From center ″	Comp.
12 41 47	72.62*	47.98	3
12 44 27	73.09*	48.44	3
12 47 15	24.66	49.31	3
12 49 39	25.37	50.02	3
12 51 49	25.86	50.51	3
12 54 34	26.86	51.51	3
12 56 52	26.80	51.45	2
12 58 22	28.54	53.19	2
13 0 19	76.95*	52.30	2
13 3 3	78.77*	54.12	3
13 5 51	29.86	54.50	3
13 7 39	30.34	54.99	2
13 14 20	32.19	56.83	3
13 20 11	33.06	57.71	3
13 24 43	33.81	58.46	3

Position angle of the wires at the longitude measures = 161°.5.

* Measured from following limb.

LATITUDE MEASURES.

From South Limb.

h m s	″	Latitude.	
13 30 7	23.30	+0.19	4

From North Limb.

13 33 59	22.88	+0.23	5

	From pr. limb.	From center.	Camp.
13 38 55	35.75	60.40	3
13 41 3	36.01	60.66	3
13 43 17	36.10	60.75	3
13 45 32	36.26	60.90	3
13 47 24	86.22*	61.57	2
13 50 24	85.98*	61.33	3
13 52 0	36.68	61.33	3
13 54 25	36.81	61.46	3

LATITUDE MEASURES — Cont.

h m s	From pr. limb. ″	From center. ″	Comp.
13 56 25	36.64	61.29	3
13 59 9	36.72	61.37	3
14 1 45	36.15	60.80	3
14 3 58	36.59	61.24	3
14 7 20	36.52	61.17	3
14 10 26	36.23	60.87	3
14 14 9	36.44	61.09	3
14 17 19	36.02	60.67	3
14 20 27	35.07	59.71	3
14 23 54	35.35	59.99	3
14 27 25	35.39	60.04	3
14 30 19	34.26	58.91	3
14 33 10	34.05	58.70	3
14 36 32	33.75	58.30	3
14 39 34	32.93	57.58	2
14 41 39	32.12	56.77	2

Position angle of the wires at the latitude measures = 71°.5.

* Measured from following limb.

TIMES OF ELONGATION COMPUTED FROM THE OBSERVATIONS.

	Before Elongation.		After Elongation.
	13 58.8		13 59.9
	13 59.8		14 0.7
	13 59.7		13 59.7
	13 59.9		13 57.5
Mean	13 59.3	Mean	13 59.4

Aug. 5.

h m s	From pr. limb. ″	From center. ″	Comp.
12 51 18	35.51	60.21	2
12 56 16	35.29	59.99	2
12 59 20	36.11	60.81	2
13 9 36	36.71	61.41	3
13 13 1	87.25*	62.54	3
13 15 6	87.46*	62.76	2
13 17 13	36.74	62.03	3
13 20 6	36.94	61.64	3
13 22 43	36.80	61.51	3
13 25 41	36.40	61.10	3
13 27 28	36.19	60.89	2
13 28 53	36.36	61.06	2
13 31 26	36.79	61.49	3
13 34 51	35.85	60.55	3
13 38 32	35.88	60.58	3
13 41 23	35.87	60.57	4

Position angle of the wires = 161°.0.

* From following limb.

Aug. 21.

h m s	From pr. limb. ″	From center. ″	Comp.
10 43 58	27.15	51.63	3
10 48 25	28.35	52.82	3
10 52 30	29.56	54.03	3
10 55 25	30.84	55.32	3
10 58 29	31.73	55.21	3
11 8 59	33.22	57.70	3
11 12 11	33.27	57.74	3
11 15 10	35.16	59.64	3
11 18 15	34.51	58.99	3
11 20 32	34.62	59.10	3
11 23 43	35.31	59.78	3
11 28 2	35.36	59.83	3
11 31 43	36.19	60.66	3
11 34 41	36.60	61.08	3
11 38 0	36.46	60.93	3
11 41 31	36.63	61.11	3
11 44 17	36.87	61.35	3
11 47 30	37.11	61.59	2

Position angle of the wires = 162°.8.

In these observations there is possibly an uncertainty of one minute in the recorded times caused by the stopping of the watch before comparison was made with standard clock. It is probable the error will be only a few seconds at most.

Aug. 25.

h m s	From pr. limb. ″	From center. ″	Comp.
10 38 42	31.01	55.36	3
10 41 26	31.67	56.03	3
10 44 39	31.92	56.28	3
10 47 46	32.52	56.88	3
10 49 47	32.84	57.20	3
10 53 52	33.37	57.73	3
10 56 42	34.27	58.63	3
10 59 18	34.75	59.10	3
11 2 17	34.66	59.02	3
11 5 16	34.76	59.12	3
11 8 32	35.24	59.60	3
11 11 29	35.61	59.97	3
11 15 22	35.57	59.93	3
11 18 49	36.33	60.69	3
11 22 30	36.52	60.88	3
11 27 27	36.89	61.25	3
11 30 14	36.35	60.71	3
11 34 42	36.58	60.94	3
11 37 17	36.15	60.51	3
11 40 14	36.10	60.46	3
11 43 24	35.92	60.27	3

Aug. 25 — Cont.

	From pr. limb.	From center.	Comp.
h m s	″	″	
11 45 46	35.60	59.96	3
11 48 6	35.36	59.71	3
11 51 32	34.47	58.83	3
11 54 44	34.83	59.19	3
11 57 11	34.08	58.44	3
12 0 14	33.97	58.33	3
12 4 54	33.73	58.09	4

Position angle of the wires = 162°.1.

Sept. 9.

	From pr. limb.	From center.	Comp.
h m s	″	″	
9 21 27	31.04	54.73	3
9 24 52	31.06	54.75	3
9 27 45	32.22	55.92	3
9 30 43	32.43	56.12	3
9 32 46	32.74	56.43	3
9 34 18	33.55	57.24	3
9 35 48	33.33	57.02	3
9 37 31	33.37	57.06	3
9 39 34	34.42	58.11	3
9 42 18	34.56	58.25	3
9 45 7	34.48	58.17	3
9 47 36	35.18	58.87	3
9 49 17	34.85	58.54	3
9 50 50	35.24	58.93	3
9 52 17	34.85	58.54	3
9 54 21	35.38	59.08	4
9 58 8	35.72	59.41	3
9 59 44	35.22	58.91	3
10 1 20	35.28	58.97	3
10 2 44	35.67	59.37	3
10 4 25	35.65	59.34	3
10 6 7	35.65	59.35	3

Sept. 9 — Cont.

	From pr. limb.	From center.	Comp.
h m s	″	″	
10 7 26	35.06	58.75	3
10 8 52	35.12	58.81	3
10 10 48	36.12	59.81	3
10 12 33	35.18	58.87	3
10 14 5	35.71	59.40	3
10 15 35	35.16	58.85	3
10 17 17	35.33	59.02	3
10 18 48	35.32	59.01	3
10 20 10	35.45	59.14	3
10 21 29	35.09	58.79	3
10 23 11	35.08	58.77	3
10 24 52	34.62	58.31	3
10 26 28	34.61	58.30	3
10 27 55	34.60	58.29	3
10 30 7	34.38	58.07	3
10 34 34	33.76	57.45	3
10 37 1	33.16	56.85	3
10 38 14	33.24	56.93	3
10 39 28	33.29	56.98	3
10 40 50	33.17	56.86	3
10 42 53	32.31	56.00	4
10 48 40	31.76	55.45	3
10 50 23	31.42	55.11	3
10 52 32	30.71	54.40	3
10 55 20	30.19	53.89	3
10 57 17	30.01	53.70	3
10 59 23	29.47	53.16	4

Position angle of the wires at the longitude measures = 162°.7.

Latitude Measures.

From South Limb.

		Latitude.	
h m s	″	″	
11 5 45	22.02	−0.19	5

From North Limb.

h m s	″	″	
11 10 57	22.28	−0.07	4
11 12 45	23.05	−0.84	3
11 14 12	22.85	−0.64	3

From South Limb.

11 16 43	21.50	−0.70	3
11 19 38	22.43	+0.22	3

Position angle of the wires at the latitude measures = 72°.8.

The observations were recorded by Mr. W. S. Adams.

This date (Sept. 9) was the tenth anniversary of the discovery of the Fifth Satellite.

Oct. 7.

Satellite and pr. limb.

	From pr. limb.	From center.	Comp.
h m s	″	″	
7 30 45	32.53	54.47	3
7 33 20	32.98	54.92	3
7 35 49	32.94	54.88	3
7 38 27	32.24	54.18	3
7 41 21	32.38	54.32	3
7 45 48	32.70	54.64	3
7 51 28	32.38	54.32	3
7 54 54	31.79	53.73	3
7 58 17	32.39	54.33	3
8 2 31	31.88	54.81	3
8 5 4	31.49	53.43	3
8 7 38	30.86	52.80	3
8 10 21	30.49	52.43	3
8 13 35	30.93	52.87	3
8 16 57	29.99	51.93	3
8 20 18	30.22	52.16	2

Position angle of the wires = 163°.3

The observations were carefully plotted, and the following elongation distances determined from the curves thus obtained:

	Apparent	Δ 5.20
	″	″
1902 July 28	61.35	47.94
Aug. 5	61.65	48.07
25	60.92	48.18
Sept. 9	59.25	48.17
Oct. 7	54.58	47.92

In all the observations the given times are $6^h\ 0^m$ slow of Greenwich M.T.

The satellite was preceding in every case.

The measures all depend upon a value for the micrometer screw of 9″.665.

The following apparent semi-diameters of *Jupiter* were used to reduce the observations:

	Equatorial	Polar
	″	″
1902 July 21	24.503	
28	24.649	23.107
Aug. 5	24.704	
21	24.476	
25	24.359	
Sept. 9	23.691	22.209
Oct. 7	21.939	

They are derived from the diameters given in *A.J.* 325.

The following are corrections to the paper in *A.J.*, No. 472, Vol. XX.

p. 126, in the table, *for* 1892 Nov. 11 *read* Nov. 12
p. 127, *for* 1898 March 6 $11^h\ 56^m\ 5^s$ *read* $11^h\ 55^m\ 5^s$.
p. 128, 1898 April 26 in the three last distances,

for	47″.88	*read* 48″.00
	47.47	47.59
	46.14	46.26

p. 128, 1899 May 1, first observation *for* 43″.38 *read* 42″.38.
p. 129, 1899 May 23, at $10^h\ 53^m\ 2^s$ *for* 43″.76 *read* 32″.76.
p. 128, 1898 April 26, the latitude observations are one measure each. The same for 1899 April 18, except the first observation. Same for April 20 for the latitude observations.

In the measures of 1898 and 1899 a preliminary value of the micrometer screw was used, which is a little larger than the final value. The measures of those years should all be reduced slightly to the amount of 0″.07 to 60″ of measured distance. As the greatest distance measured in the observations was about 32″ the largest corrections will not amount to 0″.04.

Yerkes Observatory, Williams Bay, Wis., 1903 *June.*

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ON THE SYSTEMATIC DIFFERENCE IN DECLINATION BETWEEN BRADLEY (AUWERS) AND THE CATALOGUE OF 627 STANDARD STARS (*A.J.* 531–2),

BY LEWIS BOSS.

Systematically the Catalogue of 627 Standard Stars (*A.J.* 531–2) is independent of BRADLEY's determinations. These did not enter into the computations until it became possible to ascertain and apply with care the approximate systematic corrections necessary to reduce them to the system, B. So far as the right-ascensions are concerned, this proved to be a matter of no special difficulty; but the curve of systematic correction for the declinations, as they appear in the Catalogue of AUWERS, appears to be tortuous and uncertain, in relation to the material available for its determination.

In extending the system, B, to the computation of star-positions upon which the determinations of BRADLEY must exert important effects, on account of the relatively small weight of computed μ arising from the observations of the nineteenth century alone, it is essential to know the systematic difference, B—BRADLEY, with as much certainty as possible; otherwise, the system of proper motions for the additional stars, in certain restricted zones, might sensibly differ from that defined by the proper motions of the 627 primary standards. This consideration has led me to pay far more attention to the determination of $\Delta\delta_s$ for BRADLEY's declinations than to that for any of the later catalogues.

In his reduction AUWERS assumed that the errors of graduation were materially different for the two positions of the quadrant, apparently owing to suspected increase of deformations in its plane when adjusted for observations south of the zenith. Apparently these deformations were detected by means of the transits recorded in the use of the quadrant. In my preliminary researches upon $\Delta\delta_s$ for BRADLEY's declinations, as derived from comparison with B for stars south of the zenith, my attention was naturally directed to the supposed deformation at 8° of zenith-distance south. Comparison with the Standard Catalogue did not confirm this, but did indicate a very marked positive maximum of graduation error at 18° of zenith-distance south. Other anomalies of lesser moment appeared. For illustration, we have the following mean differences (from the curve, — the observed differences are larger), $\Delta\delta_s$, from the comparison, B—BRADLEY.

At $+44°$ of declination, $-0\overset{''}{.}15$
At $+36°$ of declination, $+2.60$
At $+27°$ of declination, $+0.60$

It does not seem possible that any very large portion of the anomaly at $+36°$ can be attributed either to relative systematic error in B, or to accumulation of casual errors in BRADLEY's declinations. It seemed desirable, therefore, to investigate whether the observations with BRADLEY's quadrant are consistent with the hypothesis that the graduation error remained substantially the same in the two positions of the quadrant, as tested by the Standard Catalogue.

In the second volume of *Neue Reduction der Bradley'schen Beobachtungen* (pp. 253–410) Dr. AUWERS has published the results which he obtained for the zenith-distances measured with BRADLEY's quadrant in the two positions, North and South. If we combine with these the latitude, 51° 28′ 38″.72, adopted by AUWERS, we shall obtain the declinations as they result without the application of any correction for error of graduation, such as AUWERS has adopted (p. 252 of the *Neue Reduction*, Bd. II). This course was adopted in the present investigation, and the declinations so obtained were compared with the Catalogue of Standard Stars, B. The results of this comparison are exhibited in Table I, for quadrant North under ΔA_N in the first section of the table, and for quadrant South, in the second section, ΔA_S. The individual results have been collected in zones of 3° in width; and the signs of the differences, B—BRADLEY, are given as applicable to altitudes, considered as positive, whether north or south. Therefore, for quadrant south and for quadrant north, lower culmination, the signs represent assumed corrections, $\Delta\delta_s$, to BRADLEY's declinations; and they correspond to $-\Delta\delta_s$ for declinations observed at upper culmination.

The weights here, as throughout, are computed so as to correspond to $\pm 0''.30$ as the probable error of the unit. In

general my computations confirm the weights computed by Dr. AUWERS, except for zenith-distances greater than 75°, for which my adopted weights are less.

From inspection of these values of $\varDelta A_N$ and $\varDelta A_S$ it becomes evident that there is a very decided systematic difference between them; though the sinuosities in the trend of the two sets of numbers present a degree of resemblance which is, perhaps, quite as good as could have been anticipated, in view of the paucity of observations north of the zenith. This led to the examination of the hypothesis that the graduation-error, proper, remained practically invariable, and that the systematic difference in question is due to other causes. It seems very natural to suppose that the eccentricity of the quadrant might have been materially different in the two positions, either through wear in the pivot, or bearing, upon which the telescope turned, or through differences of strain in the fastenings for the two positions. This difference of eccentricity would give rise to a systematic difference of the form, $x \sin\zeta + y \cos\zeta$. If the wear was in the bearing, rather than in the pivot, we should expect y to be much smaller than x. It is also very probable that there may have been an appreciable alteration in the flexure of the telescope when the quadrant was set up in the position, south.

The researches of OLUFSEN (*Astr. Nach.*, Bd. 9, pp. 86–106), and those of SAFFORD (*Astr. Papers, Am. Eph.*, Vol. II, Pt. II) indicate that the declinations of the quadrant after BRADLEY's time require large and increasing corrections. From SAFFORD's results (Table X, of the work cited), I infer that, for stars south of the zenith, the part of this correction which is variable with the zenith-distance, and which he regarded as directly proportional to the zenith-distance in degrees, was roughly $+6'' \sin\zeta$ in 1767, and $+12'' \sin\zeta$ in 1787. These corrections are large enough so that no great part of them can possibly be attributed to error in the system, B_s, (declinations of the *American Ephemeris*, 1881–1900), which was employed as standard. As will be seen, further on, the progression of this error is remarkably consistent with the results found in the present investigation. From these various considerations, as well as from inspection of the differences themselves, it seemed best to assume that the systematic difference, $\varDelta A_S - \varDelta A_N$, is of the form, $k + x \sin\zeta$. This resulted in the following expression which has been adopted.

$$\varDelta A_S - \varDelta A_N = +0''.42 + 1''.20 \sin\zeta$$

The determination of this quantity was strengthened by means of a comparison, Lower *minus* Upper Culmination. Thus there is an alteration in the sine coefficient of $+1''.2$ in approximately four years; while from SAFFORD's researches the alteration appeared to be about $+6''$ in twenty years.

Modifying the values of $\varDelta A_N$ by the amount of this correction, we have the numbers in the first section of Table I, under the caption, $\varDelta' A_N$,

TABLE I. OBSERVED CORRECTIONS, $\varDelta A$, APPLICABLE TO BRADLEY'S OBSERVED ALTITUDES.

Quadrant North					Quadrant South				Mean			
δ	**	p	$\varDelta A_N$	$\varDelta' A_N$	δ	**	p	$\varDelta A_S$	δ	**	p	$\varDelta A$
°			″	″	°			″	°			″
+51.2	16	6.6	+0.13	+0.54	+51.7	8	2.6	+0.07	+51.8	24	9.2	+0.41
55.2	4	1.1	+0.82	+1.32	48.4	9	2.9	−0.05	48.2	13	4.0	+0.33
57.4	7	2.1	+0.92	+1.47	45.2	9	2.8	+1.14	45.4	16	4.9	+1.28
60.4	11	2.2	+0.89	+1.51	42.4	8	1.5	+1.20	42.5	19	3.7	+1.38
63.6	6	1.6	+0.95	+1.62	39.5	12	3.5	+2.12	39.6	18	5.1	+1.96
66.4	4	0.8	+0.82	+1.55	36.9	8	1.3	+2.11	36.8	21	2.1	+1.89
69.5	6	1.6	+1.73	+2.52	33.2	9	1.9	+2.64	33.3	15	3.5	+2.59
72.1	5	1.0	+0.70	+1.54	30.3	9	2.0	+0.78	30.5	14	3.0	+1.04
76.4	5	0.8	+0.88	+1.91	27.8	21	5.5	+1.20	27.7	26	6.3	+1.29
78.8	2	0.3	+0.03	+1.00	24.3	10	2.8	+1.49	24.2	12	3.1	+1.44
81.8	2	0.4	+0.68	+1.71	21.5	21	7.3	+1.41	21.6	23	7.7	+1.42
84.8	2	0.8	+0.17	+1.25	18.7	11	2.8	+1.76	18.6	13	3.6	+1.64
87.6	3	1.1	+1.06	+2.19	15.8	14	3.6	+2.15	15.8	17	4.7	+2.16
90.6	4	1.0	+0.44	+1.62	12.7	16	5.0	+1.81	12.6	20	6.0	+1.78
93.5	1	0.5	−1.11	+0.11	9.2	17	5.6	+1.06	9.2	18	6.1	+0.98
95.8	3	1.4	−0.32	+0.94	6.5	18	5.9	+1.30	6.7	21	7.3	+1.23
98.4	1	0.4	−0.92	−0.37	3.8	21	5.8	+1.41	3.9	22	6.2	+1.29
100.2	6	1.2	−0.03	+1.29	+ 0.5	7	2.6	+0.87	+ 0.6	13	3.8	+1.00
104.9	1	0.4	−0.26	+1.12	− 2.1	12	3.7	+1.92	− 2.1	13	4.1	+1.84
108.1	7	1.9	+0.11	+1.53	5.4	14	4.3	+0.92	5.2	21	6.2	+1.12
111.2	5	1.8	−0.26	+1.19	8.6	23	7.6	+0.86	8.5	28	9.4	+0.92
114.9	4	1.0	−0.07	+1.43	11.1	15	3.8	+0.98	11.3	19	4.8	+1.07
117.8	7	2.1	+0.24	+1.76	14.8	22	6.2	+1.35	14.9	29	8.3	+1.45
120.4	8	2.8	+0.08	+1.62	17.7	22	6.2	+1.37	17.6	30	9.0	+1.45
123.4	9	2.3	−0.47	+1.09	20.5	18	4.9	+0.85	20.4	27	7.2	+0.93
127.3	6	1.7	−0.92	+0.67	23.3	17	2.9	+0.60	23.7	23	4.6	+0.63
129.5	7	1.9	−1.25	+0.35	26.3	14	2.5	+0.38	26.4	21	4.4	+0.37
132.1	11	1.9	−1.16	+0.45	29.6	15	1.5	−0.21	29.3	26	3.4	+0.16
+135.1	11	..	−1.58	+0.04	−32.0	9	..	−1.55	−32.1	20	..	−0.56

The quantities, $\Delta' A_N$ and ΔA_S, are now comparable, according to the hypotheses made. In fact, the differences, $\Delta A_S - \Delta' A_N$, show rather suspicious accumulations of negative signs near the zenith and near 65° of zenith-distance, and of positive signs near 45° of zenith-distance. An analysis of the comparison, Lower—Upper Culmination, does not confirm these apparent systematic deviations in all cases, however; so that it may be doubted whether they are wholly real. Furthermore, taking the differences with their weights, the probable error of the unit comes out, $\pm 0''.32$, against $\pm 0''.30$, the predicted probable error. Considering all of the difficulties of the case this is probably an agreement as close as ought to have been expected. The representation would have been somewhat better through the employment of the full formula, $k + x \sin\zeta$, $+ y \cos\zeta$ to express the difference between the two positions. Yet, with material so scanty, it might prove dangerous to attempt the derivation of such a formula of correction for observations extending over less than a quadrant, since a chance distribution of accidental errors at unlucky points might lead to an illusory result.

Accordingly, the means by weight of $\Delta' A_N$ and ΔA_S were computed as they appear in the last section of Table I. From these the curve of correction, ΔA, was drawn, as it appears under ΔA_s in the first section of Table II. These mean values of ΔA_s correspond to those of "Δz," computed by Dr. Auwers, as published in the second volume of the *Neue Reduction* (p. 252). The correction to the catalogue declinations (Band III) is found from the equation:

$$\Delta\delta_s = \Delta A_s - \Delta z$$

This result is found in Table II under the caption $\Delta\delta_s$; and these are the finally adopted corrections for the declinations of Auwers' Catalogue. It is scarcely necessary to remark that the values of ΔA_s for stars, quadrant north, at upper culmination, are found by subtracting the values of ΔA_s, for corresponding zenith-distance south, from the formula, $+0''.42 + 1''.20 \sin\zeta$; and for those at lower culmination this formula is subtracted from corresponding values of ΔA_s, south.

For the zenithal arc, +49° to +54° of declination, as well as for the combination of lower with upper culminations, the values of $\Delta\delta_s$ applicable to the catalogue places must be computed by a combination of the several values of $\Delta\delta_s$, applicable for the different circumstances of observation, in proportion to the weights given in the two tables, pp. 22–33, in the introduction to the Catalogue.

In drawing the curve, from which the results for ΔA in Table II were derived, some slight modifications were introduced on account of the differences, Lower—Upper Culmination, taken from the introduction to the Catalogue (pp. 22–33), Δz having been first removed and approximate values of ΔA_s introduced.

Table II. Corrections, ΔA_s and $\Delta\delta_s$.

Quadrant South.

δ	ΔA_s	Δδ_s	δ	ΔA_s	Δδ_s	δ	ΔA_s	Δδ_s	δ	ΔA_s	Δδ_s	δ	ΔA_s	Δδ_s
°	″	″	°	″	″	°	″	″	°	″	″	°	″	″
+53	+0.26	+0.48	+37	+1.93	+2.53	+19	+1.62	+1.44	+ 1	+1.36	+0.77	−17	+1.43	+1.35
52	+0.28	+0.48	36	+2.00	+2.65	18	+1.70	+1.19	0	+1.37	+0.60	18	+1.33	+1.63
51	+0.30	+0.48	35	+2.06	+2.32	17	+1.79	+0.93	− 1	+1.37	+0.42	19	+1.18	+1.69
50	+0.33	+0.48	34	+2.13	+2.09	16	+1.88	+1.01	2	+1.36	+0.36	20	+1.02	+1.38
49	+0.40	+0.52	33	+2.17	+1.99	15	+1.90	+1.26	3	+1.32	+0.53	21	+0.88	+1.16
48	+0.54	+0.54	32	+2.07	+1.97	14	+1.80	+1.36	4	+1.28	+0.96	22	+0.78	+1.05
47	+0.79	+0.63	31	+1.65	+1.71	13	+1.68	+1.41	5	+1.22	+1.24	23	+0.69	+0.93
46	+1.01	+0.56	30	+1.32	+1.44	12	+1.52	+1.37	6	+1.14	+1.28	24	+0.59	+0.73
45	+1.17	+0.30	29	+1.25	+1.14	11	+1.33	+1.35	7	+1.03	+1.19	25	+0.50	+0.60
44.5	+1.24	−0.03	28	+1.24	+0.76	10	+1.09	+1.29	8	+0.96	+1.07	26	+0.42	+0.54
44	+1.30	−0.53	27	+1.27	+0.52	9	+1.00	+1.32	9	+0.94	+1.00	27	+0.33	+0.59
43.5	+1.36	−0.53	26	+1.31	+0.74	8	+1.02	+1.22	10	+0.98	+0.98	28	+0.26	+0.87
43	+1.42	+0.05	25	+1.36	+1.03	7	+1.06	+1.06	11	+1.05	+0.99	29	+0.18	+1.38
42	+1.52	+0.92	24	+1.41	+1.45	6	+1.12	+0.92	12	+1.14	+1.04	30	+0.10	+2.22
41	+1.63	+1.34	23	+1.45	+1.69	5	+1.17	+0.72	13	+1.23	+1.11	31	+0.03	+3.02
40	+1.72	+1.59	22	+1.48	+1.61	4	+1.24	+0.64	14	+1.34	+1.19	−32	−0.05	+3.10
39	+1.79	+1.86	21	+1.52	+1.52	3	+1.29	+0.67	15	+1.42	+1.24			
+38	+1.86	+2.24	+20	+1.56	+1.48	+ 2	+1.34	+0.76	−16	+1.45	+1.25			

Quadrant North—Above Pole.

δ	ΔA_s	Δδ_s	δ	ΔA_s	Δδ_s	δ	ΔA_s	Δδ_s	δ	ΔA_s	Δδ_s	δ	ΔA_s	Δδ_s
+49	+0.13	−0.02	+58	−0.62	−0.10	+67	−1.26	−1.14	+76	−0.35	+0.25	+85	−0.62	−0.37
50	+0.13	−0.01	59	−0.72	+0.30	68	−1.30	−0.94	77	−0.37	+0.10	86	−0.69	−0.26
51	+0.13	+0.01	60	−0.82	−0.03	69	−1.35	−0.81	78	−0.40	−0.08	87	−0.76	−0.33
52	+0.13	+0.03	61	−0.90	−0.47	70	−1.37	−0.75	79	−0.44	−0.35	88	−0.77	−0.44
53	+0.12	+0.05	62	−0.99	−0.70	71	−1.25	−0.70	80	−0.46	−0.48	89	−0.65	−0.44
54	+0.07	+0.03	63	−1.06	−0.78	72	−0.81	−0.39	81	−0.47	−0.45	+90	−0.51	−0.40
55	−0.05	−0.02	64	−1.11	−0.86	73	−0.46	−0.13	82	−0.49	−0.43			
56	−0.28	−0.15	65	−1.16	−1.00	74	−0.37	−0.01	83	−0.51	−0.44			
+57	−0.48	−0.19	+66	−1.21	−1.11	+75	−0.34	+0.15	+84	−0.55	−0.45			

Quadrant South — Below Pole.

δ	ΔA_s	$\Delta\delta_s$	δ	ΔA_s	$\Delta\delta_s$	δ	ΔA_s	$\Delta\delta_s$	δ	ΔA_s	$\Delta\delta_s$	δ	ΔA_s	$\Delta\delta_s$
°	″	″	°	″	″	°	″	″	°	″	″	°	″	″
+45	−1.67	+0.10	+55	−0.79	+0.38	+64	−0.27	−0.67	+73	−0.13	−0.61	+82	−0.12	−0.30
47	−1.51	−0.01	56	−0.69	+0.37	65	−0.36	−0.81	74	−0.08	−0.73	83	−0.16	−0.17
48	−1.43	−0.10	57	−0.54	+0.41	66	−0.44	−0.92	75	−0.02	−0.73	84	−0.20	−0.06
49	−1.34	−0.05	58	−0.37	+0.44	67	−0.50	−0.98	76	.00	−0.64	85	−0.23	+0.05
50	−1.27	.00	59	−0.21	+0.27	68	−0.53	−1.00	77	+0.01	−0.52	86	−0.23	+0.13
51	−1.17	+0.10	60	−0.11	−0.03	69	−0.49	−0.94	78	+0.01	−0.40	87	−0.13	+0.17
52	−1.09	+0.20	61	−0.08	−0.23	70	−0.41	−0.82	79	+0.01	−0.36	88	+0.13	+0.28
53	−0.99	+0.28	62	−0.10	−0.39	71	−0.29	−0.66	80	−0.03	−0.37	89	+0.43	+0.42
+54	−0.89	+0.36	+63	−0.17	−0.53	+72	−0.20	−0.57	+81	−0.06	−0.37	+90	+0.51	+0.40

In fact, the drawing of the curve was a process of trial and error, in successive approximations, until what was deemed a fairly satisfactory representation of the various classes of material was obtained.

The curve for ΔA was not usually pushed to the indicated maxima and minima of the residuals, since it was felt that some of these might be due to unlucky combinations of residuals having like signs.

In the subjoined statement appears a digest of the comparison, Lower *minus* Upper Culmination. In the first column, following the argument, number of stars concerned, and the weight, are given the means of "*u*—*o*" taken from the tables of AUWERS (pp. 24-33, Bd. III), and in the last column those which arise from the present discussion after taking into account $\Delta\delta_s$, as it appears in Table II.

LOWER *minus* UPPER CULMINATION.

δ	**	*p*	"*u*—*o*"	L—U
°			″	″
+46.5	44	(3.8)	−0.14	−0.64
49.5	54	6.4	−0.05	−0.36
52.5	34	5.4	−0.06	.00
55.5	45	5.2	−0.10	+0.37
58.5	49	5.6	−0.48	−0.17
61.5	37	4.5	−0.19	+0.02
64.5	33	3.9	+0.11	+0.31
67.5	20	2.4	+0.48	+0.54
70.5	19	2.3	−0.23	−0.28
73.5	14	1.6	+0.39	−0.08
76.5	15	1.5	+0.37	−0.39
79.5	12	1.2	+0.14	+0.02
82.5	13	1.1	−0.44	−0.32
85.5	10	1.3	+0.62	+0.95
+88.5	4	0.6	−1.13	−0.40

It will be seen that the amended results indicate an agreement between the declinations from Lower and Upper Culmination, respectively, as good as could have been anticipated, perhaps, when all the difficulties are considered. An intimate study of the details of this comparison confirms me in the opinion that the graduation-error of the quadrant was substantially the same in the two positions; though it is not at all improbable that very small differences might have existed.

Before leaving this part of the subject, certain remarks may not be out of place.

It should be recalled that, in the first use of the quadrant, north, the observed zenith-distances may have been systematically different from those determined after the new balancing of the quadrant, Dec. 2, 1750.

Some part of the differences, $\Delta A_S - \Delta' A_N$, which are expressed in the formula $+0''.42 + 1''.20 \sin \zeta$, may be due to systematic errors, in the system, B; but I should be greatly surprised if it should turn out that half of this difference shall hereafter be found chargeable to this source.

There are evidences of outstanding systematic differences between the corrections for zenith-distances, north and south, as reconciled in this discussion, which are rather larger than the weights would lead one to anticipate. Allusion to these has already been made. The difference at large zenith-distances may easily be due to a real difference in the refractions at low altitudes, north and south. Between declinations, 57° s.p. and 73° s.p., also, there is a considerable discordance between observed and computed ΔA_N. This amounts to $+0''.28$ (weight, 9.6), in the sense of a further correction for the declinations corrected according to Table II. But if the curve of ΔA had been deflected at this point to produce a better agreement between computed, ΔA, and observed, ΔA_N, then a larger discrepancy in L—U would have resulted. The mean value of L—U for the zone in question is only $+0''.05$ (weight, 18.7), to be sure, and this might readily admit of some increase; but in the zone, +60° s.p. to 69° s.p., where this increase could most naturally be made, the corresponding value of L—U is already $+0''.24$ (weight, 10.8), and this would be increased by any further reduction of the discordance in question. It thus appears that the standard stars for this zone do not very well represent the generality of BRADLEY's quadrant observations within those limits.

In order to show clearly what outstanding discrepancies exist when the assumption is made, as in this discussion, that the error of graduation remained practically the same for the two positions of the quadrant, we have the following schedule. The residuals are gathered in general means, including both Lower and Upper Culminations. If, now, we suppose that the adopted corrections of Table II have been applied to the catalogue declinations, comparison of

the Standard Catalogue with the observations of quadrant north, alone, indicates that further mean corrections to the catalogue declinations are required as follows:

Outstanding Discrepancies.

δ	p	$\Delta'\delta$
°		″
+51.0	8.5	−0.01
55.7	7.2	−0.16
61.1	8.7	+0.17
67.9	5.2	+0.09
73.1	4.1	+0.04
80.3	2.3	−0.13
+86.6	4.8	−0.13

So far as these quantities are concerned, they do not appear to interpose any very practical objection to the adoption of the corrections contained in Table II.

For quadrant south the adopted mean curve of correction represents the observations substantially as well as would a curve based upon southern observations alone.

This investigation was undertaken more with the object of ascertaining the best practicable systematic corrections for the Bradley-Auwers star positions, rather than for the purpose of deciding upon the peculiarities of Bradley's quadrant. Yet it seems very probable that the deductions set forth in the foregoing brief abstract have some resemblance to the actual facts concerning the graduation-error of the Bird quadrant. The somewhat regular recurrence of large positive corrections at zenith-distances of approximately 18°, 36°, 54° and 67°, wears rather a suspicious aspect, as pointing to some systematic source in the method of marking the graduations (or in fastening the quadrant to the pier). These maximum points seem to be quite consistently indicated in the observations both of quadrant north and quadrant south, as exhibited in Table I. If, after removal of such errors, as collimation, eccentricity, flexure and error in adopted refraction, the quantities, ΔA, are to be considered as mainly due to errors of graduation, it can scarcely be objected to them that they are exceptionally large. It may be doubted whether a single quadrant, under a single microscope, of many of the modern transit-circles would show much more perfect graduation.

Adopting the values of ΔA, as exhibited in the foregoing, the declinations were next tested for systematic errors of the form, $\Delta\delta_\alpha$. For this purpose the declinations, quadrant north, were divided into two series, the first containing all observations at zenith-distances less than 58°.5 (decl. 70° s.p.), and the second, all observations from declination, 70° s.p. to 47° s.p. In a similar way the observations, quadrant south, were divided into two series, of which the common boundary is the equator. Table III represents the results, together with the means for quadrant north and south, respectively.

Table III. Observed Corrections for $\Delta\delta_\alpha$.

−30° to 0°			0° to +54°			−30° to +54°		
α	p	$\Delta\delta_\alpha$	α	p	$\Delta\delta_\alpha$	α	p	$\Delta\delta_\alpha$
h		″	h		″	h		″
0.5	2.1	−0.53	0.5	3.1	+0.37	0.5	5.2	+0.01
1.4	0.7	+0.14	1.5	4.2	+0.07	1.5	4.9	+0.08
2.4	1.0	−0.20	2.6	4.2	−0.01	2.5	5.2	−0.05
3.6	1.7	−0.38	3.5	5.3	−0.28	3.5	7.0	−0.30
4.7	1.3	+0.30	4.6	3.8	+0.33	4.6	5.1	+0.32
5.3	2.2	+0.04	5.4	2.1	−0.11	5.3	4.3	−0.04
6.6	1.0	−0.09	6.3	3.2	−0.06	6.4	4.2	−0.07
7.9	0.2	+1.12	7.4	2.5	+0.44	7.4	2.7	+0.48
. .	. .	. . .	8.5	2.4	−0.01	8.5	2.4	−0.01
9.6	0.5	+0.94	9.6	3.2	+0.11	9.6	3.7	+0.22
10.7	1.0	+0.55	10.4	2.8	+0.10	10.5	3.8	+0.17
11.5	1.0	+0.57	11.4	4.4	+0.29	11.4	5.4	+0.34
12.6	1.6	−0.39	12.4	2.0	−0.56	12.5	3.6	−0.48
13.4	1.4	−0.13	13.5	1.7	−0.46	13.5	3.1	−0.31
14.6	2.8	−0.74	14.5	1.8	−0.57	14.6	4.6	−0.68
15.5	3.7	−0.24	15.4	3.0	−0.19	15.5	6,7	−0.21
16.4	2.3	+0.15	16.5	1.7	−0.05	16.4	4.0	+0.03
17.5	2.4	+0.09	17.5	4.0	−0.36	17.5	6.4	−0.19
18.6	1.7	+0.08	18.7	2.1	−0.02	18.7	3.8	−0.06
19.5	1.8	+0.60	19.5	4.0	+0.05	19.5	5.8	+0.22
20.4	3.7	+0.67	20.5	3.7	+0.29	20.5	7.4	+0.48
21.5	3.7	−0.12	21.4	2.0	+0.46	21.5	5.7	+0.08
22.5	3.0	−0.14	22.6	2.5	.00	22.5	5.5	+0.06
23.5	3.4	−0.10	23.5	2.2	+0.09	23.5	5.6	−0.03

+49° to 70° s.p.			70° s.p. to 40° s.p.			49° to 40° s.p.		
α	p	$\Delta\delta_\alpha$	α	p	$\Delta\delta_\alpha$	α	p	$\Delta\delta_\alpha$
h		″	h		″	h		″
0.3	4.2	−0.12	0.0	0.7	+0.26	0.3	4.9	−0.09
1.9	1.8	+0.39	1.5	0.6	−0.35	1.8	2.4	+0.20
3.6	0.9	−0.56	3.8	1.0	−0.77	3.7	1.9	−0.67
5.9	1.9	+0.01	6.0	2.9	+0.20	6.0	4.8	+0.13
8.1	1.4	+0.69	7.8	2.3	+0.11	7.9	3.7	+0.33
9.8	2.1	+0.51	9.7	1.3	+0.47	9.8	3.4	+0.50
12.2	3.7	+0.09	12.2	0.8	+0.38	12.2	4.5	+0.13
13.9	2.2	+0.51	13.8	1.3	−0.38	13.9	3.5	+0.18
16.0	1.3	+0.11	16.2	0.7	−0.26	16.0	2.0	+0.03
17.9	2.7	+0.06	. .	. .	. . .	17.9	2.7	+0.04
19.7	3.4	−0.10	20.4	0.5	−0.47	19.8	3.9	−0.14
21.8	3.1	−0.69	22.2	1.5	−0.71	21.9	4.6	−0.70

For the declinations south of the zenith the general agreement of the mean values of $\Delta\delta_\alpha$ in the two divisions seems to warrant the consolidation of them into a single series. For the representation of this series of observed corrections I adopt,

$$+0''.08 \sin\alpha + 0''.07 \cos\alpha - 0''.25 \sin 2\alpha - 0''.05 \cos 2\alpha$$

For the observations, quadrant north, the material is much more scanty than for quadrant south; and there seem to be some anomalies in the observations at great zenith-distances, to which Auwers has made allusion in the introduction to his Catalogue (pp. 34–35). On this point Dr. Auwers remarks (p. 35): "Die letzte Columne der obigen Tafel, welche die mittleren $\delta_w - \delta_s$ für die tieferen Sterne enthält, würde einen Fehler von jährlicher Periode andeuten, aber die näheren Circumpolarsterne schliessen

eine solche Annahme direct aus; die Störung, auf welche noch bei der Ableitung der Polhöhe zurückzukommen sein wird, scheint vielmehr auf einzelne Abschnitte der Beobachtungen bei Quadrant Nord beschränkt gewesen zu sein." This may serve to account for some of the other anomalies already noticed. I have, therefore, considered it advisable to apply a uniform correction, $\Delta\delta_n$, to all observations, quadrant north, derived from the combined result of the two divisions. I have adopted the correction,

$$+0''.08 \sin\alpha - 0''.30 \cos\alpha,$$

employing the argument, true right-ascension, whether the star was observed above, or below the pole. In a somewhat summary analysis of the material I have also found as an expression for this correction, $+0''.08 \sin\alpha - 0''.20 \cos\alpha + (0''.01 \sin\alpha - 0''.12 \cos\alpha)\,\mathrm{tg}\,\delta$; but this improves the representation very little, and does not wholly remove the anomalies found in the observations beyond 75° of zenith-distance. In the course of this discussion I compared the declinations observed, quadrant south, with those which were observed, quadrant north, below the pole, from +45° to 54° 23′ of declination. The column next the last in the subjoined table contains the difference, L—U, uncorrected for $\Delta\delta_n$, and the last column contains the same corrected for the combined values of $\Delta\delta_n$, quadrant north and south.

h	p	L—U	L—U (corr'd.)
		″	″
0.0	0.6	+1.47	+1.22
2.0	1.2	+0.64	+0.54
3.8	1.9	+0.16	+0.15
5.7	0.7	−0.17	−0.20
6.5	0.5	−2.37	−2.41
9.9	0.2	+0.13	+0.27
12.0	0.2	−0.26	+0.15
13.7	0.3	−0.81	−0.31
17.3	0.9	−1.03	−0.97
20.0	1.8	+0.35	−0.05
21.8	0.7	+1.29	+0.80

This shows the nature of the anomalies, and exhibits the improvement produced by the adoption of the formulas of correction $\Delta\delta_n$. Probably it would be better to assign weight zero for observations having zenith-distance greater than 80°. There appears to be no particular trace of these anomalies for zenith-distances less than 75°.

The corrections contained in Table II, together with the adopted values of $\Delta\delta_n$, are now in use at this observatory, in the computations for extension of the Catalogue of 627 Standard Stars to include a much greater number of what might be termed secondary standard stars.

ELEMENTS AND EPHEMERIS OF COMET *c* 1903 (BORRELLY),*

By H. R. MORGAN and ELEANOR A. LAMSON.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent Naval Observatory.]

The following elements were deduced from three normal places derived from observations made at Lick and Washington Observatories, on June 22, 23, 24, 30, July 1, 2, 7, 8 and 9:

Elements.

$$T = 1903 \text{ August } 27.60410 \text{ Gr. M.T.}$$

$$\begin{aligned} \pi &= 60^\circ\ 52'\ 34'' \\ \Omega &= 293\ \ 32\ \ 53 \\ i &= 84\ \ 59\ \ 50 \\ q &= 0.32966 \end{aligned}$$

Residuals (O—C): $\cos\beta\,\Delta\lambda = +3''.7\ ,\ \Delta\beta = +2''.1$

Heliocentric Coordinates.

$$\begin{aligned} x &= r[9.610061] \sin(206^\circ\ 0'\ 46'' + v) \\ y &= r[9.962176] \sin(\ 13\ 53\ 44 + v) \\ z &= r[9.998661] \sin(105\ 51\ 40 + v) \end{aligned}$$

Ephemeris.

1903 G.M.T.	α	δ	log Δ	Light
	h m s	° ′		
July 28.5	12 17 36	+57 14.2	9.6446	8.8
Aug. 1.5	11 43 42	51 18.5	9.7264	7.4
5.5	11 22 55	46 40.1	9.7991	6.7
9.5	11 7 32	42 50.7	9.8635	6.4
13.5	10 54 25	39 24.5	9.9208	6.7
17.5	10 42 3	35 58.0	9.9722	7.3
21.5	10 29 56	32 6.8	0.0179	8.2
25.5	10 18 31	27 27.8	0.0570	8.6
29.5	10 9 18	21 54.3	0.0878	7.4
Sept. 2.5	10 3 34	15 49.1	0.1098	5.4
6.5	10 1 8	9 42.9	0.1254	3.7
10.5	10 1 3	+ 3 51.1	0.1372	2.5
14.5	10 2 28	− 1 42.3	0.1471	1.8
18.5	10 4 52	6 58.0	0.1562	1.3
22.5	10 7 54	11 57.7	0.1648	1.0
26.5	10 11 20	16 42.9	0.1735	0.8
30.5	10 15 2	−21 15.1	0.1824	0.6

Brightness on June 22 is taken as the unit.

* *From Supplement to No. 544.*

DEFINITIVE ORBIT OF COMET 1891 IV,

By HENRY A. PECK.

This comet was discovered by Barnard October 2, 1891. It is described as moderately bright, about one minute in diameter, with scarcely any nucleus. It was far south, and was always observed with difficulty by its discoverer, disappearing from his sight in a week. It was observed at no other northern observatory, but was followed, however, for nearly two months at Cordoba. These two series of observations, together with a short one from Sydney, are all that have appeared. In *A.N.* 3237 Hind has published an orbit based upon the Cordoba observations of October 19, November 12 and December 3. As will be seen from the following, these elements satisfy the southern observations with a fair degree of approximation, but leave much to be desired for those of Barnard.

The Hind elements are as follows:

$$T = 1891 \text{ Nov. } 13.54555 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 269^\circ\, 34'\, 59''.5 \\ \Omega &= 218\;\; 0\;\; 13.4 \\ i &= 77\;\; 59\;\; 54.7 \end{aligned}\right\} 1891.0$$

$$\log q = 9.9872737$$

$$\begin{aligned} x &= [9.9021683]\, r \sin[188^\circ\, 48'\, 43''.03 + v] \\ y &= [9.8927485]\, r \sin[135\;\, 53\;\, 30.45 + v] \\ z &= [9.9382463]\, r \sin[253\;\, 10\;\, 25.45 + v] \end{aligned}$$

The ephemeris positions are

		α apparent h	m	s	δ apparent °	′	″	ab. t	log Δ
Oct.	1	7	20	9.91	−24	53	51.8	0.00554	9.982
	2		25	36.67	26	21	33.9	551	980
	3		31	12.81	27	49	17.8	548	978
	4		36	58.63	29	16	49.3	546	977
	5		42	54.40	30	43	54.0	545	976
	6		49	0.36	32	10	16.9	544	975
	7		55	16.77	33	35	42.9	544	975
	8	8	1	43.79	34	59	56.8	544	975
	9		8	21.63	36	22	43.9	545	976
	10		15	10.38	37	43	49.8	547	977
	11		22	10.12	39	3	0.4	548	978
	12		29	20.86	40	20	2.8	550	980
	13		36	42.56	41	34	44.6	553	982
	14		44	15.06	42	46	54.6	556	985
	15		51	58.15	43	56	21.8	560	987
	16		59	51.51	45	2	57.5	564	990
	17	9	7	54.75	46	6	33.2	568	993
	18		16	7.33	47	7	2.2	573	997
	19		24	28.67	48	4	18.6	578	0.001
	20		32	58.00		58	18.5	583	005
	21		41	34.54	49	48	58.4	589	009
	22		50	17.35	50	36	16.7	595	014
	23		59	5.42	51	20	12.6	601	018
	24	10	7	57.66	52	0	46.3	608	023
	25		16	52.92		37	59.4	615	028
	26		25	49.99	53	11	54.7	622	033
	27		34	47.63		42	35.7	629	038
	28		43	44.58	54	10	6.7	637	043
	29	10	52	39.63	−54	34	33.1	0.00645	0.048

		α apparent h	m	s	δ apparent °	′	″	ab. t	log Δ
Oct.	30	11	1	31.55	−54	56	0.7	0.00653	0.054
	31		10	19.17	55	14	35.9	661	059
Nov.	1		19	1.40		30	25.7	669	065
	2		27	37.16		43	37.1	677	070
	3		36	5.50		54	18.0	686	075
	4		44	25.59	56	2	35.9	694	080
	5		52	36.63		8	38.8	703	086
	6	12	0	37.99		12	34.4	711	091
	7		8	29.10		14	30.4	720	096
	8		16	9.55			34.7	728	101
	9		23	38.97		12	54.3	737	107
	10		30	57.12		9	36.9	745	112
	11		38	3.83		4	49.5	754	117
	12		44	59.01	55	58	38.8	763	122
	13		51	42.67		51	11.1	772	127
	14		58	14.84		42	32.7	780	132
	15	13	4	35.65		32	48.8	789	136
	16		10	45.23		22	5.7	797	141
	17		16	43.79		10	27.8	806	146
	18		22	31.55	54	58	0.4	814	151
	19		28	8.78		44	47.7	823	155
	20		33	35.74		30	54.0	831	159
	21		38	52.72		16	23.0	839	163
	22		44	0.00		1	18.5	847	167
	23		48	57.91	53	45	43.4	855	171
	24		53	46.76		29	41.3	863	175
	25		58	26.87		13	14.6	871	179
	26	14	2	58.55	52	56	26.2	879	183
	27		7	22.09		39	18.3	886	187
	28		11	37.81		21	53.1	893	191
	29		15	45.99		4	12.5	900	194
	30		19	46.91	51	46	18.4	907	197
Dec.	1		23	40.83		28	12.3	914	200
	2		27	28.04		9	55.8	921	203
	3		31	8.78	50	51	30.2	928	207
	4		34	43.29		32	56.8	934	210
	5		38	11.84		14	16.7	940	213
	6		41	34.62	49	55	31.2	947	215
	7		44	51.88	−49	36	40.7	0.00953	0.218

The following star-positions referred to the mean equinox and ecliptic of 1891.0 have been used:

No.	α h	m	s	δ °	′	″	Authority
1	7	30	55.65	−27	52	29.4	Wash.M.Cir.Z. 84, No.103
2	7	36	36.55	29	17	43.9	Arg. General Catal.
3	7	44	18.72	30	29	4.8	Arg. Gen. Catal.; Yarnall
4	7	50	15.24	32	12	16.0	Washington Mer. Tr. Zone 217, No. 64. Arg. Zone Catal.
5	7	54	55.50	33	36	56.5	Washington Mer. Cir. Zone 166, No. 11. Arg. Zone Catal.
6	8	5	2.95	35	8	9.0	Arg. General Catal.; 2d Washington; Stone; Cape 1850.
7	8	8	24.47	36	39	44.1	Arg. General Catal.; 2d Washington; Stone; Cape 1850.
8	8	8	31.08	36	20	24.5	Washington Mer. Circle Zone 171, No. 26.
9	8	10	0.19	36	37	16.9	Arg. General Catal.
10	8	15	7.56	37	50	8.6	Washington Mural Zone 234, No. 26; Argentine Zone Catal.
11	8	17	27.55	39	16	25.5	Arg. General Catal.; 2d Washington; Stone; Cape 1850.
12	8	21	30.87	39	31	6.7	Arg. General Catal.
13	9	32	55.48	48	51	59.9	Arg. General Catal.; Stone; Cape '40; '50; '85; '90; Madras Gen. Catal.
14	9	38	55.67	49	38	37.5	Arg. Zone Catal.
15	9	40	55.25	49	30	29.3	Arg. General Catal.
16	9	49	50.45	−50	37	55.7	A.G.C.; Stone; Cape '40-'50

No.	α	δ	Authority
	h m s	° ′ ″	
17	9 55 49.11	51 19 37.3	Arg. General Catal.
18	10 1 42.46	51 52 32.4	Arg. General Catal.
19	10 3 6.12	52 0 11.4	Arg.G.C.; Stone; Cape '50
20	10 32 24.70	53 40 22.1	Arg. General Catal.
21	10 47 26.95	54 33 35.5	Arg.G.C.; Stone; Cape '50
22	12 25 20.44	56 22 22.0	3 comp. with γ *Crucis*
23	12 31 9.12	56 11 15.7	Arg. General Catal.
24	12 40 7.61	55 53 31.6	A.G.C.; Stone; Cape '40, '50
25	12 50 57.99	55 42 59.5	A.G.C.; Madras Gen. C.
26	13 12 41.67	55 24 11.8	4 comp. with No. 28
27	13 13 29.41	55 11 29.4	Arg. General Catal.
28	13 14 3.27	55 13 41.2	Arg. General Catal.; Stone
29	13 17 2.95	54 49 4.0	Arg. General Catal.
30	13 55 20.15	53 27 58.7	Arg. Zone Catal.

No.	α	δ	Authority
	h m s	° ′ ″	
31	14 2 39.92	52 55 6.9	Arg. General Catal.; Stone
32	14 20 5.28	52 1 57.8	Arg. General Catal.
33	14 25 2.57	51 29 12.5	Arg. General Catal.; Stone
34	14 36 30.13	50 12 19.6	Arg. Zone Catal.
35	14 37 0.58	50 43 37.3	Arg. General Catal.
36	14 43 25.23	49 38 27.2	Arg. Zone Catal.

The following new proper motions have been used:

No. 21	$\Delta\alpha = -0^s.005$	$\Delta\delta = 0''.000$
24	$+0.002$	-0.044

In comparing the observations with the ephemeris, the time of observation has been corrected for aberration, and then reduced to the meridian of Greenwich.

Date	Place	α apparent	π	O—C Δα cos δ	δ apparent	π	O—C Δδ	*
		h m s		″	° ′ ″	″	″	
Oct. 3.03695	Mt. Hamilton	7 31 25.20	−0.26	− 6.3	−27 52 18.5	+8.0	+21.7	1
4.00455	"	37 0.36	.36	3.0	29 17 3.5	7.7	17.3	2
5.00022	"	42 54.02	.38	10.8	30 43 52.3	7.7	10.1	3
5.99582	"	48 58.91	.41	5.7	32 15 28.3	7.7	(−5′ 25″.3)	4
7.00977	"	55 20.62	.42	− 3.7	33 36 46.0	7.9	− 5.5	5
8.01554	"	. . . .	. .	. .	35 1 12.7	8.2	+10.0	6
8.03347	"	8 1 57.58	.34	+ 3.7	. . . .	.	. .	6
9.00940	"	8 26.27	.39	+ 5.6	36 23 34.6	+7.9	+ 3.4	8
9.15258	Sydney	9 23.28	.63	− 7.4	34 52.9	−3.1	17.1	7
9.25637	"	10 0.09	.45	− 0.7	42 18.3	−0.8	15.0	9
10.01027	Mt. Hamilton	15 15.76	.41	+ 8.8	37 44 42.7	+7.9	4.4	10
11.15711	Sydney	23 17.39	.65	− 3.7	39 14 56.5	−3.2	15.8	11
11.23089	"	48.70	.51	− 4.8	20 49.1	0.8	9.6	12
Oct. 19.76897	Cordoba	9 31 0.54	.74	+ 1.6	48 46 2.6	2.0	+ 3.0	13
19.78790	"	9.51	.72	− 5.4	47 10.6	1.3	− 4.1	13
20.79807	"	39 49.73	.72	− 7.2	49 38 56.6	1.0	+ 3.1	15
20.82112	"	40 2.20	.68	− 1.2	40 3.7	−0.3	+ 5.5	14
20.83521	"	9.03	.64	− 5.5	46.2	+0.2	+ 6.4	14
21.81367	"	48 41.18	.70	+ 9.5	50 27 42.0	−0.5	+ 0.7	16
22.84073	"	57 41.21	.65	− 4.2	51 13 37.3	+0.3	−10.8	17
23.81578	"	10 6 20.14	.72	+ 1.3	53 32.5	−0.5	0.0	18
23.83821	"	32.72	.67	+ 6.9	54 24.7	+0.2	1.7	19
26.81974	"	33 11.00	.73	− 3.9	53 37 17.9	−0.6	− 0.1	20
28.82513	"	51 7.12	.72	+ 1.3	54 30 28.9	−0.5	+ 0.4	21
Nov. 8.82389	Cordoba	12 22 20.91	.70	− 6.8	56 13 10.5	−1.3	+ 7.6	22
9.82341	"	29 41.73	.70	+ 3.7	10 21.8	1.4	− 4.6	23
10.82869	"	36 52.35	.69	+ 1.8	5 49.5	1.3	6.4	24
11.81539	"	43 44.08	.68	+ 1.5	55 59 51.7	1.7	0.2	24
12.80694	"	50 25.90	.67	− 3.4	52 46.2	2.0	5.0	25
15.80859	"	13 9 34.66	.64	−11.3	24 13.7	2.1	2.6	28
15.82961	"	43.96	.64	+ 2.3	23 1.8	1.5	4.0	26
16.80983	"	15 36.98	.63	− 0.8	12 44.2	2.1	1.7	27
17.81480	"	21 28.07	.62	− 4.2	0 20.3	2.0	− 0.1	29
Nov. 23.81328	Cordoba	52 54.34	.57	+ 2.4	53 32 34.4	2.1	+ 6.4	30
25.81480	"	14 2 9.58	.55	+ 1.6	52 59 29.2	2.0	+ 3.4	31
28.80785	"	14 59.46	.52	+ 0.8	7 31.5	2.2	+ 3.7	32
30.80338	"	22 55.84	.51	− 0.2	51 31 45.8	2.2	− 1.3	33
Dec. 3.80628	"	34 2.75	.49	+ 0.6	50 36 30.6	2.1	+ 0.3	35
4.80494	"	37 31.63	.48	− 4.6	17 57.2	2.1	− 3.7	34
6.79378	"	44 11.66	.46	− 2.7	49 40 34.7	2.3	− 0.4	36
6.81202	"	15.46	−0.47	− 0.4	32.7	−1.9	(+1′ 40″.9)	36

The observations have in general received the weight unity. Those in ( ) have been rejected entirely, and three others have had their weight reduced. The hour-angle of the second Sydney observation on Oct. 9 has been corrected by subtracting twenty minutes from the time of observation. For the first observation on Dec. 6 the difference $*-\mathcal{C}$ in declination has been made to read $-2' 4''.6$, instead of $-24''.56$, as printed.

Combining in the usual manner, the following normal places result, referred to the mean obliquity and equinox of 1891.0.

	α	δ	O—C $\Delta\alpha\cos\delta$	O—C $\Delta\delta$
	h m s	° ′ ″	″	″
Oct. 8.0	2 1 42.67	−34 59 45.79	−2.33	+10.81
23.0	9 59 4.61	51 20 5.64	0.62	+ 0.74
Nov. 13.0	12 51 41.03	55 51 0.89	1.35	− 1.89
Dec. 1.0	14 23 38.60	51 27 58.20	0.31	+ 1.20

By a very rough preliminary computation it was ascertained that the large residual in declination for Oct. 8 could be made to practically vanish by slight changes in the elements most intimately connected with the perihelion passage. The further work is therefore based upon the elements

$$T = \text{Nov. } 13.54150 \text{ G.M.T.}$$
$$\omega = 269^\circ\ 34'\ 31''.6$$
$$\log q = 9.9872612$$

and these corrected elements leave the residuals

$\partial\alpha\cos\delta$	$\partial\delta$
″	″
−0.12	−0.28
−0.56	−1.68
−2.05	−1.09
+0.09	+3.61

Using Schönfeld's notation the equations of condition are

$\partial\kappa$	$\kappa\sqrt{2}\partial T$	∂q	$\partial\lambda$	$\partial\nu$			Wt.
+650	−803	+ 65	+169	−182	=	− 12 + 227 $\partial\epsilon$	3
559	701	−247	−204	+118	=	56 + 111	3
263	278	414	539	+ 7	=	−205 + 1	2
+132	− 63	351	−501	−230	=	+ 9 + 1	2
−549	+377	605	+741	800	=	− 28 − 53	3
+ 23	− 87	207	832	479	=	−168 + 20	2.5
430	452	347	471	− 7	=	−109 + 2	2
535	459	549	236	+108	=	+361 − 40	2

the units being the third decimal place for the coefficients and the hundredths of a second for the absolute terms.

From these equations of condition the following normal equations are deduced:

$\partial\kappa$	$\kappa\sqrt{2}\partial T$	∂q	$\partial\lambda$	$\partial\nu$		
+4221	−4406	− 500	− 944	+1188	=	+106 + 667 $\partial\epsilon$
−4406	+4826	+ 816	+ 400	− 682	=	+ 33 − 811
− 500	+ 816	+2830	−1379	+1619	=	+ 21 + 92
− 944	+ 400	−1379	+5230	−2676	=	−105 − 49
+1188	− 682	+1619	−2676	+2764	=	+340 + 12

The elimination equations are

$$\partial\kappa - 1044\,\kappa\sqrt{2}\,\partial T - 118\,\partial q - 224\,\partial\lambda + 281\,\partial\nu = +25 + 160\,\partial\epsilon$$
$$\kappa\sqrt{2}\,\partial T + 1258\,\partial q - 2487\,\partial\lambda + 2360\,\partial\nu = +606 - 449\,\partial\epsilon$$
$$\partial q - 315\,\partial\lambda + 446\,\partial\nu = -61 + 127\,\partial\epsilon$$
$$\partial\lambda - 207\,\partial\nu = +69 - 20\,\partial\epsilon$$
$$\partial\nu = +169 - 155\,\partial\epsilon$$

The most probable values of the unknown quantities are

$$\partial\kappa = +6''.09 \quad -0.239\,\partial\epsilon$$
$$\kappa\sqrt{2}\,\partial T = +5.94 \quad -0.431$$
$$\partial q = -1.03 \quad +0.180$$
$$\partial\lambda = +1.04 \quad -0.052$$
$$\partial\nu = +1.69 \quad -0.155$$

and the corrections to the elements are

$$\partial T = +0^d.00118 \quad -[5.9340]\,\partial\epsilon$$
$$\partial\omega = +6''.45 \quad -0.271\,\partial\epsilon$$
$$\partial\Omega = -1.74 \quad +0.158\,\partial\epsilon$$
$$\partial i = -1.05 \quad +0.053\,\partial\epsilon$$
$$\partial q = -0.0000050 \quad +[3.9409]\,\partial\epsilon$$

where $\partial\epsilon$ is understood as expressed in seconds of arc. If a parabolic form is assumed for the orbit, the resulting elements and their probable errors are

$$T = \text{Nov. } 13.54268 \pm 0.00179 \text{ G.M.T.}$$
$$\left.\begin{array}{l} \omega = 269^\circ\ 34'\ 38''.0 \quad \pm 7''.0 \\ i = 77\ 59\ 53.6 \quad \pm 1.4 \\ \Omega = 218\ \ 0\ 11.7 \quad \pm 3.2 \end{array}\right\} 1891.0$$
$$\log q = 9.9872590 \pm 42 \text{ (units of 7th place)}$$

$$x = [9.9021712]\,r\sin(188^\circ\ 48'\ 21''.80 + v)$$
$$y = [9.8927475]\,r\sin(135\ 53\ \ 7.10 + v)$$
$$z = [9.9382445]\,r\sin(253\ 10\ \ 4.33 + v)$$

A comparison of the residuals obtained by computing an ephemeris for the dates of the normal places, with those that result from direct substitution in the equations of condition is given as proof of the numerical accuracy of the solution.

$\Delta\alpha\cos\delta$ Eq. of Cond.	Elements.	$\Delta\delta$ Eq. of Cond.	Elements.
″	″	″	″
+0.89	+0.86	+0.78	+0.86
−0.04	−0.10	−1.57	−1.57
−1.88	−1.85	−1.87	−1.88
+0.21	+0.19	+2.08	+2.06

Several characteristics of these residuals suggest that the parabola may not be the true form of the orbit. Certainly the one represented by these elements is little, if any, better than that produced by the supposition that the corrections to the longitude of the node and the inclination are zero. It will be noticed that $\Sigma(pvv)$ has only dropped from 45″.1 to 33″.1 and that the probable errors of the elements in all cases except that of $\log q$ exceed the corrections themselves. A direct solution of the equations, retaining the eccentricity as an unknown quantity indicates a tendency toward an ellipse. The coefficient of $\partial\epsilon$ in the last equation becomes so small, however, that its theoretical weight is practically at the vanishing point.

If the results given above are substituted in the weighted equations, the sum of the squares of the residuals becomes a minimum for $\partial\epsilon = -139''.5$ and $\Sigma(pvv)$ becomes only 9″.7, the individual residuals being

$\Delta\alpha\cos\delta$	$\Delta\delta$
″	″
−0.16	−0.58
−0.74	+0.73
+0.73	+1.18
+0.79	+0.45

The range of uncertainty is quite large so that this value of $\partial\epsilon$ can not by any possibility be considered as fixed within a limit of ±40″. The ellipse above indicated has an eccentricity of 0.999324 and the other elements are

$$T = \text{Nov. } 13^{\text{d}}.5547 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 269^\circ\ 35'\ 16'' \\ i &= \ \ 77\ \ 59\ \ 46 \\ \Omega &= 217\ \ 59\ \ 50 \end{aligned}\right\}1891.0$$

$$\log q = 9.987204$$

Syracuse University, June 19, 1903.

ON THE APPARENT ELLIPTICITY OF *MARS*,

By E. E. BARNARD.

At the opposition of *Mars* in 1894 the disc of the planet appeared decidedly elliptical with the 36-inch of the Lick Observatory. On two dates I made settings for the position angle of the apparent equation.

	d	h	m		°
1894 Oct.	15	14	30	P.A. =	62.6
Nov.	4	11	50		43.8
					53.2

These are discordant. It is, however, difficult to measure the position angle of an ellipticity of this kind. I was not looking for ellipticity of the disc, and was surprised when it was noticed. Thinking there might be some deception in the matter, I have never referred to the observations.

At the opposition of this spring, I was again struck with this peculiarity, and on four dates made settings for the position of the apparent equator.

	d	h	m		°
1903 Mar.	25	10	0	P.A. =	122.9
	30	11	0		133.2
Apr.	4	10	0		121.0
	6	9	30		122.8
					125.0

From the ephemerides of *Mars* printed in the *Monthly Notices, R.A.S.*, for April 1894, and June 1902, by Mr. Marth and Mr. Crommelin, respectively, I find the position of the Martian equator for the approximate dates of observation, and residuals C−O,

	Comp.	C—O
	°	°
In 1894	54.5	+1½
In 1903	120.0	−5

The accordances with the known position of the equator leads me to think the apparent ellipticity may be real. If it is real, the ellipticity of the planet must be decidedly greater than the theoretical value. It is a well known fact that observations differ greatly in respect to the polar compression of *Mars*. Some of the larger values obtained would make it readily apparent to the eye.

In the observations the eyes were placed in different positions with respect to the direction of the ellipticity, which remained unchanged and decided in character.

Yerkes Observatory, Williams Bay, Wis., 1903 *April* 25.

SUSPECTED VARIABLE NEAR *R CYGNI*,

By ORMOND STONE.

July 15, 1903, a star of $10^{\text{M}}.8$, 14^{s} preceding and 0′.8 north of 7045 *R Cygni*, was seen by Mr. Chas. P. Olivier and Mr. G. F. Paddock with the 26-inch equatorial of this Observatory. No star is shown in this place on Hagen's chart. July 18 it was estimated at $14^{\text{M}}.5$, and July 19 at $14^{\text{M}}.7$.

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DOUBLE-STAR MEASURES,

By JOHN A. MILLER and W. A. COGSHALL.

The stars in the following list are those which were noted as double by the Berlin observers while making the observations for the Catalog of the *Astronomische Gesellschaft* (Zone 20°–25°), no measures of which have hitherto been published. The list was prepared by Professor S. W. Burnham. The measures were made with the 12-inch refractor of Kirkwood Observatory, to which is attached a micrometer by Warner and Swasey.

The position angle for each night is the mean of four settings, and the distance the mean of three settings.

The stars which have been noted as double by the Berlin observers, but which seem to us single, have been examined on two or more nights, and pronounced single after having been observed at least once when seeing was steady. The magnitude given is the average of estimates made at the time the stars were measured. The letter C. follows the measures made by Mr. Cogshall, and the letter M. those made by myself. The positions given are for 1900.

John A. Miller.

DM. 20°18. A.G. 56. $8^M.9$; $9^M.4$.

$\alpha = 0^h\,11^m\,49^s.99$; $\delta = +21°\,13'\,11''.9$

t	θ_0 °	ρ_0 ″	
1902.793	135.4	1.73	
1902.804	133.8	1.63	
1903.030	133.7	1.92	
1902.876	134.3	1.79	M.

DM. 23°135. A.G. 305. $9^M.0$; $10^M.0$.

$\alpha = 0^h\,53^m\,21^s.73$; $\delta = +24°\,8'\,36''.6$

1902.785	109.4	3.92	
1902.804	112.1	3.80	
.845	112.5	3.91	
1902.811	111.3	3.88	C.

DM. 25°139. A.G. 314.

$\alpha = 0^h\,56^m\,27^s.77$; $\delta = +23°\,15'\,21''.7$

1902.785	244.5	4.61	
.804	243.3	4.18	
.826	242.3	4.66	
1902.805	243.4	4.48	M.

DM. 20°154. A.G. 330. $9^M.8$; $9^M.9$.

$\alpha = 1^h\,0^m\,11^s.43$; $\delta = +20°\,31'\,8''.9$

1902.785	201.9	0.83	
.804	206.2	0.85	
1902.795	204.1	0.84	C.

DM. 20°410. A.G. 757. $9^M.4$.

$\alpha = 2^h\,25^m\,17^s.44$; $\delta = +21°\,3'\,40''.5$

1902.017	247.9	4.41	
.115	246.9	4.50	
.131	246.9	4.65	
1902.088	247.2	4.52	C.

DM. 20°507. A.G. 907.

$\alpha = 3^h\,0^m\,8^s.72$; $\delta = +20°\,28'\,34''.2$

t	θ_0 °	ρ_0 ″	
1902.804	20.2	0.80	
.826	22.7	0.89	
1903.227	31.1	0.93	
1902.922	24.7	0.87	M.

DM. 21°442. A.G. 989. 9^M ; 11^M.

$\alpha = 3^h\,14^m\,59^s.59$; $\delta = +21°\,17'\,42''.4$

1902.131	287.9	3.91	
.826	284.0	4.00	
1903.162	283.4	3.54	
1902.706	285.3	3.82	M.

DM. 22°620. A.G. 1298. 9^M ; 10^M.

$\alpha = 3^h\,56^m\,7^s.79$; $\delta = +23°\,3'\,57''.4$

1902.017	174.2	1.69	
.151	170.7	1.60	
1903.068	170.4	1.35	
1902.412	171.7	1.55	M.

DM. 24°772. A.G. 1662.

$\alpha = 5^h\,4^m\,36^s.57$; $\delta = +25°\,1'\,18''.5$

1902.151 Not double.

Marked "obl." in one zone in A.G. Catal.

DM. 22°978. A.G. 1856. 8^M ; $9^M.5$.

$\alpha = 5^h\,34^m\,0^s.32$. $\delta = +22°\,28'\,46''.3$

1902.017	142.8	7.10	
1903.022	144.0	7.07	
1903.068	142.5	6.93	
1902.702	143.1	7.03	C.

DM. 21°1008. A.G. 1992.

$\alpha = 5^h\,43^m\,37^s.35$; $\delta = +21°\,47'\,47''.9$

1902.195 Not double.

Marked "dupl. seq. maj." in A.G. Catal.

DM. 20°1216. A.G. 2113. $8^M.8$.

$\alpha = 5^h\,55^m\,6^s.40$, $\delta = +20°\,13'\,48''.5$

t	θ_0 °	ρ_0 ″	
1902.115	100.4	14.08	
.131	103.4	14.35	
.151	96.2	14.91	
1903.023	100.7	14.14	
1902.355	100.2	14.37	C.

DM. 20°1259. A.G. 2175. $8^M.7$; 10^M.

$\alpha = 6^h\,0^m\,46^s.04$; $\delta = +20°\,6'\,50''.9$

1902.195	198.5	1.50	
.214	199.4	1.53	
1902.205	199.0	1.51	M.

DM. 24°1161. A.G. 2242. 9^M ; $9^M.2$.

$\alpha = 6^h\,6^m\,41^s.64$; $\delta = +24°\,27'\,2''.7$

1902.151	181.2	1.85	
.195	179.4	2.07	
1903.068	182.5	1.63	
1902.471	181.0	1.85	M.

DM. 22°1280. A.G. 2308. $8^M.7$.

$\alpha = 6^h\,13^m\,43^s.79$; $\delta = +22°\,9'\,5''.5$

1902.151	48.8	1.73	
.195	50.2	1.51	
.208	49.1	1.66	
1902.185	49.4	1.63	C.

DM. 24°1270. A.G. 2392. 9^{M} ; $9^{M}.1$.

$\alpha = 6^h\,21^m\,21^s.90$; $\delta = +24°\,35'\,32''.0$

t	θ_0	ρ_0	
	°	″	
1902.151	209.5	2.81	
.208	206.8	2.52	
1903.151	210.0	2.18	
1902.503	208.8	2.50	M.

DM. 23°1480. A.G. 2559. $8^{M}.5$; 9^{M}.

$\alpha = 6^h\,38^m\,33^s.02$; $\delta = +23°\,32'\,54''.1$

1902.151	76.4	1.45	
.195	77.7	1.46	
.208	76.2	1.60	
1902.188	76.8	1.50	C.

DM. 21°1445. A.G. 2696.

$\alpha = 6^h\,51^m\,46^s.95$; $\delta = +21°\,8'\,57''.1$

1902.195 Not double.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 24°1508. A.G. 2739. 9^{M} ; $9^{M}.2$.

$\alpha = 6^h\,56^m\,52^s.57$; $\delta = +24°\,36'\,5''.6$

1902.208	23.3	1.64	
.214	20.6	1.28	
1903.151	21.5	1.29	
.162	19.9	1.69	
1902.684	21.3	1.48	M.

DM. 22°1655. A.G. 2911.

$\alpha = 7^h\,16^m\,12^s.30$; $\delta = +22°\,50'\,5''.6$

1902.195 Not double.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 22°1797. A.G. 3152. 9^{M} ; $10^{M}.5$.

$\alpha = 7^h\,45^m\,57^s.99$; $\delta = +22°\,30'\,44''.6$

1902.195	329.8	10.92	
.208	329.5	11.36	
.214	331.6	11.01	
1902.206	330.3	11.10	C.

DM. 22°1678. A.G. 2941. $8^{M}.7$; $10^{M}.3$.

$\alpha = 7^h\,20^m\,0^s.95$; $\delta = +22°\,17'\,8''.1$

1902.214	176.1	1.55	
.271	175.6	1.59	
1903.022	174.1	1.58	
1902.502	175.3	1.57	C.

DM. 20°2095. A.G. 3396.

$\alpha = 8^h\,23^m\,40^s.46$; $\delta = +20°\,45'\,41''.1$

1903.208 Not double.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 23°1978. A.G. 3449. $9^{M}.2$; $10^{M}.2$.

$\alpha = 8^h\,31^m\,4^s.14$; $\delta = +23°\,35'\,49''.3$

1902.291	7.9	1.61	
1903.151	5.8	1.52	
1903.162	7.5	1.65	
1902.868	7.1	1.59	M.

DM. 25°1997. A.G. 3558.

$\alpha = 8^h\,42^m\,21^s.26$; $\delta = +24°\,57'\,10''.5$

1902.291 Not double. W.A.C.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 23°2004. A.G. 3559. 9^{M} ; $9^{M}.1$.

$\alpha = 8^h\,44^m\,37^s.28$; $\delta = +23°\,30'\,8''.7$

t	θ_0	ρ_0	
	°	″	
1903.153	71.4	1.71	
.208	73.7	1.61	
.227	75.7	1.78	
1903.196	73.6	1.70	M.

DM. 24°2053. A.G. 3688. 9^{M} ; $9^{M}.5$.

$\alpha = 9^h\,7^m\,47^s.23$; $\delta = +24°\,28'\,10''.5$

1902.227	318.5	4.94	
.271	316.7	4.66	
.307	318.8	4.18	
1902.268	318.0	4.59	M.

DM. 24°2089. A.G. 3766.

$\alpha = 9^h\,22^m\,59^s.25$; $\delta = +24°\,14'\,25''.7$

1902.271. Not double.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 21°2128. A.G. 3876.

$\alpha = 9^h\,51^m\,22^s.52$; $\delta = +21°\,15'\,12''.6$

1902.304 Uncertain.
.373 Probably elongated.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 23°2288. A.G. 4150.

$\alpha = 10^h\,54^m\,27^s.48$; $\delta = +23°\,45'\,2''.2$

1902.271 Not double.

Marked "dupl., med.," in one zone, and single in three zones in A.G. Catal.

DM. 22°2387. A.G. 4323.

$\alpha = 11^h\,34^m\,19^s.28$; $\delta = +21°\,52'\,0''.9$

1902.271 Not double.

Marked "comp. $9^{M}.5$, 1–2 sec.?" in one zone in A.G. Catal.

DM. 23°2471. A.G. 4544. $8^{M}.8$; $9^{M}.4$.

$\alpha = 12^h\,27^m\,45^s.41$; $\delta = +23°\,33'\,47''.3$

1902.271	312.2	0.88	
.307	320.4	0.76	
1903.206	311.8	0.99	
1902.595	314.8	0.88	M.

DM. 21°2434. A.G. 4563.

$\alpha = 12^h\,31^m\,49^s.01$; $\delta = +20°\,47'\,14''.8$

1902.373 Probably double.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 23°2528. A.G. 4672.

$\alpha = 12^h\,57^m\,42^s.56$; $\delta = +23°\,29'\,4''.2$

Single. Marked "dupl. 2–3 sec. comp. less than 9^{M}" in A.G. Catal.

DM. 23°2530. A.G. 4674.

$\alpha = 12^h\,58^m\,7^s.23$; $\delta = +23°\,10'\,32''.4$

Single. Marked "dupl. pr." in A.G. Catal.

DM. 24°2532. A.G. 4681. 9^{M} ; $9^{M}.5$.

$\alpha = 12^h\,58^m\,48^s.34$; $\delta = +24°\,10'\,50''.3$

t	θ_0	ρ_0	
	°	″	
1902.307	127.8	3.17	
.373	. .	2.04	
.411	129.5	2.25	
1903.206	127.7	2.54	
.266	128.4	2.59	
1902.713	128.3	2.52	C.

DM. 24°2542. A.G. 4702.

$\alpha = 13^h\,3^m\,14^s.85$; $\delta = +23°\,56'\,48''.7$

Not double. Marked "dupl. maj.?" in two zones in A.G. Catal.

DM. 21°2531. A.G. 4789. $9^{M}.1$; $9^{M}.5$.

$\alpha = 13^h\,22^m\,21^s.71$; $\delta = +20°\,58'\,52''.6$

1902.307	305.3	1.16	
.444	305.7	1.31	
.504	300.2	1.16	
1903.206	305.2	1.50	
1902.615	304.1	1.28	M.

DM. 24°2588. A.G. 4798. $8^{M}.8$; 12^{M}.

$\alpha = 13^h\,23^m\,55^s.70$; $\delta = +24°\,5'\,55''.3$

1902.307	245.4	3.69	
.444	248.3	2.51	
.515	248.7	2.43	
1902.422	247.5	2.88	C.

DM. 23°2682. A.G. 5046.

$\alpha = 14^h\,15^m\,33^s.21$; $\delta = +23°\,30'\,49''.5$.

1902.504 Double, but too close to meas.!

Marked "dupl.?" in one zone.

DM. 21°2798. A.G. 5383. 9^{M} ; 10^{M}.

$\alpha = 15^h\,35^m\,22^s.90$; $\delta = +21°\,36'\,8''.3$.

1902.444	129.5	2.80	
.520	131.7	2.47	
1903.227	128.4	2.68	
1902.730	129.9	2.65	M.

DM. 20°3216. A.G. 5528.

$\alpha = 16^h\,4^m\,19^s.89$; $\delta = +20°\,40'\,10''.6$.

1902.518 Not double.

Marked "dupl.?" in one zone.

DM. 24°3048. A.G. 5715.

$\alpha = 16^h\,40^m\,18^s.09$; $\delta = +23°\,59'\,7''.3$.

This is a nebula. Marked "multiple" in Catal.

DM. 24°3080. A.G. 5764. 9^{M} ; $11^{M}.5$.

$\alpha = 16^h\,50^m\,28^s.76$; $\delta = +24°\,41'\,22''.3$.

1902.411	209.6	2.20	
.502	213.9	2.12	
.504	209.4	1.84	
.695	210.9	2.09	
1902.528	210.9	2.06	M.

DM. 23°3151. A.G. 6056. 9^{M} ; $9^{\mathrm{M}}.3$.

$\alpha = 17^h\,33^m\,39^s.01$; $\delta = +22^\circ\,58'\,11''.2$.

t	θ_0	ρ_0	
	°	″	
1902.411	172.4	2.65	
.504	170.5	2.81	
.698	173.4	3.02	
1902.538	172.1	2.83	M.

DM. 20°3540. A.G. 6078. 9^{M} ; $9^{\mathrm{M}}.5$.

$\alpha = 17^h\,36^m\,19^s.02$; $\delta = +20^\circ\,19'\,49''.3$.

1902.411	127.7	2.33	
.504	128.5	2.46	
.531	133.0	2.30	
1902.482	129.7	2.36	C.

DM. 21.3386. A.G. 6410. 9^{M} ; $9^{\mathrm{M}}.2$.

$\alpha = 18^h\,15^m\,46^s.91$; $\delta = +21^\circ\,17'\,20''.7$.

1902.515	191.6	1.56	
.698	197.0	1.34	
1903.266	197.0	1.43	
1902.400	193.9	1.43	
1902.970	194.9	1.44	M.

DM. 24°3423. A.G. 6481.

$\alpha = 18^h\,23^m\,27^s.24$; $\delta = +24^\circ\,18'\,58''.7$.

1902 Not double.

Marked "dupl. pr. med." in Catal.

DM. 24°3798. A.G. 7129.

$\alpha = 19^h\,31^m\,53^s.62$; $\delta = +24^\circ\,30'\,24''.6$.

1902.559 Not double.

Marked "obl.?" in one zone in A.G. Catal.

DM. 21°3994. A.G. 7387. 9^{M} ; $10^{\mathrm{M}}.4$.

$\alpha = 19^h\,53^m\,57^s.56$; $\delta = +21^\circ\,52'\,4''.8$.

1902.515	278.2	1.21	
.824	273.3	0.96	
.848	275.2	1.31	
1902.729	275.6	1.16	M.

DM. 24°4017. A.G. 7504.

$\alpha = 20^h\,4^m\,41^s.12$; $\delta = +25^\circ\,2'\,41''.6$.

1902.583 Uncertain.

Marked "dupl.??" in one zone in Catal.

DM. 24.4202. A.G. 7824. $8^{\mathrm{M}}.6$; $10^{\mathrm{M}}.2$.

$\alpha = 20^h\,33^m\,50^s.64$; $\delta = +24^\circ\,49'\,52''.4$.

t	θ_0	ρ_0	
	°	″	
1902.693	218.7	10.41	
.706	218.3	10.30	
.766	217.5	10.66	
1902.722	218.2	10.46	M.

DM. 24°4235. A.G. 7921. 9^{M} ; $9^{\mathrm{M}}.1$.

$\alpha = 20^h\,41^m\,16^s.66$; $\delta = +24^\circ\,19'\,55''.5$.

1902.695	355.7	1.43	
.766	357.5	1.81	
.818	359.3	1.80	
1902.760	357.5	1.68	M.

DM. 20°4822. A.G. 8079. $8^{\mathrm{M}}.8$; $10^{\mathrm{M}}.4$.

$\alpha = 21^h\,0^m\,5^s.62$; $\delta = +20^\circ\,29'\,6''.2$.

1902.695	173.4	7.64	
.766	172.9	7.38	
.818	176.8	7.63	
1902.760	174.4	7.55	M.

DM. 22°4455. A.G. 8332. 9^{M} ; $9^{\mathrm{M}}.5$.

$\alpha = 21^h\,35^m\,0^s.29$; $\delta = +22^\circ\,54'\,27''.4$.

1902.695	153.2	8.74	
.818	153.1	8.76	
.826	154.1	8.53	
1902.780	153.5	8.68	M.

DM. 21°4611. A.G. 8379. $9^{\mathrm{M}}.2$; $9^{\mathrm{M}}.5$.

$\alpha = 21^h\,40^m\,53^s.11$; $\delta = +21^\circ\,28'\,46''.8$.

1902.695	356.4	1.88	
.818	358.1	1.66	
.826	359.2	1.69	
1902.780	357.9	1.74	M.

DM. 20°5007. A.G. 8383.

$\alpha = 21^h\,41^m\,25^s.38$; $\delta = +20^\circ\,42'\,31''.5$.

t	θ_0	ρ_0	
	°	″	
1902.695	233.4	2.05	
.766	232.6	2.19	
.826	234.4	2.05	
1902.762	233.5	2.10	M.

DM. 21°4711. A.G. 8560.

$\alpha = 22^h\,8^m\,26^s.61$; $\delta = +21^\circ\,18'\,1''.6$.

1902.785	230.6	5.05	
.826	230.6	5.00	
1903.088	232.4	4.82	
1902.900	231.2	4.96	M.

DM. 21°4718. A.G. 8568. $8^{\mathrm{M}}.8$; $9^{\mathrm{M}}.8$.

$\alpha = 22^h\,10^m\,6^s.15$; $\delta = +21^\circ\,27'\,13''.0$.

1902.785	19.1	1.68	
.804	20.6	2.26	
.826	19.4	1.94	
1903.088	26.1	1.64	
1902.876	21.3	1.88	C.

DM. 23°4600. A.G. 8733.

$\alpha = 22^h\,40^m\,2^s.83$; $\delta = +23^\circ\,51'\,12''.0$.

1902 Single.

Marked "obl?" in one zone in Catal.

DM. 22°4769. A.G. 8840. 9^{M} ; $9^{\mathrm{M}}.5$.

$\alpha = 22^h\,58^m\,54^s.80$; $\delta = +22^\circ\,37'\,2''.1$.

1902.785	228.3	2.04	
.807	224.8	1.80	
1902.796	226.6	1.92	C.

DM. 22°4936. A.G. 9171.

$\alpha = 23^h\,54^m\,1^s.71$; $\delta = +22^\circ\,58'\,56''.2$.

1902.766 Single.

Marked "dupl.?" in Catal.

Indiana University, Bloomington, Ind.

OBSERVATIONS OF THE STAR *KRUEGER* 60,

MADE WITH THE 40-INCH TELESCOPE,

BY E. E. BARNARD.

In 1890 Professor Burnham, at the Lick Observatory, measured a list of stars noted by Krueger as double in his catalogue of the *Astronomische Gesellschaft*.

In the *Astronomical Journal*, 486, p. 47, Mr. Eric Doolittle gives a list of measures of No. 60 of this list, and shows that the stars *A* and *B* had greatly changed their places when compared with the measures of Professor Burnham in 1890.

In concluding, Mr. Doolittle says: "Though the measures are so few, they indicate that there is here a faint star with a large proper motion, attended by a minute companion, which either has a large proper motion of its own, or else is in rapid revolution about the primary."

When Mr. Doolittle's measures were published, it appeared to me wholly improbable that two faint stars, so near each other, should have such motions without being physically connected. In *A.J.* 488, p. 64, I have printed some measures which I obtained in 1900 of these objects. These clearly showed the motion assigned to the stars by Mr. Doolittle.

Acting upon the assumption that *A* and *B* must be a binary system, I have since carefully measured these stars, and the measures of the present year clearly show that the motion of *B* with reference to *A* cannot be rectilinear even when compared with my measures of 1900, 1901 and 1902 alone. If compared with the measures of Professor

Burnham in 1890 it will be seen that his distance AB must be nearly 2″ too small to even suggest rectilinear motion.

Following are a continuation of the measures of 1900 printed in *A.J.* 488, p. 64.

Observations in 1901.

A and *B.*

			°	″		
1901.729	Sept.	23	130.05	3.37		
.731		24	129.11	3.39	9.5	10.5
.748		30	131.50	3.37		
.751	Oct.	1	131.65	3.25		
.783		13	129.69	3.31		
.805		21	130.59	3.26	9.2	10.5
.827		29	130.59	3.22		
1901.768			130.45	3.31	9.3	10.5

A and *C.*

			°	″	
1901.729	Sept.	23	59.65	37.17	
.731		24	59.84	37.19	9.5
.748		30	59.86	37.29	
.751	Oct.	1	59.52	37.14	
.783		13	59.63	37.42	
.805		21	59.48	37.35	9.7
.827		29	59.49	37.33	
1901.768			59.64	37.27	9.6

A and *D.*

			°	″	
1901.783	Oct.	13	23.56	21.53	15
.805		21	23.04	21.71	15.3
.827		29	23.07	21.78	
1901.805			23.22	21.67	15.1

A and *E.*

			°	″	
1901.729	Sept.	23	98.93	68.18	12
.731		24	99.14	68.47	12
.748		30	99.06	68.62	Single distances. ½ weight.
.751	Oct.	1	98.92	68.20	
.783		13	99.01	68.30	
.805		21	98.91	68.37	11.3
.827		29	98.83	68.61	
1901.768			98.90	68.39	11.8

A and *F.*

			°	″	
1901.729	Sept.	23	275.36	39.60	14
.731		24	274.78	39.52	14
.751	Oct.	1	275.20	39.60	
.783		13	275.33	39.51	15
.805		21	275.34	39.52	15
.827		29	275.53	39.55	
1901.771			275.26	39.55	14.5

Observations in 1902.

A and *B.*

			°	″
1902.744	Sept.	29	127.25	3.33
.764	Oct.	6	128.17	3.43
.766		7	126.73	3.35
.786		14	126.06	3.35
1902.765			127.30	3.37

A and *C.*

			°	″
1902.744	Sept.	29	59.64	38.27
.764	Oct.	6	59.69	38.09
.766		7	59.64	38.31
.786		14	59.60	38.49
1902.765			59.64	38.29

Observations in 1903.

A and *B.*

			°	″			
1903.380	May	19	122.36	3.32			
.418	June	2	122.34	3.42			
.437		9	123.85	3.40	9.5	11.0	
.454		15	123.49	3.37			
.473		22	123.47	3.33			
.492		29	124.90	3.36			
.495		30	125.96	3.26			
.511	July	6	121.31	3.51			Very bad seeing. Difficult.
.514		7	123.18	3.30			
1903.464			123.43	3.36	9.5	11.0	

A and *C.*

			°	″	
1903.380	May	19	59.55	38.59	
.396		25	59.57	38.67	
.418	June	2	58.89	38.83	Excessively bad seeing.
.437		9	59.19	38.75	
.454		15	59.35	38.70	
.473		22	59.59	38.73	
.492		29	59.57	38.73	
.495		30	59.22	38.73	
.511	July	6	59.54	38.83	
.514		7	59.70	38.70	
1903.457			59.42	38.73	

A and *E.*

			°	″	
1903.437	June	9	97.89	69.14	13
.454		15	97.99	69.35	
.473		22	97.87	69.18	
.495		30	97.96	69.40	
.511	July	6	98.07	69.45	
.514		7	97.78	69.49	
1903.481			97.93	69.34	13

A and *F.*

			°	″	
1903.473	June	22	277.24	38.46	13
.495		30	276.84	38.45	12.5
.511	July	6	277.06	38.33	12
.514		7	277.09	38.50	
1903.498			277.06	38.43	12.5

A and *D.*

			°	″	
1903.511	July	6	24.68	22.74	
.514		7	24.09	22.82	15
1903.512			24.39	22.78	15

Following is a complete list of all the measures of these stars that are known to me. Professor Burnham has kindly supplied me with his yet unpublished measures.

A and *B*.

	°	″	M	M		
1890.79	178.8	2.32	9.0	12.0	1*n*	β
1898.45	140.7	3.19	9.1	10.5	5*n*	Doolittle
1900.74	134.0	3.18	9.1	11.1	4*n*	Doolittle
1900.94	133.4	3.25	9.1	10.5	4*n*	Barnard
1901.37	131.4	3.35	..	..	4*n*	β
1901.77	130.4	3.31	..	..	7*n*	Barnard
1902.76	127.4	3.37	..	..	3*n*	Barnard
1902.81	126.5	3.36	..	..	4*n*	β
1903.43	123.5	3.31	..	..	5*n*	β
1903.46	123.4	3.36	9.5	11.0	9*n*	Barnard

A and *C*.

	°	″		M		
1890.79	56.3	26.82	..	9.3	1*n*	β
1898.45	58.7	34.39	..	9.4	5*n*	Doolittle
1900.74	59.2	36.18	..	9.4	4*n*	Doolittle
1900.94	59.3	36.71	..	9.4	4*n*	Barnard
1901.37	58.6	36.55	..	..	4*n*	β
1901.77	59.6	37.27	..	..	7*n*	Barnard
1902.76	59.7	38.23	..	..	3*n*	Barnard
1902.81	59.4	38.29	..	..	4*n*	β
1903.43	58.5	38.61	..	..	5*n*	β
1903.46	59.4	38.73	..	..	10*n*	Barnard

A and *D*.

	°	″		M		
1900.94	21.0	21.29	..	15.5	2*n*	Barnard
1901.80	23.2	21.67	..	..	3*n*	Barnard
1903.51	24.4	22.78	..	15.0	2*n*	Barnard

A and *E*.

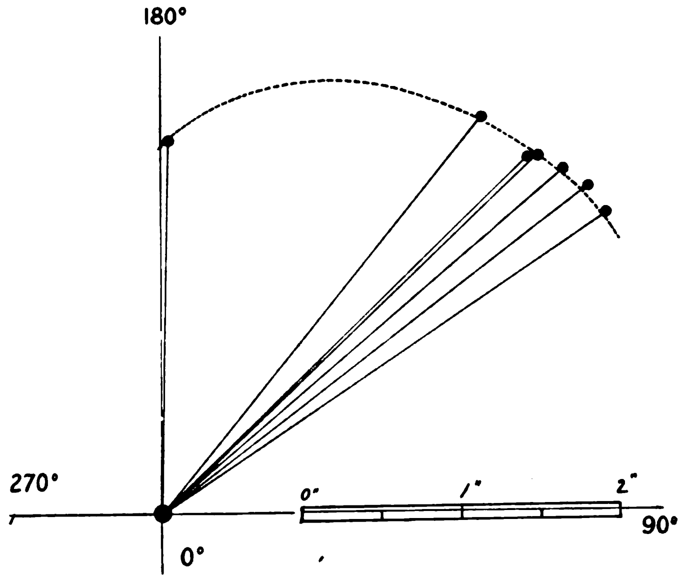
	°	″		M		
1900.94	99.0	67.83	..	13.0	3*n*	Barnard
1901.34	98.5	67.08	..	..	3*n*	β
1901.77	98.9	68.39	..	..	7*n*	Barnard
1903.48	97.9	69.34	..	13.0	6*n*	Barnard

A and *F*.

	°	″		M		
1901 77	275.3	39.55	..	..	6*n*	Barnard
1903.50	277.1	38.43	..	12.5	4*n*	Barnard

In the inclosed diagram I have plotted the observations of the stars *AB*. The measures will be easily identified on the diagram, and to prevent confusion I have not inserted the dates. Professor BURNHAM's measures of 1901, 1902 and 1903 were received after the diagram was made, and have not been inserted. They would not materially change the results. I have roughly drawn a curve through the measures to give some rough idea of what may be the form of the path of *B*.

In inspecting this diagram, it might be said that if we allow a very large error in BURNHAM's distance of 1890 — which is improbable — that the observations made since 1898 can be represented by a straight line. It will be seen, however, that the position angle is diminishing at a uniform rate, while the distance is essentially stationary. This could only approximately occur for a short time in rectilinear motion when the stars were at their nearest approach. Assuming this to be the case, then to reconcile my last observation with this idea, BURNHAM's distance of 1890 would have to be increased as much as 3″ or 4″, which is asking too much.



The mean angular motion previous to 1898 was about 5°. Since then it has been between 3° and 4°. The change of angle between 1902 and 1903 seems too large, but the measures appear to be good in both years, and they are verified by BURNHAM's measures.

From the observations of *AC* Mr. DOOLITTLE determined the motion *A* to be 0″.93 in the direction 247°.9. Using my measures of this year and BURNHAM's of 1890, I make the motion to be 0″.951 in the direction of 246°.3. Reduced to rectilinear coordinates this motion is

In α $-0\overset{''}{.}871$ or $-1\overset{s}{.}606$
In δ -0.382

From the foregoing observations and remarks it would appear that we have here a very rare case of a binary system, where the components are of the 9th and 11th magnitudes, with an apparent distance of over three seconds of arc, which in all probability has a period well within one hundred years, and which has a large apparent proper motion through space.

These considerations would lead one to think that this star may possibly be relatively near to our solar system. With this last idea in view, I have made observations of *A* with reference to some of the surrounding stars in the hope of getting some evidence of parallax. Some of these measures are included in the present paper. The measures so far made, show that the parallax can not be large.

On May 25, 1903, the position of *A* was measured with reference to the star Helsingfors–Gotha A.G.C. 13177.

$\Delta\alpha = 0^m\ 14^s.13$ from 10 transits.
$\Delta\delta = 2'\ 41''.26$ from 2 measures.
A was north preceding.

This gives for the position of A (corrected for motion),

$$1903.0 \quad \alpha = \quad 22^h\ 24^m\ 32^s.69$$
$$1903.0 \quad \delta = +57°\ 12'\ 38''.5$$

In my paper on this object in *A.J.* 488, p. 64, the last distance for AB is printed $2''.23$. It should have been $3''.23$. In this same paper I have erroneously called the system of AB, β1291. Professor BURNHAM has already assigned this number to another star.

While observing these stars in 1901 I noticed a rather wide double, very much like AB, and whose position for 1855.0 is

$$\alpha = 22^h\ 19^m\ 42^s.8 \quad , \quad \delta = +57°\ 6'.3$$

Yerkes Observatory, Williams Bay, Wis., 1903 *July* 10.

The following measures have been made of this object:

1901.731	Sept. 24	247°.04	2″.99	9ᴹ.5	12ᴹ.0
.805	Oct. 21	247.19	3.11	9.0	11.0
1901.768		247.11	3.05	9.2	11.5
1903.473	June 22	248°.79	3″.09		
.511	July 6	247.62	3.04		
.514	7	246.68	3.25		
1903.499		247.70	3.13		

There seems to be no change in this star.

There is also another small double in this region which I have so far only located as being $15' \pm$ preceding Krueger 60.

1902	Sept. 29	260°.15	1″.37	10ᴹ	11ᴹ
1903	June 15	262.50	1.41		

ON THE RELATIVE VALUES OF THE MICROMETERS AND THEIR TEMPERATURE-COEFFICIENTS AT THE SIX INTERNATIONAL LATITUDE STATIONS,

BY H. KIMURA.

The large mean deviation of the reduction to the group mean at Tschardjui, given on page 120 of "*Resultate des Internationalen Breitendienstes,*" Bd. I, by Prof. ALBRECHT, arouses my doubt that it partly comes from the error of the adopted value of the micrometer. The existence of such errors for all the stations has been proved by the calculation of Mr. NAKANO, who had earlier noticed the same matter. His results given below, as A in Table I, are obtained by the following process: First, for each pair were formed the differences between the individual values of the reduction to the group-mean and the mean of all, the data being taken from pp. 114–118 of the above-named report. From the differences thus formed for the six pairs in each group belonging to each station by the method of least-squares, the following values A were derived, the coefficients of the unknown quantity A being $(\Delta z - \Delta z_m)$, where Δz is the semi-difference of the zenith-distances of two stars in each pair, and Δz_m, the mean of all Δz in the group.

TABLE I.
(unit $0''.0001$).

Group	Mizusawa		Tschardjui		Carloforte		Gaithersburg		Cincinnati		Ukiah	
	A	ΔA	A	ΔA	A	ΔA	A	ΔA	A	ΔA	A	ΔA
I	− 23	− 7	−186	+ 44	+170	+ 15	− 83	− 40	+ 28	−48	+ 80	+ 9
II	− 95	− 79	− 50	+180	+ 64	− 91	− 86	− 43	+ 88	+12	+ 86	+15
III	− 35	− 19	−146	+ 84	+130	− 25	−112	− 69	+ 66	−10	+137	+66
IV	−155	−139	− 62	+168	+330	+175	−283	−240	+ 71	− 5	+ 90	+19
V	− 37	− 21	− 8	+222	+102	− 53	−174	−131	+ 22	−54	+ 74	+ 3
VI	+ 28	+ 44	−181	+ 49	+ 66	− 89	− 38	+ 5	+ 51	−25	+ 90	+19
VII	+ 21	+ 37	−280	− 50	+231	+ 76	− 33	+ 10	+ 59	−17	+ 70	− 1
VIII	+ 36	+ 52	−293	− 63	+ 51	−104	+ 53	+ 96	+111	+35	+ 48	−23
IX	+ 18	+ 34	−319	− 89	+ 96	− 59	+ 48	+ 91	+ 66	−10	+ 90	+19
X	+ 30	+ 46	−582	−352	+259	+104	+ 99	+142	+104	+28	+115	+44
XI	+ 15	+ 31	−376	−146	+146	− 9	+109	+152	+162	+86	− 16	−87
XII	+ 11	+ 27	−282	− 52	+222	+ 67	− 16	+ 27	+ 83	+ 7	− 16	−87
Mean	$-0''.0016$		$-0''.0230$		$+0''.0155$		$-0''.0043$		$+0''.0076$		$+0''.0071$	

The mean values in the last line are the relative constant errors of the micrometers.

Now, from the above results, I have tried to find the temperature-coefficients of the micrometers for the six stations. It is, however, a quite difficult matter to determine their absolute values, because the general conditions of the temperature-variations at all the stations resemble each other, and hence their mutual dependencies cannot be wholly avoided. Thus I have aimed here to find the relative amounts of the coefficients which have been

brought as near to the absolute as possible. The mathematical form of A is $\Delta M - \left(\Delta\alpha . t - \frac{\Sigma\Delta\alpha . t}{6}\right)$, where ΔM is the relative constant correction of the value of the micrometer; $\Delta\alpha$, the correction of the temperature-coefficient; t, the mean temperature for a group; and $\frac{\Sigma\Delta\alpha . t}{6}$, the mean of $\Delta\alpha . t$ for all the stations in each group. ΔA, the difference from the mean given in Table I, is therefore in the expression of $\Delta\alpha . (t-t_m) - \frac{\Sigma\Delta\alpha . (t-t_m)}{6}$, where t_m is the mean of t for each station.

For finding $\Delta\alpha$ from these differences, I have, first of all, assumed that, — in the three stations, Mizusawa, Gaithersburg and Cincinnati, where the temperature-coefficients have already been applied, — the employed coefficients are correct, namely, $\Delta\alpha$ = zero. Then the mean of ΔA for these three stations will give nothing more than $-\frac{\Sigma\Delta\alpha . (t-t_m)}{6}$. Apply this value of $\frac{\Sigma\Delta\alpha . (t-t_m)}{6}$ to all ΔA's, and find $\Delta\alpha$ for all the stations by the method of least-squares. Next form $\frac{\Sigma\Delta\alpha . (t-t_m)}{6}$ with $\Delta\alpha$ newly found, and add them to the original ΔA. From the sums, by least-squares, the following values of the temperature-coefficients are obtained:

TABLE II.

For 1′	Mizusawa	Tschardjui	Carloforte	Gaithersburg	Cincinnati	Ukiah
$\Delta\alpha$	−0″.00005	−0″.00192	−0″.00066	+0″.00063	−0″.00018	−0″.00173
Adopted α	−0.00205	0	0	−0.00142	−0.00141	0
Corrected α	−0″.00210	−0″.00192	−0″.00066	−0″.00079	−0″.00159	−0″.00173

For the comparison, I give here the calculated A with the above numbers.

TABLE III.

Calculated A (unit 0″.0001).

Group	Mizusawa	Tschardjui	Carloforte	Gaithersburg	Cincinnati	Ukiah
I	−25	−182	+132	− 50	+ 66	+ 76
II	−45	−126	+139	−125	+ 58	+114
III	−59	− 44	+132	−153	+ 46	+ 93
IV	−61	+ 19	+138	−164	+ 44	+ 77
V	−45	− 85	+151	−137	+ 65	+ 65
VI	−22	−203	+169	−100	+ 77	+ 90
VII	−14	−280	+172	− 49	+ 74	+111
VIII	+ 5	−355	+177	+ 7	+ 87	+ 95
IX	+24	−406	+177	+ 55	+101	+ 66
X	+34	−415	+172	+ 91	+110	+ 24
XI	+31	−386	+162	+ 88	+107	+ 13
XII	+ 3	−285	+143	+ 32	+ 84	+ 38

Now, after correcting the reductions to the group-mean by the calculated A, as the relative errors of the values of the micrometers, I have found the mean deviations for the six stations to be

Mizusawa	±0″.033	Gaithersburg	±0″.028
Tschardjui	±0.045	Cincinnati	±0.034
Carloforte	±0.046	Ukiah	±0.026

and those for the order of the differences of the zenith-distances to be

0′ — ±1′.0	±0″.032
±1.0 — ±2.0	±0.038
±2.0 — ±3.0	±0.035
±3.0 — ±4.0	±0.033
±4.0 — ±5.0	±0.033
±6.0 — ±7.0	±0.036
±7.0 — ±8.0	±0.061

These numbers show that the corrected individual value of the reduction to the group-mean has no considerable dependence either upon the stations or upon the differences of the zenith-distances.

It will however be noted that this discussion would fail if there is any abrupt displacement or readjustment of the focus. At my station, in the autumn of 1901, the readjustment of the focus was made, but fortunately the scale-reading on it remained sensibly unchanged.

The corrections for the errors of the values of the micrometer upon the final mean values (from the six stations), of the reductions to the group-mean, of the group-differences, and of the corrections to the aberration-constant, are slight in the present case, rarely rising to ±0″.02. But at those stations in which the errors of the values of the micrometers are pretty large, the final latitudes may be thereby affected directly and considerably, because, although Δz_m's were chosen so as to be zero at a certain epoch, they will become pretty considerable, as time elapses, on account of the precession. Thus I dare to say that, for the latitude-work of the high precision at the present time, such systematic errors of the values of the micrometers ought not to be wholly neglected.

There exists still another kind of corrections for the calculation of the latitude-variation. They come from some terms of the nutation of short period which are neglected in the *Jahrbuch*, the maximum double amplitude being 0″.06. These might of course have a slight effect upon the mean latitude of a group, especially in cases when the group was observed on the days for which the corrections of this kind have mostly the same sign.

Mizusawa International Latitude Station, 1903 *June*.

OBSERVATIONS OF COMET *c* 1903 (BORRELLY),

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By THEO I. KING.

[Communicated by Rear Admiral C. M. CHESTER, U.S.N., Superintendent.]

Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
1903 h m s			m s	′ ″	h m s	° ′ ″			s	″
June 29 15 2 52.3	1	29 , 6	+1 40.09	+ 6 33.4	21 42 52.63	+ 0 55 6.5	*n*8.3458	0.7315	+2.65	+15.7
30 14 53 16.7	2	30 , 6	+0 56.88	+ 0 54.8	21 40 53.07	+ 2 33 58.7	*n*8.4703	0.7148	+2.69	+15.4
July 1 15 5 8.1	3	30 , 6	+1 16.18	− 4 27.6	21 38 35.79	+ 4 19 22.1	7.8216	0.6960	+2.73	+15.1
2 14 59 28.5	4	30 , 6	+3 4.92	+ 2 46.7	21 36 1.35	+ 6 13 58.6	7.9222	0.6741	+2.77	+14.9
6 15 32 42.0	5	30 , 6	+2 42.58	+ 7 9.6	21 21 19.32	+16 12 16.5	9.1429	0.5379	+2.94	+13.8
7 14 40 7.9	6	30 , 6	+0 40.32	− 1 6.2	21 16 21.76	+19 15 39.7	8.7066	0.4686	+2.99	+13.5
7 15 2 25.3	7	30 , 6	−1 22.40	− 4 49.1	21 16 16.62	+19 18 45.8	8.9901	0.4724	+2.99	+13.4
8 14 57 50.9	8	30 , 6	+2 39.44	+ 6 52.8	21 10 16.43	+22 48 8.3	9.0468	0.3943	+3.05	+13.2
9 14 28 35.4	9	30 , 6	+1 6.94	+ 1 42.3	21 3 20.32	+26 34 4.3	8.8714	0.2751	+3.10	+12.9
13 13 1 32.1	10	14 , 3	−0 23.00	+ 6 34.6	20 18 23.04	+44 45 26.7	8.2874	*n*9.9639	+3.40	+13.0
15 11 50 1.6	11	30 , 6	+1 38.90	+ 2 40.8	19 37 31.33	+54 27 10.6	*n*8.7406	*n*0.3742	+3.52	+14.6
17 9 49 8.5	12	20 , 4	−2 15.62	+13 16.9	18 34 28.85	+62 39 50.0	*n*9.4535	*n*0.5199	+3.38	+17.0
21 9 32 35.5	13	24 , 5	−4 48.89	+ 5 20.9	15 14 4.51	+68 30 34.0	9.8358	*n*0.5207	+0.74	+17.6
23 9 59 29.1	14	29 , 6	−0 41.19	+ 9 28.2	13 53 3.68	+65 59 44.4	9.9976	*n*9.8306	−0.15	+13.1
28 8 44 49.2	15	30 , 6	−3 35.45	−11 17.1	12 16 49.37	+57 7 44.1	9.9049	0.0996	−0.20	+ 5.1
Aug. 5 9 0 24.1	16	30 , 6	−1 25.91	+ 0 55.0	11 22 36.37	+46 35 49.7	9.8189	0.6495	+0.14	− 1.7
11 8 43 0.6	17	20 , 4	+2 58.13	+ 1 51.8	11 0 37.11	+41 3 22.0	9.7677	0.7247	+0.31	− 5.0

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	21 41 9.89	+ 0 48 17.4	Nicolajew, A.G. 5509	10	20 18 42.64	+44 38 39.1	Bonn, A.G. 14116
2	21 39 53.50	+ 2 32 48.5	Albany, A.G. 7593	11	19 35 48.91	+54 24 15.2	Camb.(U.S.), A.G. 6117
3	21 37 16.88	+ 4 23 34.6	Albany, A.G. 7580	12	18 36 41.09	+62 26 16.1	Hels.-Gotha, A.G. 9906
4	21 32 53.66	+ 6 10 57.0	Leipzig II, A.G. 10845	13	15 18 52.66	+68 24 55.5	Christiania, A.G. 2298
5	21 18 33.80	+16 4 53.1	Berlin A, A.G. 8723	14	13 53 45.02	+65 50 3.1	Christiania, A.G. 2079
6	21 15 38.45	+19 16 32.4	Berlin A, A.G. 8695	15	12 20 25.02	+57 18 56.1	Hels.-Gotha, A.G. 7156
7	21 17 36.03	+19 23 21.5	1 Peg., Newc. Fund. Cat.	16	11 24 2.14	+46 34 56.4	Bonn, A.G. 8060
8	21 7 33.94	+22 41 2.3	Berlin B, A.G. 8126	17	10 57 38.67	+41 1 35.2	Bonn, A.G. 7881
9	21 2 10.28	+26 32 9.1	Camb.(Eng.) A.G. 12121				

BROOKS'S PERIODICAL COMET, *NOVA GEMINORUM.*

A dispatch from Prof. CAMPBELL, received Aug. 19, at Harvard College Observatory, states that BROOKS's periodical comet was found by AITKEN at the Lick Observatory, on Aug. 18, in the following position:

$$1903 \text{ Aug. } 18.8500 \text{ Gr. M.T.}, \quad \alpha = 21^h\,10^m\,11^s.3 \quad , \quad \delta = -27°\,4'\,19''$$

Also, that the spectrum of *Nova Geminorum* was observed on Aug. 17, by CURTIS, to be of the nebular type.

CORRIGENDUM.

No. 544, p. 154, col. 1, line 19 from top; *for* +82°.0 *put* +882°.0.

CONTENTS.

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MEAN RESULTS OF THE MEASURES OF 227 DOUBLE STARS,

BY ERIC DOOLITTLE.

The double star work at the Flower Observatory during the past three years has consisted of the measurement of somewhat more than 1000 pairs. These pairs were selected as follows; (1), about 550 BURNHAM stars, including those which are in rapid motion, or on which there are no recent measures; (2), about 300 HOUGH stars, the intention being to re-measure during the next few years all of the stars discovered by Dr. HOUGH; and (3), about 150 miscellaneous pairs, including the binaries of which Dr. SEE has computed the orbits, and several neglected stars of STONE, the Washburn Observatory, Berlin, etc. Each star was measured on an average of from three to four nights, each night's observation consisting of at least four measures of double distance, and four of position angle.

These measures will in time be published as Vol. II, Part II, of the Publications of the Flower Observatory; as this volume will probably not appear for some time, it is thought best to publish now a few of the results.

In the following table, the stars are arranged in the order of their right-ascensions, and to save space the right-ascensions and declinations are not written. The fifth column refers to the notes at the foot of the table, and the sixth shows the number of nights of observation on each star. Stars for which orbits have been computed are marked with an asterisk.

Star	Date	θ	ρ	Notes	n	Star	Date	θ	ρ	Notes	n	Star	Date	θ	ρ	Notes	n
		°	″					°	″					°	″		
β484	1902.11	156.4	1.80	*a, o*	3	β396	1902.71	66.9	1.27	*a, o*	3	OΣ52	1901.01	117.1	0.73	*v*	3
β253	1901.76	46.5	0.60	*a, o*	6	Ho.213	1901.86	203.0	0.34	*t, y*	2	β84	1901.90	23.1	0.69	*t*	3
β485	1901.81	300.3	0.41	*c*	5	β1228	1900.83	278.1	0.80	*f, y*	2	β878	1901.99	69.4	1.51	*g*	1
β1026	1900.80	336.6	0.42	*a, k*	2	β235 Aa	1900.84	95.9	0.80	*b*	2	β533	1901.88	47.5	0.61	*c*	3
β998	1900.78	115.2	1.18	*k*	3	β1162	1900.90	148.8	0.40	*a, b*	2	β1181	1901.09	86.7	0.45	*o*	3
β1015	1900.82	119.4	0.58	*f*	3	β1100	1900.86	41.2	0.49	*t*	2	β538	1900.84	132.0	1.79	*g*	3
β779	1901.07	254.1	1.05	*c*	4	β503	1900.78	132.2	5.81	*g*	3	β1184	1900.96	269.4	0.53	*z*	3
β1157	1902.71	86.9	1.73	*o*	3	β1229	1900.81	291.7	1.20		2	β743	1901.07	249.5	0.71	*o*	3
β394	1901.29	278.9	1.12	*a, o*	4	β1163	1901.85	215.1	0.34	*v, y*	2	β542	1900.82	188.8	1.42	*g, y*	3
β107	1900.78	355.8	5.96	*g, h*	3	β783	1902.73	318.1	0.86	*a, o*	3	β1004	1900.80	138.2	1.76	*c*	3
β1158	1902.81	150.1	0.41	*b, d*	3	β870	1901.85	53.6	1.14	*c*	2	OΣ80	1901.63	178.0	0.71	*c*	3
β780	1902.71	141.3	2.70	*a, o*	2	β509	1900.90	254.7	0.68	*k*	3	*OΣ82	1901.57	117.7	0.66		3
β395	1902.80	113.6	0.57	*t*	1	β1016	1900.90	207.8	0.58	*z*	3	β789	1901.79	320.9	1.24	*a, o*	2
β257	1900.80	235.0	0.62	*k*	2	β1001	1900.80	4.3	1.16	*z*	1	β882	1900.85	225.8	2.35	*g*	4
β866	1900.80	70.8	1.55	*a, o*	2	β260	1902.78	237.9	0.77	*b, d*	3	β1044	1900.92	227.8	0.83	*t, y*	4
β1160	1901.86	117.1	1.28	*a, o*	2	β515	1902.71	241.0	1.50	*a, o*	2	*β883	1901.98	70.3	0.22		2
β232 AB	1900.87	336.2	0.30	*v*	1	β873	1902.77	24.3	2.13	*a, o*	3	β552	1901.98	203.3	0.56	*v*	2
AC	1900.84	294.2	28.37	*b*	4	β1172	1900.82	241.0	1.71	*t, y*	2	β313	(No trace of companion)				4
β781	1902.86	28.8	1.10	*a, o*	4	β518	1902.78	142.0	1.55	*a, o*	4	OΣ92	1901.00	255.7	2.97	*b*	3
β496	1901.82	2.7	5.44	*a, o*	2	β519	1900.95	47.4	0.80	*t*	2	β1238	1900.91	6.5	1.30	*z*	2
β498	1902.76	154.3	2.82	*a, o*	5	β306	1901.75	20.1	3.17	*a, o*	4	β1047	1900.87	46.3	0.30	*v*	1
β1028	1900.80	260.5	2.27	*t*	2	β83	1901.98	101.7	1.03	*c, e*	3	β886	1900.97	255.5	0.99	*b*	2
β499	1900.80	347.3	52.77	*t*	3	β307	(No trace of duplicity.)					β191	1901.07	24.3	3.55	*a, o*	4
β302	1900.82	105.5	0.61	*b*	3	β525	1902.38	143.3	0.32	*v*	5	β1048	1901.05	352.8	2.33	*z*	3
Ho. 493	1902.34	20.2	35.14	*h, y*	3	β400	1899.76	52.8	23.00	*h, y*	8	β1049	1900.89	297.6	0.68	*z*	2

Star	Date	θ	ρ	Notes	n	Star	Date	θ	ρ	Notes	n	Star	Date	θ	ρ	Notes	n
		°	″					°	″					°	″		
β892	1901.61	273.3	1.25	*a, o*	4	*Σ1728	1903.31	190.0	0.68		8	β1129	1900.55	337.5	0.40	*z*	2
β1055	1903.13	336.2	1.52	*z*	4	β609	1901.83	359.1	0.81	*a, z*	3	β142	1900.56	336.5	1.61	*b, d*	3
β1058	1901.97	260.2	0.34	*a,v,y*	1	β221	1901.24	48.9	1.62	*k*	3	β827	1900.55	266.0	0.89	*k*	2
β1242	1901.85	120.1	0.42	*a, z*	3	β800	1901.34	113.2	2.76	*d*	3	β658	1901.60	302.5	0.51	*t*	2
Ho.513	1901.41	3.3	1.51	*t*	3	β610	1902.07	15.9	4.04	*a, z*	8	β439	1901.61	240.1	3.12	*c*	4
β1021	1900.92	85.0	0.63	*z*	3	Ho.260	1902.34	224.3	0.70	*b*	4	β986	1901.61	238.5	4.53	*t*	4
β194	1901.08	275.6	1.14	*a, o*	4	β114	1902.21	144.4	1.46	*a, b*	4	β670	1901.60	44.2	0.55	*c, e*	3
OΣ149	1902.01	271.4	0.91	*v*	3	*Σ1768	1903.49	133.2	1.28		8	*β151	1900.82	13.5	0.63		2
OΣ154	1900.96	123.0	26.45		4	β612	1901.35	237.0	0.36	*v*	2		1901.80	14.7	0.63		3
**Sirius*	1903.12	127.8	6.31		1	β115	1901.34	229.2	1.70	*a, t*	3		1902.76	16.0	0.51		5
OΣ157	1902.05	336.4	0.71	*c*	2	Ho.542	1902.34	263.6	0.50	*z, y*	3	β1209	1901.67	292.9	0.47	*z*	2
β899	1900.93	270.1	0.75	*f*	2	β807	1903.05	240.1	1.14	*a, z*	4	β152	1901.56	97.5	0.58	*c*	3
β900	1900.95	276.2	1.60	*a, z*	2	β346	1902.51	250.1	1.40	*b, y*	5	β367	1901.79	140.3	0.52	*v*	3
β1279	1901.10	13.4	1.08	*a, z*	3	*Σ1888	1903.44	186.3	2.49		14	β68	1901.57	152.4	1.91	*o*	3
OΣ170	1901.77	108.7	1.51		3	β350	1901.51	158.8	1.19	*k*	3	β368	1901.59	87.1	0.70	*c, h*	3
β330	1901.10	216.0	1.29	*a, o*	4	Ho.60	1902.35	35.7	0.37	*m*	4	β251	1901.61	232.4	3.05	*z*	3
β1024	1901.08	96.9	1.34	*t, g*	3	β227	1903.25	174.8	2.25	*c, h*	3	β271	1901.76	238.8	3.12	*b, d*	3
β578	1902.10	45.5	2.28	*c, e*	7	β228	1903.26	221.8	1.05	*c*	3	β164	1901.67	244.1	0.72	*m*	3
β332	1901.64	171.4	0.90	*f, h*	4	*Σ1937	1901.57	7.0	0.87		4	β684	1901.65	123.6	1.05	*c*	3
*β101	1901.13	301.7	0.61		4		1903.34	15.2	0.99		7	β1212	1901.79	273.6	0.60	*f, h*	3
β902	1901.10	243.1	1.25	*a, g*	4	*Σ1938	1903.40	67.4	1.01		7	Ho.608	1902.15	119.2	0.52	*z*	5
β581 AB	1900.91	289.7	0.44	*b, v*	3	*OΣ298	1903.40	186.0	1.27		10	β1213	1901.44	310.7	0.85	*t*	3
AC	1900.96	195.5	4.72	*b, v*	4	*Σ1967	1903.45	115.1	0.74		10	Ho.610	1902.15	239.9	0.67	*z*	5
*Σ1196	1901.05	361.9	1.17		7	β620	1901.15	163.0	0.67	*a, k*	3	β842	1902.74	119.7	1.29	*e, z*	3
	1903.18	354.0	1.21		4	β946	1902.40	147.9	1.64	*a, g*	5	β375	1902.71	308.5	0.81	*a, z*	3
β205	1903.04	230.5	0.70	*v*	6	β621	1901.36	56.8	0.54	*v*	3	β1215	1902.76	271.9	1.68	*a, z*	3
β587	1903.21	141.1	0.83	*c*	3	*Σ2032	1903.39	214.4	4.51		8	Ho.179	1902.74	258.0	0.51	*n*	5
β589	1901.48	216.3	3.22	*h*	4	β813	1901.44	168.2	1.04	*a, z*	3	β376	1902.40	150.7	3.67	*a, z*	3
*Σ1356	1901.09	113.1	0.79		6	β815	1901.34	339.3	9.15	*b, d*	3	Ho.180	1902.73	227.6	0.68	*t*	4
	1903.24	116.9	0.80		3	β817	1901.52	328.8	1.16	*a, z*	4	β1216	1901.79	319.2	0.57	*m*	3
β909	1901.25	89.6	6.28	*h*	3	*Σ2084	1903.42	205.5	1.23		7	β379	1902.22	333.7	1.27	*a, z*	4
OΣ215	1902.53	210.2	0.88	*c*	4	β821	1901.44	313.2	1.28	*a, z*	4	β172	1902.30	7.4	0.73	*c*	6
β1281	1901.19	67.9	0.96	*t*	3	β823	1903.46	11.8	1.03	*b, e*	3	β291	1901.81	176.9	0.44	*b*	2
β1073	1902.36	41.9	3.31	*h*	3	β1118	1901.43	247.2	0.52	*v*	2	β844	1902.31	317.2	3.14	*a, z*	3
β1074	1901.27	203.9	2.62	*t*	3	β628	1903.30	352.7	0.49	*c*	4	β175	1902.37	318.6	1.39	*a, z*	3
β915	1901.30	230.2	1.47	*h*	4	β128	1902.25	324.3	4.05	*m*	4	Ho.295	1902.74	327.7	0.32	*t*	2
β597	1901.64	40.6	0.75	*g*	4	β1250	1901.47	68.3	2.02	*b*	4	Ho.296	1902.74	73.9	0.25±	*t*	2
*Σ1523	1901.21	148.7	2.37		6	β1089	1901.38	346.9	0.79	*c*	2	β710	1901.80	241.0	0.42	*p*	3
	1903.31	142.2	2.42		6	*Σ2173	1903.41	326.8	1.19		8	β711	1902.27	44.0	0.95	*v*	6
OΣ237	1901.15	263.3	1.28	*c, d*	3	β961	1901.58	139.8	8.15	*a, z*	4	β851	1902.24	161.1	2.32	*a, h*	2
β602	1901.37	81.7	0.55	*t*	1	β1251	1901.48	65.1	1.30	*c*	4	OΣ489	1901.99	49.5	1.04	*v*	3
β603	1901.27	313.2	0.80	*c*	3	Ho.560	1902.34	89.7	0.47	*t*	3	Ho.197	1902.70	103.2	0.43	*t*	3
β794	1902.79	184.1	0.43	*v*	4	Ho.70	1902.41	106.5	0.43	*t*	3	β716	1901.77	205.3	1.72	*z*	3
β919	1901.35	12.9	4.46	*t*	3	*A.C.7	1901.49	59.8	1.77		5	β79	1901.65	85.1	0.93	*c, e*	3
β28	1901.86	7.6	2.14	*b*	4		1902.46	63.0	1.86		3	β80	1901.84	12.1	0.34	*v*	3
Ho.537	1902.34	174.3	1.21	*t, y*	4		1903.35	65.4	1.78		5	Ho.301	1901.75	351.2	1.49	*c*	4
β607	1901.68	315.7	1.02	*z*	4	β358	1902.91	204.1	4.49	*a, z*	4	β1221	1902.77	145.8	1.93	*z*	6
*Σ1670	1901.19	328.8	5.99		6	*Σ2262	1903.45	257.3	2.13		6	β1149	1901.79	305.2	0.60	*z*	2
	1903.29	328.0	6.05		4	Ho.565	1902.42	66.4	0.37	*a, z*	4	β720	1901.84	170.3	0.43	*v*	3
OΣ256	1901.87	81.1	0.68	*b*	3	*A.C.15	1903.37	332.2	1.54		7	OΣ500	1902.09	333.0	0.69	*b*	3
β1082	1902.79	93.7	1.41	*b*	4	β1091	1900.52	29.4	0.36	*i*	3	β723	1902.79	168.1	3.64	*z*	5
β929	1901.31	223.8	0.61	*m*	3	β641	1900.56	344.1	1.08	*c*	3	β858	1901.80	262.1	0.66	*c, d*	2
β930	1901.40	119.5	2.90	*t*	3	β648	1901.57	221.7	1.23	*v*	5	β1223	1902.81	298.0	1.28	*z*	6
β799	1901.37	248.1	0.76	*b, d*	3	β1204	1901.59	13.8	0.42	*t*	2	β1152	1901.83	100.3	0.74	*z*	3
												β861	1902.74	177.2	1.39	*a, z*	6

NOTES.

(*a*) No recent measures; (*b*) The angle has certainly increased; (*c*) The angle has certainly decreased; (*d*) The distance has increased; (*e*) The distance has decreased; (*f*) The angle has probably increased; (*g*) The angle has probably decreased; (*h*) Probable increase of distance; (*i*) Probable decrease of distance; (*k*) The suspected change is not confirmed by these measures. (*m*) Observations very discordant; probably fixed. (*n*) Observations very discordant; motion is probable. (*o*) Fixed; (*t*) The character of the motion is uncertain; (*v*) Rapid motion; (*y*) Further measures are much needed; (*z*) The pair is probably fixed.

The Flower Observatory, 1903 *Aug.* 20.

MEASURES OF *SIRIUS*, *ξ BOOTIS* AND F. 70 *OPHIUCHI*,

BY ERIC DOOLITTLE.

The following measures were made with the 18-inch refractor of the Flower Observatory; the fourth column states whether, while making the measures, the line joining the eyes was Parallel to (*P*), or at Right-angles to (*R*), the line joining the stars, and the fifth column gives the number of measures of position angle and double distance on each night.

Sirius.

On only one night during the past year was the companion seen with perfect distinctness. The diffraction rings were then clear and steady, and such satisfactory measures could be secured that I do not think it probable that the resulting angle can be in error by more than five degrees.

	°	″	
1903.123	121.8	6.51	*P* (*a*)
	127.2	6.55	*P* (*a*)
	127.9	6.30	*P* (*a*)
	129.3	6.16	*P* (*a*)
	130.9	6.14	*R* (*b*)
	129.2	6.27	*R* (*b*)
	127.9	6.30	*R* (*b*)
	128.6	6.22	*R* (*b*)
1903.123	127.85	6.31	1*n*

(*a*), Lamp to the right. (*b*), Lamp to the left. Seeing not quite so good as at first.

The position for this date computed from Dr. SEE's elements* is

127°.76 6″.16

The agreement is thus very exact.

ξ Bootis.

	°	″		
1903.269	187.14	2.53	*R*	4
1903.291	187.05	2.35	*R*	4
1903.334	184.75	2.39	*R*	6
1903.414	187.22	2.47	*P*	4
1903.416	184.83	2.65	*R*	4
1903.433	186.14	2.48	*R*	4
1903.463	185.65	2.45	*R*	4
1903.485	186.63	2.51	*R*	4
1903.490	186.30	2.57	*R*	6
1903.496	187.35	2.33	*R*	4
1903.499	185.95	2.39	*R*	4
1903.504	187.24	2.47	*P*	4
1903.534	186.35	2.65	*R*	4
1903.542	186.01	2.56	*R*	4
1903.441	186.33	2.49		14*n*

* There is a misprint in Dr. SEE's article on this star published in *A.J.* 418. The value of *a* should be 8″.0316, instead of 8″.0136. This error does not occur in the "*Evolution of the Stellar Systems.*"

The Flower Observatory, 1903 *Sept.* 6.

The residuals, (O—C), from Dr. SEE's ephemeris are

1903.44 +30°.5 +1″.22

It is thus evident that the companion is departing from the orbit in a remarkable manner. A trial orbit indicates that the angles obtained by HERSCHEL in 1780 and 1792 are considerably too large, and that the period will largely exceed the value of 128.0 years assigned by Dr. SEE. According to Dr. SEE's elements, the companion should pass the periastron this year, but the above measures indicate that the periastron passage will not occur before 1905.7. It should be noticed that measures during the next four years will be of the utmost value in fixing the values of the elements.

F. 70 *Ophiuchi.*

	°	″		
1903.414	203.92	1.81	*R*	4
1903.416	202.86	1.84	*R*	8
1903.433	202.24	1.87	*R*	8
1903.463	201.73	1.83	*R*	4
1903.485	199.67	2.00	*P*	4
1903.490	197.35	1.82	*R*	4
1903.534	196.73	2.05	*P*	6
1903.551	198.48	1.86	*R*	4
1903.638	200.33	1.79	*R*	4
1903.668	195.64	1.89	*R*	4
1903.671	199.36	1.91	*P*	4
1903.675	198.05	1.93	*R*	4
1903.537	199.70	1.88		12*n*

The residuals, (O—C), from the orbits of Dr. SEE and Dr. SCHUR, and from my own orbit, (*A.J.* 400), together with the residuals from my measures of 1897–1900 are as follows:

	SEE °	SCHUR °	°	SEE ″	SCHUR ″	″	*n*
1897.54	−8.44	−10.56	−3.13	+0.03	−0.28	+0.11	13
1899.45	−5.21	−11.59	+2.79	+0.08	−0.30	+0.02	3
1900.57	−6.81	−15.95	+1.12	+0.21	+0.06	+0.14	6
1903.56	−2.62	−17.21	−0.27	+0.25	+0.24	+0.03	12

The steady departure from SCHUR's orbit still continues; it is remarkable that the residuals from this orbit are still increasing, though it is now seven years since the companion passed periastron. While the measures now agree reasonably well with the positions predicted by Dr. SEE and myself, neither of these ephemerides satisfy the prior observations with the exactness which might reasonably be expected in a star of this character.

It may be added that during the past three years I have examined the pair on every night when the definition was unusually good. Even when stars of 0″.2 were separable, the components of F. 70 *Ophiuchi* appeared round with all powers.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. Chester, U.S.N., Superintendent.]

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
(20) *Massalia.*										
d h m s			m s	' "	h m s	° ' "			s	"
May 17 10 29 30	1	30 , 6	+0 8.11	+9 47.7	14 49 17.73	−15 50 8.3	n8.929	0.853	+2.92	− 6.2
17 10 52 21	2	30 , 6	−2 20.96	+5 14.3	14 49 16.76	−15 50 4.4	n8.578	0.854	+2.92	− 6.0
19 9 59 41	3	17 , 4	+1 56.55	−3 22.1	14 47 30.08	−15 41 48.5	n9.099	0.850	+2.91	− 6.5
21 10 11 39	3	30 , 6	+0 10.82	+4 59.6	14 45 44.36	−15 33 26.7	n8.913	0.852	+2.92	− 6.4
21 10 18 11	4	30 , 6	+0 22.04	+2 19.4	14 45 44.15	−15 33 25.7	n8.836	0.852	+2.92	− 6.5
(68) *Leto.*										
May 9 10 51 7	5	29 , 6	+0 7.64	+0 7.4	15 32 44.76	−21 2 30.5	n9.293	0.868	+3.00	− 2.5
13 10 21 49	6	30 , 6	+0 22.17	−0 39.9	15 28 56.83	−20 59 48.0	n9.333	0.866	+3.04	− 3.0
21 11 15 38	7	30 , 6	−1 36.61	+0 15.3	15 21 7.90	−20 52 5.6	n8.371	0.879	+3.11	− 3.8
28 10 28 58	8	30 , 6	+1 22.04	+0 55.6	15 14 33.20	−20 44 5.8	n8.701	0.878	+3.13	− 4.7
June 1 9 49 15	9	30 , 6	+0 46.39	−4 33.2	15 11 1.68	−20 39 26.0	n8.971	0.876	+3.13	− 5.1
(69) *Hesperia.*										
June 2 10 29 7	10	30 , 6	+0 10.62	−5 17.0	15 40 45.98	− 9 29 23.4	n8.781	0.816	+2.94	− 1.4
3 10 8 2	10	29 , 6	−0 31.77	−2 45.8	15 40 3.59	− 9 26 52.2	n8.970	0.815	+2.94	− 1.4
14 10 11 29	11	30 , 6	+0 47.88	−9 28.2	15 32 59.84	− 9 5 23.6	8.176	0.814	+2.94	− 1.2
15 10 37 57	11	27 , 6	+0 14.13	−8 9.2	15 32 26.09	− 9 4 4.5	8.888	0.812	+2.94	− 1.1
(39) *Laetitia.*										
June 2 9 39 40	12	30 , 6	+1 6.62	−4 57.0	15 42 34.73	− 3 50 24.5	n9.202	0.772	+2.84	− 0.8
3 9 24 5	12	30 , 6	+0 20.66	−3 31.3	15 41 48.77	− 3 48 58.8	n9.254	0.771	+2.84	− 0.8
8 9 39 42	13	30 , 6	−0 17.06	−6 24.3	15 38 6.04	− 3 44 13.3	n9.022	0.772	+2.85	− 0.3
8 10 1 44	12	25 , 5	−3 22.83	+1 13.5	15 38 5.30	− 3 44 13.3	n8.792	0.773	+2.86	− 0.1
14 11 34 26	14	30 , 6	+0 48.95	+1 0.1	15 34 3.89	− 3 43 45.5	9.242	0.771	+2.84	− 0.2
(129) *Antigone.*										
June 3 10 48 59	15	30 , 6	−0 46.82	+2 2.2	16 33 29.84	− 1 31 56.2	n9.065	0.754	+2.87	+ 2.7
8 10 56 14	16	30 , 6	+0 48.30	+0 38.2	16 29 33.34	− 1 43 20.9	n8.745	0.756	+2.90	+ 3.0
14 12 20 22	17	30 , 6	−0 43.74	+0 27.7	16 25 5.31	− 2 5 9.1	9.218	0.758	+2.94	+ 3.4
14 12 37 30	18	30 , 6	−1 40.36	−2 10.4	16 25 4.89	− 2 5 11.4	9.293	0.757	+2.94	+ 3.4
15 11 10 49	19	30 , 6	+0 51.69	−3 42.3	16 24 26.17	− 2 9 20.3	8.588	0.760	+2.94	+ 3.3
(17) *Thetis.*										
May 21 12 13 10	20	30 , 6	−1 2.68	−0 40.6	17 6 26.55	−14 13 49.6	n9.077	0.843	+2.90	+ 4.2
28 12 57 52	21	30 , 6	+0 12.03	+2 37.0	17 0 32.09	−14 11 12.3	8.608	0.845	+3.02	+ 4.2
28 12 58 10	22	30 , 6	−0 3.75	+1 13.4	17 0 32.03	−14 11 11.6	8.614	0.845	+3.02	+ 4.2
June 2 11 19 12	23	30 , 6	−2 24.39	−3 15.4	16 56 0.78	−14 11 55.1	n9.049	0.843	+3.08	+ 4.1
3 11 20 8	24	30 , 6	+2 21.72	+0 57.3	16 55 4.46	−14 12 18.5	n9.002	0.844	+3.10	+ 3.8
(43) *Ariadne.*										
June 18 10 41 5	25	30 , 6	−2 15.62	+1 24.9	16 48 44.75	−22 58 18.8	n8.694	0.888	+3.44	+ 3.6
21 9 23 22	26	20 , 8	+0 0.30	+0 40.7	16 46 15.71	−22 43 54.4	n9.262	0.877	+3.45	+ 3.2
(432) *Pythia.*										
June 18 11 32 39	27	27 , 5	−1 52.50	+4 25.8	16 55 48.22	−19 16 40.1	8.662	0.872	+3.35	+ 4.4
21 10 9 49	28	24 , 8	+0 38.51	−1 38.3	16 52 52.88	−19 39 56.8	n8.991	0.871	+3.37	+ 4.0
(405) *Thia.*										
June 21 12 2 14	29	30 , 6	+3 55.37	+3 22.3	17 23 44.24	−19 35 42.0	8.872	0.872	+3.39	+ 6.4
30 11 31 43	30	40 , 8	+0 11.12	+2 13.8	17 16 10.20	−18 31 19.9	8.999	0.866	+3.41	+ 6.2
30 11 39 1	31	29 , 6	−0 34.61	+1 29.6	17 16 9.94	−18 31 17.9	9.060	0.865	+3.42	+ 6.3
July 1 10 8 24	32	25 , 5	−3 30.72	−3 55.1	17 15 28.75	−18 25 9.7	n8.814	0.867	+3.42	+ 6.5

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(57) *Mnemosyne.*					
d h m s			m s	′ ″	h m s	° ′ ″			s	″
July 18 10 45 9	33	30 , 6	−2 51.88	−0 52.4	19 51 38.13	+ 1 6 8.2	*n*9.211	0.730	+3.26	+17.7
21 10 55 4	34	30 , 6	+0 20.46	+2 35.5	19 49 23.66	+ 0 59 24.3	*n*9.069	0.731	+3.29	+18.1
22 10 1 30	34	30 , 6	−0 22.56	+0 7.1	19 48 40.65	+ 0 56 56.0	*n*9.319	0.732	+3.30	+18.2
24 10 17 10	35	30 , 6	−0 24.18	+5 38.4	19 47 11.05	+ 0 51 20.1	*n*9.211	0.733	+3.31	+18.5
25 10 56 26	36	20 , 4	+2 13.93	+2 59.2	19 46 25.28	+ 0 48 13.6	*n*8.892	0.733	+3.31	+18.5
					(270) *Anahita.*					
July 18 11 39 3	37	29 , 6	+2 42.50	+7 50.3	20 38 21.58	−14 43 10.5	*n*9.188	0.843	+3.34	+19.9
21 11 33 14	37	29 , 6	+0 1.98	+3 49.7	20 35 41.10	−14 47 10.8	*n*9.137	0.844	+3.38	+20.2
22 10 41 13	38	30 , 6	+1 24.22	+4 40.8	20 34 47.92	−14 48 37.3	*n*9.356	0.836	+3.39	+20.2
23 10 54 40	38	24 , 8	+0 27.43	+3 2.4	20 33 51.14	−14 50 15.7	*n*9.286	0.840	+3.40	+20.2
24 11 14 50	39	25 , 5	+3 29.35	−5 50.6	20 32 53.22	−14 51 58.8	*n*9.160	0.844	+3.43	+20.2
Aug. 7 10 7 6	40	30 , 6	−1 9.86	−3 35.5	20 19 31.35	−15 21 1.6	*n*9.158	0.847	+3.55	+20.3
7 10 28 43	41	30 , 6	+0 50.06	−3 33.8	20 19 30.58	−15 21 3.0	*n*9.002	0.850	+3.55	+20.2
9 10 15 26	42	30 , 6	+1 42.94	−7 1.2	20 17 44.34	−15 25 35.3	*n*9.032	0.850	+3.56	+20.0
9 10 33 23	43	30 , 6	−0 1.70	+7 39.8	20 17 43.70	−15 25 36.7	*n*8.852	0.851	+3.56	+20.1
11 10 59 44	44	30 , 6	+1 17.72	−7 2.1	20 16 0.42	−15 30 11.7	7.483	0.853	+3.56	+19.9

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	14 49 6.70	−15 59 49.8	W., A.G.Z.52, 114, 209	23	16 58 22.09	−14 8 43.8	Wash. A.G.Z. 43, 122
2	14 51 34.80	−15 55 12.7	" " 51, 114, 209	24	16 52 39.64	−14 13 19.6	Rad. 1890, 4407
3	14 45 30.62	−15 38 19.9	Newcomb's Fund.Catal.	25	16 50 56.93	−22 59 47.3	Cape 1885, 1184
4	14 45 19.19	−15 35 38.6	" " "	26	16 46 11.96	−22 44 38.3	Rad. 1890, 4380
5	15 32 34.12	−21 2 35.4	Cincinnati 1885, 2632	27	16 57 37.37	−19 21 10.3	Cincinnati 1885, 2797
6	15 28 31.62	−20 59 5.1	" " 2626	28	16 52 11.00	−19 38 22.5	" " 2789
7	15 22 41.40	−20 52 17.1	" " 2613	29	17 19 45.48	−19 39 10.7	" " 2842
8	15 13 8.03	−20 44 56.7	" " 2591	30	17 15 55.67	−18 33 39.9	U.S.N. Obs. Tr.Cir.Pos.
9	15 10 12.16	−20 34 47.7	" " 2586	31	17 16 41.13	−18 32 53.8	" " "
10	15 40 32.42	− 9 24 5.0	Wien, A.G.Z. 57, 141	32	17 18 56.05	−18 21 21.1	Rad. 1890, 4530
11	15 32 9.02	− 8 55 54.2	" " 258, 326	33	19 54 26.75	+ 1 6 42.9	Nicolajew, A.G. 5027
12	15 41 25.27	− 3 45 26.7	U.S.N. Obs. Tr.Cir.Pos.	34	19 48 59.91	+ 0 56 30.7	" " 5006
13	15 38 20.25	− 3 37 48.7	" " "	35	19 47 31.92	+ 0 45 23.2	Newcomb's Fund.Catal.
14	15 33 12.10	− 3 44 45.4	" " "	36	19 44 8.04	+ 0 44 55.9	Nicolajew, A.G. 4985
15	16 34 13.79	− 1 34 1.1	Nicolajew, A.G. 4184	37	20 35 35.74	−14 51 20.7	Rad. 1890, 5564
16	16 28 42.14	− 1 44 2.1	" " 4166	38	20 33 20.31	−14 53 38.3	Wash., A.G.Z. 68, 129
17	16 25 46.11	− 2 5 40.2	" " 4151	39	20 29 20.44	−14 46 28.4	" " 72, 129
18	16 26 42.31	− 2 3 4.4	" " 4153	40	20 20 37.66	−15 17 46.4	Rad. 1890, 5487
19	16 23 31.54	− 2 5 41.3	" " 4140	41	20 18 36.97	−15 17 49.4	Wash., A.G.Z. 66, 131
20	17 7 26.33	−14 13 13.2	Wash., A.G.Z. 46, 120	42	20 15 57.84	−15 18 54.1	" " 64, 131
21	17 0 17.04	−14 13 53.5	" " 43, 120	43	20 17 41.84	−15 33 36.6	" " 63, 131, 219
22	17 0 32.76	−14 12 29.2	" " 43, 120	44	20 14 39.14	−15 23 29.5	" " 64, 131

OBSERVATIONS OF BROOKS'S COMET (1881 V),

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By C. W. FREDERICK.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1903 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Aug. 20 11 37 39	1	27 , 6	−1 22.66	−4 55.4	21 1 27.31	−27 4 26.5	8.820	0.903	+3.73	+21.5
21 11 20 21	2	23 , 5	+1 55.95	+2 38.7	21 0 43.53	−27 4 12.9	8.576	0.903	+3.75	+20.9

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	21 2 46.24	−26 59 52.3	C.G.C. 28982	2	20 58 43.83	−27 7 12.5	C.G.C. 28872

THE WHITE SPOT ON *SATURN*,

By E. E. BARNARD.

This object, observations of which were given in *A.J.* 542–543, promises to be of unusual interest, because its period of rotation is much longer than that previously attributed to *Saturn*.

It is not often that the average observer has the chance to determine the rotation period of *Saturn*, for conspicuous spots on the planet are very rare — the last one, a conspicuous white spot, was observed by HALL at Washington in December, 1876. The observations of that spot gave a period of $10^h\ 14^m\ 23^s.8 \pm 2^s.30$, according to Professor HALL (*A.N.* 2146).

It would seem that the only determination previous to HALL's, was by Sir WM. HERSCHEL in 1794, when he found the period to be $10^h\ 16^m\ 0^s.4$, which might be in error by 2^m. This period was determined from some peculiarity of the belts, and not from a definite spot.

Since HALL's observations in 1876, no conspicuous spot has appeared on the planet, though faint spots have been reported by one or two observers, but they could not be seen by others.

The present opportunity, therefore, is a rare one, and the period of the planet (or of the spot, since the spots of *Saturn* doubtless have different proper motions) ought to be well determined this time, for the object is both conspicuous and distinct.

At the second observation of this spot — that of June 24 — it was seen that the period of $10^h\ 14^m$ would not fit the observations, and that the rotation time must be decidedly longer. It was later found to be about $10^h\ 39^m$.

In *A.N.* 3883, K. GRAFF of the Hamburg Observatory, by combining the observation here of June 23 with an observation at Hamburg on June 26, and one at Bamberg, by HARTWIG, on the same date, found the period to be $10^h\ 39^m.01$.

This period is some 25 minutes longer than HALL's. It is therefore important that the spot be observed as carefully as possible, so that its exact period may be determined.

It is well known that the different spots on *Jupiter* have different rotation periods in general, but there has never been observed any such great difference as that indicated above for *Saturn*.

At an observation here of this spot on Aug. 2, with the 12-inch, it was very distinct and easy. The probability is, therefore, that it may last out the season of *Saturn's* present apparition.

There seem to be several spots, and it is well to avoid confusion in their identification.

Following is a continuation of the observations of the original white spot. The observations of June 23 and 24 will be found in *A.J.* 542–543.

1903 *July* 6 (transit $12^h\ 41^m$).

A note says: "The observation refers to the following of two spots."

July 13 $12^h\ 30^m$ no spots visible at this time.
13 20 there is a luminous spot following the center. It is long and irregular.
14 5 the main body not quite in transit.
14 9 in transit.
14 14 in transit.
14 16 I think it is past transit.
14 17 certainly past transit. There is a smaller, and not so distinct a spot, joining this, following and separate from the larger by a dark patch.

Adopted time of transit $14^h\ 11^m$.

July 14 the spot above (observed on the 13th) identified with certainty. Following are observations of its transit:

$11^h\ 19^m$ not quite in transit.
11 24 not quite in transit.
11 25 in transit.
11 27 in transit.
11 29 I think it is past transit.
11 30 uncertain yet.
11 32 a little uncertain yet.
11 34 it is certainly past transit.

Adopted time of transit $11^h\ 26^m$.

There is a small spot following. No other spots visible on the disc. At $11^h\ 41^m$ the preceding spot was conspicuously past transit. Distance between the spots = $3''.5$.

At $12^h\ 30^m$ the two spots are past transit, and no others visible.

Aug. 2 $13^h\ 46^m$ a luminous spot very nearly in transit.
13 49 perhaps not yet in transit.
13 52 in transit.
13 55 in transit.
13 56 in transit, fairly well seen; small and distinct.
13 58 perhaps past transit.
14 0 perhaps past transit; a little uncertain.
14 2 it seems to be past now, but a little uncertain.
14 8 past, but not decidedly so.
14 10 decidedly past transit.

The time of transit would be close to $13^h\ 57^m$.

It was slightly elongated.

No other spot visible on the disc.

I believe that these are all observations of the original white spot, though there may be some uncertainty on account of there being two spots at this point.

The following observations are also of white spots, but they do not seem to be of the original spot.

June 30 13^h 19^m a definite white spot, quite well defined at its following end; in transit at the above time. "If this is the spot seen before, it is not near so conspicuous as it was."

July 7 13 20 a small, luminous, round spot, $1\frac{1}{2}''$ in diameter, in transit. It is on a cold steel blue or bluish gray narrow belt. No other spot visible.

13 28 it is, I think, past transit.

July 20 11 24 a small, oblong, white spot, in transit. There is a small white spot preceding it, which is on the narrow bluish belt, and breaks the continuity of the belt. No other spots visible.

July 20 11^h 55^m no other spots visible.

July 21 12 35 there seems to be a white spot past transit.

July 27 12 30 can see no spots.

If the observations of Aug. 2 are of the original white spot, a rough approximation would make the period

$$10^h\ 38^m.8$$

The observations of the spot on July 13 and 14 (where identification was absolutely certain) are consistent with the above period.

I hope to make an exact determination of the periodic time, at the close of the observations of these spots.

Yerkes Observatory, Williams Bay, 1903 *Aug.* 6.

THE RING NEBULA IN *LYRA*, AND THE DUMB-BELL NEBULA IN *VULPECULA*, AS GREAT SPIRALS,

By J. M. SCHAEBERLE.

In No. 539 of this *Journal* the announcement was made that the Ring nebula in *Lyra*, as shown by photographs taken with my reflector, is a spiral. An examination of the plate published in the same number of the *Journal* will show that each of the two main branches, on leaving the center of the nebula, divide up into a number of small arcs, as though the several observed stream-lines represented the debris along the paths of different out-going masses which originally left a common origin at the same instant in diametrically opposite directions, *but with various radial velocities.**

The *inner* streams of each branch after completing something more than a semi-circle intersect the *outer* streams of the opposite branch before the latter have completed their first quadrant. The overlapping of these streams causes an ever-widening nebulous area of greater density, which, added to the effects of the inclination of the plane of the nebula to the line of sight naturally reaches a maximum at points near the minor axes. The outer streams of each branch, after completing the first 180°, form with the superposed inner streams which are at a greater angular distance from the origin, the inner boundary of the main nebula, and at still greater angular distances from the origin these compound heavier streams† gradually approach the outer boundary of the nebula and disappear in the neighborhood of the two ends of the major axis after having completed something more than five quadrants for the outer streams of each branch, and probably several revolutions for the inner streams.

* That each of any two fluid or gaseous masses at the instant of separation should break up into parts having various radial velocities would seem to be much more probable than that the whole mass of each half should have a common velocity.

† Two superposed gaseous spirals having a common origin, but different initial velocities, would produce a spiral structure composed of arcs of greater density where these spirals overlap, connected by the less dense areas formed by the separated individual streams.

In photographic prints so timed that only a faint nebulosity is visible near the extremities of the major axis, the appearance of the whole nebula is so strikingly similar in form to the Dumb-bell nebula in *Vulpecula* that it might easily be mistaken for the latter object by one who had not made a special study of these nebulas. (This resemblance led me to examine the Dumb-bell nebula photographically with the result mentioned farther on.)

If the outer boundary of the Ring nebula represents the extreme distance which the inner streams have reached at the present visual stage of the nebula, and if the arrangement of the matter composing the nebula results from the action of gravitational forces, then theory requires that the outer streams of each branch should exist at still greater distances from the origin. These outer streams, no longer reinforced by the inner ones, would require greater optical power to render them visible.

Photographs of long exposure which I have taken at various times under ordinary atmospheric conditions show much streaky nebulosity in various position angles, concave towards the center of the Ring nebula.

On August 21, 1903, while making the preliminary adjustments, it was at once apparent that the atmosphere was most unusually pure and steady. A series of exposures from 2^m up to 35^m duration were made, and the resulting negatives reveal the fact that the various exterior nebulosities, including BARNARD's small nebula,* and practically all the neighboring stars, are apparently part of the same great spiral of which the well-known Ring nebula forms but a comparatively small central part.

The enlargement of the negative which had an exposure of 35^m, sent with this paper, may not be suitable for reproduction. On this particular occasion a much longer exposure might well have been given with better results.

* See *Astr. Nach.*, No. 3200. This nebula is visible on negatives having an exposure of one minute.

The general curvature of these exterior streams is unmistakably that of a clock-wise spiral whose maximum visible diameter is about one-quarter of a degree. One gets the impression that if these exterior streams were brighter a structure similar to the Whirlpool nebula (M. 51) would result; the nucleus of the latter being regarded as a ring nebula on a small scale.

As a result of the photographic examination of the Dumb-bell nebula, mentioned above, the very first series of negatives, taken several weeks ago, revealed, unmistakably, that this object is a great *counter* clock-wise spiral, at least half a degree in diameter, the well-known nebula occupying the central area. This central area (plainly elliptical in outline on the photographs) seems to be formed in much the same way as the above-described Ring nebula, except that the several streams into which the main branches divide are much more divergent near the origin. The several exterior streams, which are very regular, and like the Ring-nebula, studded with faint stars, appear to make but little more than a complete revolution up to points 15′ distant from the origin,* while the inner streams probably make several revolutions before reaching the bright exterior boundary of the Dumb-bell only about 4′ from the center.

I have been waiting for a good night to make a long exposure on this nebula, but as this opportunity may not soon be available, a photograph, which may possibly be reproduced so as to show the spirals, is sent herewith. It is a 15-diameter enlargement of a negative exposed for 20 minutes on July 22, 1903.

Additional interest will attach to planetary nebulas in general if it shall be found that the well-known objects of this class are but parts of structures similar to the two considered in the present article.

*The streams probably extend much farther from the center, but in my instrument the aberrational effects become so large that the results are uncertain at greater distances from the optical axis.

Ann Arbor, 1903 *Sept.* 1.

OBSERVATIONS OF 7793 *SS CYGNI*,

BY ZACCHEUS DANIEL.

SS Cygni has recently acted in a very strange manner. The maximum in February was anomalous in form, and similar to that of December, 1899, a curve of which was published by J. A. PARKHURST in *Popular Astronomy*, V. 8, p. 46, and the *Astrophysical Journal*, V. 12, p. 268. The observations indicate a stand-still at a point where the rise is usually very rapid. This maximum was also observed by HARTWIG (*A.N.* 3866). After a short period of normal brightness following the April maximum, I found it on May 10, apparently, somewhat brighter, but in a few days it returned to normal. On May 25, however, it was certainly above normal, but a few days later it had become faint again. That was probably the end of a short maximum. The next maximum was also short. The rise began on June 21. On the decrease, a stand-still occurred at $10^M.5$. The maximum in July is noteworthy in that the decrease was somewhat checked toward the end, the approach to normal brightness being comparatively slow. This has also been observed at several previous maxima.

Following are my observations this year to date. They were made by ARGELANDER's method, with apertures ranging from 12.7 cm. to 58.4 cm. The magnitudes are on the visual scale given in the *Astrophysical Journal*, V. 12, p. 260. The decimals of a day are in Greenwich M.T.

1903	J.D.	Mag.	1903	J.D.	Mag.
Jan. 1	6116.553	8.73	June 29	6295.744	9.49
6	121.533	10.30	30	296.691	9.87
10	125.526	11.32	July 1	297.776	10.02
18	133.521	11.28	3	299.816	10.52
Feb. 5	151.515	10.29	4	300.802	10.59
6	152.524	10.13	6	302.743	11.32
9	155.517	9.35	9	305.656	11.37
10	156.506	9.35	10	306.737	11.37
12	158.507	8.76	15	311.657	11.27
14	160.503	8.86	16	312.684	11.27
Apr. 8	213.771	8.66	19	315.716	11.17
9	214.889	8.76	21	317.839	11.27
22	227.864	11.22	27	323.858	8.36
May 10	245.776	11.05	28	324.708	8.46
13	248.673	11.12	29	325.828	8.66
15	250.712	11.17	31	327.833	8.66
17	252.721	11.22	Aug. 1	328.665	8.76
25	260.802	9.74	2	329.831	8.96
28	263.780	10.92	5	332.618	9.70
June 2	268.742	11.37	7	334.653	10.26
3	269.767	11.37	9	336.625	10.98
12	278.641	11.42	10	337.583	11.02
13	279.724	11.32	11	338.826	11.12
21	287.708	10.98	12	339.805	11.27
26	292.781	8.76	14	341.790	11.22
27	6293.660	8.96	17	6344.810	11.32

The Observatory, Princeton, N.J., 1903 *Aug.* 28.

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ORBIT OF COMET 1900 II,

By J. M. POOR.

1. Introduction.

The definitive orbit of this comet has already been published as it was determined by Manoel Soares de Mello e Simas from a very thorough discussion of nearly all the published observations.* The following elements of the same comet had been deduced, and were nearly ready for the printer when De Mello's paper was received. Publication has been delayed that the discussion of the perturbations might be carried considerably farther than was at first intended, though most of the computations were completed by the writer while Fellow at Princeton University. The writer congratulates himself as well as the astronomical public in general on the close agreement of the results as assuring confidence in the numerical accuracy of the computations, though this work would not have been undertaken had occupation of the same field by another been known. The following paper is abridged in order to avoid repetition so far as is consistent with the satisfactory statement of results.

This comet, independently discovered on July 23, 1900, by Borrelly, at Marseilles, and Brooks of Geneva, U.S.A., was observed for a little more than three months, while an attempted observation by Aitken at Mt. Hamilton, five months after discovery, though interrupted by rising fog, showed the comet to be nearly in its predicted position, and indicated that the orbit was nearly parabolic.

2. Elements, Ephemeris, and Stars.

The elements here employed for comparison and correction are based on those computed by Scheller and Wedemeyer (*A.N.* 3660). They are

$$T = 1900 \text{ August } 3.19930 \text{ Gr. M.T.}$$

$$\left.\begin{aligned} \omega &= \;\; 12^\circ\, 25'\, 34''.80 \\ \Omega &= 328\;\; 0\;\; 26.20 \\ i &= \;\; 62\;\; 30\;\; 44.00 \end{aligned}\right\} \text{Ecliptic 1900.0}$$

$$\log q = 0.0063830$$

* *Astronomische Abhandlungen als Ergänzungshefte zu den Astronomischen Nachrichten herausgegebenen* von Prof. Dr. H. Kreutz, Nr. 4, 1 Teil.

which give

$$\begin{aligned} x &= r\,(9.9457964)\ \sin(\;\; 86^\circ\, 20'\, 32''.37 + v) \\ y &= r\,(9.6867100)\ \sin(283\;\; 8\;\; 27.03 + v) \\ z &= r\,(9.9966354)\ \sin(\;\;\; 0\;\; 9\;\; 27.35 + v) \end{aligned}$$

Throughout the computation Greenwich mean time has been employed, and solar coordinates have been taken from the *American Ephemeris and Nautical Almanac.*

Comparisons with the ephemeris from these elements were made by interpolation for the date of observation corrected for aberration-time from positions computed at intervals of twelve or twenty-four hours, as the case might require, except for observations in right-ascension between August 20.5 and August 29.5, and in declination between August 23.5 and August 27.5, when the motion of the comet near the pole made it seem best to compute the place for the time of each observation directly from the elements.

As far as possible A.G. star places have been employed, but for polar stars *Carrington* has been used. Several star places are the result of comparison with A.G. stars or Berlin J.B. stars, and others are the result of meridian observations published with the comet observations. D.M., Cincinnati, Nos. 12 and 13, Berlin J.B. 1900, Gr. Obs. 1896, Romberg, and Hamburg zones have been used, and some positions have been modified by Pulkowa Obs., Yarnall, Greenwich Ten-Year Catal., Radcliffe (1860.0), and Gr. Obs. 1898. Sixteen A.G. Dorpat places were kindly furnished in manuscript by Profs. Levitzky and Scharbe of Dorpat Observatory. Romberg was consulted for me by Mr. D. T. Wilson at Cincinnati, and *Fedorenko* by Prof. Searle at Harvard, though the latter positions were not employed. Proper motions have been applied whenever found. The position of the star 100 of De Mello's list, to which he calls attention, was found to be R.A. = $3^h\, 34^m\, 33^s.54$; Decl. = $71^\circ\, 58'\, 42''.1$ as deduced from positions furnished from Dorpat. The reduction of observations depending on the star *Carrington* 685 (De Mello's star 120) indicated a proper motion in declination, and its comparison with Nos. 708 and 720 of the same Catalogue, made at Princeton

by the computer, yielded as a result an annual proper motion in R.A. of $-0^s.0057$, and in Decl. of $-0''.1338$ (equator 1903.0). Its position for 1900.0, as deduced from these observations, was found to be R.A. $= 4^h\ 46^m\ 37^s.92$; Decl. $= 81°\ 49'\ 57''.5$.

In reducing stars to the epoch 1900.0 the constants of STRUVE and PETERS have been uniformly employed, and above 75° declination rigorous formulas were used.

3. Observations and Weights.

It was intended to collect all observations, and a request for such as had not been published was inserted in *A.N.* 3753. The usual journals were examined, but by an oversight all observations published in *Comptes Rendus* only remained unknown to the computer until the appearance of DE MELLO's work. These included three observations made at Algiers, the valuable series of Bordeaux and Lyons, with the less numerous series made at Marseilles, Paris and Toulouse. To those omitted must also be added the series made at Washington, and published in *A.J.* 535, too late for consideration in this discussion, and likewise omitted by DE MELLO, as were also two unpublished observations made at Pulkowa, which were kindly furnished by the observer, A. SOKOLOW. For completeness the latter are here inserted.

		Comet—Star		Parallax				
1900 Gr. M.T.		R.A.	Decl.	R.A.	Decl.	R.A.	Decl.	Comp. Star
		m s	′ ″	s	″	h m s	° ′ ″	
September	6.29000	+2 27.65	−5 27.3	+1.67	+2.5	13 24 3.06	78 58 32.7	A.G. Kasan 2333
	11.33524	+0 35.58	−2 7.2	+1.11	+3.8	13 46 53.67	76 2 46.7	" " 2428

In reducing observations parallax factors have all been recomputed by means of BAUSCHINGER's Tables and reductions from mean to apparent place have been made by the use of constants of the *American Ephemeris and Nautical Almanac.*

For convenience in weighting, observations have been divided into three groups. Group 1 consists of those cases where more than five observations were made with a filar micrometer by one observer. Group 2 is made up of those series more than five in number made by a single observer with a ring or bar micrometer; and Group 3 includes all series less than six in number by a single observer. The only apparent exceptions to these rules are two cases in Group 3 in which the series numbered six, but these were subsequently reduced to five or less by rejected observations. Such rejections were determined as follows:

First. Because recent or accurate catalogue-places of companion stars were not found, or because no catalogue place was found: Arcetri, Aug. 19 (comp.-star *Fedorenko*); Hamburg (Sch.), Sept. 19; Hamburg (M), Sept. 19; Kiel; July 28; Königsberg (S), Aug. 23; Paris, July 24; Pola (H), July 25; Pola (M), July 25. Letters in parenthesis indicate the observer as explained in the list below.

Second. Because of serious discrepancy with other observations made at nearly the same time: Denver (L), July 28, Decl.; Denver (H), July 30 and all other observations in declination; Heidelberg (C), July 25, Decl.; Heidelberg (V), Aug. 18, Decl.; Kiel, July 25, Decl.; Kremsmünster, Aug. 19, Decl.; Lemberg, Aug. 10 and all other observations in declination; Leipzig, July 25, Decl.; Göttingen, July 25; Utrecht (V), Aug. 16, Decl. and Sept. 5, R.A.; Geneva, July 26, Decl.; Padua, Aug. 1, second obs. Decl. In no case was the attempt made to harmonize by changing the date of an observation.

In determining weights each remaining residual was first given unit weight, and curves, one for each coordinate, were constructed by means of normals at convenient intervals. These curves were assumed to represent the true path of the comet with which the residuals of each observer were compared, thus giving "secondary" residuals from the first powers of which the computed weight of an observation was found by the formulas

$$r = 0.8453\,\frac{[v]}{m} \quad \text{and} \quad p = \frac{c^2}{r^2}$$

in which c, the mean value of r for all observers whose observations numbered more than 5, was found to be $0^s.16$ in R.A., and $1''.9$ in Decl.

Observations of Group 1 were then given the integral weights 2, 3, or 4, according to the numerical value of the weight as computed. In the same way, those of Group 2 received weights ½ or 1, while the weights of Group 3 were assigned arbitrarily because of the small number in each series.

From the results of comparisons with the curves it was thought best to apply corrections to observations as follows:

Correction.

Observatory	R.A.	Decl.
	s	″
Arcetri,	−0.25	. .
Geneva,	+0.20	+2.3
Kiel,	. .	+2.9
Kremsmünster,	−0.34	. .

The following table contains details outlined above with references to journals where observations were found.

Observatory	Observer		Group	R.A. Obs.	R.A. p	Decl. Obs.	Decl. p	Journal
Algiers	Sy		1	8	4	8	3	*B.A.*, Vol. 18
Arcetri	Abetti		1	42	2	42	2	*A.N.* 3674, 3687. Arcetri Pub., No. 15
Besançon	Chofardet	= C	1	12	4	12	4	*A.N.* 3656, 3691. *B.A.*, Vol. 18
	Sallet	= S	3	2	1	2	1	*A.N.* 3656
Copenhagen	Pechule		3	3	1	3	1	*A.N.* 3654, 3655
Denver	Heller	= H	3	6	1	6	0	*A.J.* 508
	Ling	= L	1	22	3	22	3	*A.J.* 492, 503
Geneva	Pidoux		1	8	2	8	2	*A.N.* 3660
Göttingen	Schur		3	1	0	1	0	*A.N.* 3655
Hamburg	Scheller	= Sl	1	13	3	13	4	*A.N.* 3655, 3724
	Schorr	= Sch.	3	3	1	3	1	*A.N.* 3724
	Messow	= M	3	6	1	6	1	*A.N.* 3724
Heidelberg	Courvoisier	= C	3	4	1	4	1	*A.N.* 3654, 3720
	Valentiner	= V	3	4	1	4	1	*A.N.* 3720
Jena	Knopf		2	6	1	6	1	*A.N.* 3692
Kiel	Ristenpart		2	8	1	8	1	*A.N.* 3654, 3655
Königsberg	Cohn	= C	3	1	1	1	1	*A.N.* 3655
	Struve	= S	1	12	4	12	4	*A.N.* 3760
Kremsmünster,	Schwab		2	8	1	8	1	*A.N.* 3760
Leipzig	Hayes		3	1	1	1	0	*A.N.* 3654
Lick	Crawford	= C	3	4	1	4	1	*A.J.* 484
	Aitken	= A	1	10	3	10	2	*A.J.* 490
	Perrine	= P	3	3	2	3	2	*A.J.* 494
Lemberg	Ernst		2	15	½	15	0	*A.N.* 3655, 3740
Nicolaeff	Kortazzi		1	11	2	11	3	*A.N.* 3677
Northampton	Byrd		3	2	1	2	1	*A.J.* 495
Paris	Bigourdan		3	1	0	1	0	*A.N.* 3654
Padua	Autoniazzi		1	13	3	13	3	*A.N.* 3678
Pola	Höhl	= H	3	3	½	3	½	*A.N.* 3661
	Marchetti	= M	3	1	0	1	0	*A.N.* 3661
Poughkeepsie	Whitney		3	2	1	2	1	*A.J.* 495
Pulkowa	Sokolow		3	2	2	2	2	Correspondence.
Rome	Millosevich		3	4	2	4	2	*A.N.* 3655, 3745
Strassburg	Kobold		1	10	4	10	4	*A.N.* 3654, 3726
Utrecht	Nijland	= N	1	12	3	13	3	*A.N.* 3654, 3719
	Veenstra	= V	1	11	2	11	2	*A.N.* 3719
Vienna	Palisa	= P	3	2	½	2	½	*A.N.* 3655, 3718
	Holetschek	= H	3	3	½	3	½	*A.N.* 3713

4. Perturbations.

Perturbations taking into account the action of *Mercury*, *Venus*, *Earth*, *Mars*, *Jupiter* and *Saturn* were computed in rectangular coordinates referred to the ecliptic, with July 29.0 as the date of osculation. A uniform interval of 40 days was adopted for *Mars*, *Jupiter* and *Saturn*, while for *Mercury*, *Venus* and *Earth*, the interval was 10 days until September 17, after which it was 20 days. Referred to the equator as fundamental plane the perturbations in rectangular coordinates deduced were as follows:

Perturbations (Seventh decimal place).

Date		δx	δy	δz
July	19	− 1	0	0
	29	0	0	0
Aug.	8	− 11	+ 0	− 3
	18	− 30	+ 4	− 12
	28	− 55	+ 10	− 25
Sept.	7	− 86	+ 20	− 43
	17	−121	+ 33	− 67
Sept.	27	−160	+ 49	− 96
Oct.	7	−201	+ 67	−131
	17	−244	+ 88	−172
	27	−288	+110	−219

5. Normal Places and Least-Square Solution.

For the construction of normal places the list of observations was divided into eight sections at the following dates: July 29.0, Aug. 3.0, Aug. 11.0, Aug. 20.0, Sept. 1.0, Sept. 14.0, and Oct. 10.0. To find a normal place with its corresponding time and weight, the following equations were applied to the observations of each section:

$$t_0 = \frac{[pt]}{[p]}, \quad (\cos\delta\,.\,\Delta\alpha)_0 = \frac{[p\,.\cos\delta\,.\,\Delta\alpha]}{[p]},$$

$$\Delta\delta_0 = \frac{[p\,.\,\Delta\delta]}{[p]} \text{ and } p_0 = [p]$$

The normal places thus found, including corrections for perturbations, are as follows:

No.	Date	R.A.	$(\cos\delta.\Delta\alpha)_0$	p_0	Decl.	$(\Delta\delta)_0$	p_0
		° ′ ″	″		° ′ ″	″	
1	July 26.7	41 37 49.84	−3.98	115	20 51 8.27	− 0.04	114
2	31.7	43 9 30.24	−2.24	116	36 9 55.38	− 2.88	113.5
3	Aug. 8.1	46 51 52.18	−2.90	76.5	57 41 49.99	− 2.26	70
4	16.1	56 36 8.51	−3.28	72	75 14 47.31	+ 3.50	65
5	24.84	116 19 3.02	+5.46	61	85 42 33.68	+ 3.52	65
6	Sept. 8.62	204 2 46.37	+8.68	50	77 33 48.50	− 9.82	54
7	22.0	213 22 48.83	+8.37	64	71 23 36.87	−13.42	60
8	Oct. 19.4	224 5 18.06	+4.11	44	65 58 25.52	−16.37	41

For the least-square solution the elements were referred to the mean equator of 1900.0 and differential coefficients computed by means of the formulas given in Klinkerfues, *Theoretische Astronomie, Zweite Auflage.*

The elements so referred are

$T = $ 1900 Aug. 3.19930 Gr. M.T.

$$\left.\begin{aligned} \omega' &= 0^\circ\ 9'\ 27''.35 \\ \Omega' &= 331\ \ 43\ \ 41.17 \\ i' &= 82\ \ 52\ \ 37.55 \end{aligned}\right\}\ \text{Equator 1900.0}$$

$\log q = 0.0063830$

Equations of the form

$$\cos\delta.\frac{\partial\alpha}{\partial\Omega'}d\Omega' + \cos\delta.\frac{\partial\alpha}{\partial i'}di' + \cos\delta\frac{\partial\alpha}{\partial\omega'}d\omega' + \cos\delta.\frac{\partial\alpha}{\partial T}dT + \cos\delta.\frac{\partial\alpha}{\partial q}dq + \cos\delta.\frac{\partial\alpha}{\partial e}de = \Delta\alpha.\cos\delta$$

and $$\frac{\partial\delta}{\partial\Omega'}d\Omega' + \frac{\partial\delta}{\partial i'}di' + \frac{\partial\delta}{\partial\omega'}d\omega' + \frac{\partial\delta}{\partial T}dT + \frac{\partial\delta}{\partial q}dq + \frac{\partial\delta}{\partial e}de = \Delta\delta$$

each made homogeneous and multiplied by the square root of its weight, after putting $x = d\Omega'$, $y = di'$, $z = d\omega'$, $t = 10^4 dT$, $u = 10^6 dq$ and $w = 10^4 de$, become the following:

Equations of Condition.

Right-Ascension.

1	+7.7683 x	+1.4477 y	−2.9631 z	+0.4528 t	−4.6772 u	−1.0195 w	=	−42.6807
2	+7.7522	+0.4459	−0.3558	−0.1698	−4.7957	−0.3213	=	−24.1255
3	+5.0272	−0.5654	+2.7374	−0.8271	−3.6921	+0.3836	=	−25.3646
4	+1.8913	−0.4116	+4.7915	−1.2005	−3.2057	+0.2469	=	−27.8317
5	−8.1736	+4.6175	+2.2463	−0.2434	−1.4365	−3.1798	=	+42.6440
6	−4.4486	+3.7784	−5.4038	+1.2800	+1.6025	−1.1228	=	+61.3769
7	−3.1791	+3.5780	−6.9729	+1.4305	+1.8474	+1.3630	=	+66.9600
8	−1.3642	+2.0688	−6.6233	+1.0666	+1.5173	+6.6962	=	+26.2627

Declination.

1	− 8.0679 x	−1.8922 y	+20.6923 z	−10.2847 t	−0.2164 u	−19.6231 w	=	− 0.4271
2	−13.7908	−0.9145	+17.7338	− 8.7714	−0.8230	− 5.5256	=	− 30.6825
3	−14.4487	+1.8634	+ 8.0771	− 3.8545	−0.8764	+ 4.5982	=	− 18.9085
4	−13.4567	+4.3590	+ 2.2498	− 1.0104	−0.3191	+ 3.0084	=	+ 28.2179
5	− 6.8807	+3.4364	− 4.7621	+ 1.0979	+2.1503	+ 0.3760	=	+ 28.3792
6	+ 4.5611	−4.7897	− 1.8458	− 0.2072	+1.0876	+11.6799	=	− 72.1620
7	+ 3.5866	−5.7932	− 1.2974	− 0.4515	+0.7335	+19.0539	=	−103.9509
8	+ 1.3023	−5.4001	− 0.9393	− 0.3970	+0.3366	+24.4664	=	−104.8192

in which coefficients are given in natural numbers.

These differential coefficients were checked according to the method given in the above reference with very slight modification. To check

$$\frac{\partial\alpha}{\partial T},\ \frac{\partial\alpha}{\partial q},\ \frac{\partial\alpha}{\partial e},\ \frac{\partial\delta}{\partial T},\ \frac{\partial\delta}{\partial q}\ \text{and}\ \frac{\partial\delta}{\partial e}$$

as well as the computation of the positions to which residuals had been applied in finding the normal places, increments appropriate for each date were successively applied to T, q, and e, and new positions computed from the slightly changed elements. The remaining coefficients were checked by finding $d\log\xi$, $d\log\eta$ and $d\log\zeta$ where the notation is that of the reference.

To illustrate (Klinkerfues, page 710),

$dx = -y\,d\Omega'$ or $d\log\xi = -\dfrac{m}{206265}\dfrac{y}{\xi}(d\Omega')''$ where $(d\Omega')''$ is given in seconds of arc. This in units of the seventh place becomes

$$d\log\xi = [1.82336_n]\frac{y}{\xi}(d\Omega')''$$

where numbers are expressed in logarithms. Similar expressions for

$$d \log \xi\ ,\ d \log \eta \text{ and } d \log \zeta$$

were found in terms of $(d\Omega')''$, $(di')''$, and $(d\omega')''$.

Whether the numerical values of $\log \xi$, $\log \eta$ and $\log \zeta$ were to be increased or decreased in a particular case was easily determined by inspection of the equations.

From the equations of condition normal and elimination equations given below were obtained. As a Brunsviga computing machine was employed, coefficients are given in natural numbers.

Normal Equations.

$$\begin{array}{rrrrrrcr}
+1603.4289\,w & -580.7418\,z & +287.9879\,x & +216.6062\,t & -235.4222\,y & +59.0753\,u & = & -5097.5544 \\
-580.7418 & +1007.7049 & -506.0848 & -439.4582 & -82.1140 & -85.1039 & = & -1361.9109 \\
+287.9879 & -506.0848 & +975.6906 & +248.6591 & -188.6705 & -78.1787 & = & -2284.0715 \\
+216.6062 & -439.4582 & +248.6591 & +207.4411 & +37.9990 & +27.0762 & = & +684.7583 \\
-235.4222 & -82.1140 & -188.6705 & +37.9990 & +179.8256 & -2.0807 & = & +2406.0835 \\
+59.0753 & -85.1039 & -78.1787 & +27.0762 & -2.0807 & +87.1817 & = & +604.2810
\end{array}$$

Elimination Equations.

$$\begin{array}{rrrrrrcr}
w & -0.362187\,z & +0.179608\,x & +0.135089\,t & -0.146824\,y & +0.036843\,u & = & -3.179158 \\
 & z & -0.503882\,x & -0.452748\,t & -0.209917\,y & -0.079897\,u & = & -4.023466 \\
 & & x & +0.038600\,t & -0.319781\,y & -0.167550\,u & = & -4.137199 \\
 & & & t & +0.214254\,y & -0.371981\,u & = & +2.643481 \\
 & & & & y & -1.241788\,u & = & +0.612815 \\
 & & & & & u & = & +28.268782
\end{array}$$

Whence

$$\begin{array}{rlrl}
u = & +28.269 & x = & +11.81 \\
y = & +35.72 & z = & +14.18 \\
t = & +5.5065 & w = & +3.2929 \text{ and } [nn.6] = 807.6
\end{array}$$

Reversed elimination gave identical results and also quantities by means of which were obtained

$$\begin{array}{ll}
r_w = \pm 1.0484 & r_t = \pm 1.6997 \\
r_z = \pm 2.60 & r_y = \pm 4.70 \\
r_x = \pm 2.21 & r_u = \pm 3.691
\end{array}$$

Substitution in the equations of condition gave as check $[pvv] = 807.6$. We have, therefore, the following corrections to the equatorial elements

$$\begin{array}{rlllrll}
d\Omega' = & +11''.81 & \pm 2''.21 & & dT = & +0^d.000551 & \pm 0^d.000170 \\
di' = & +35.72 & \pm 4.70 & & dq = & +0.0000283 & \pm 0.0000037 \\
d\omega' = & +14.18 & \pm 2.60 & & de = & +0.0003293 & \pm 0.0001048
\end{array}$$

Collecting results the definitive equatorial elements are

Epoch of osculation, July 29.0, 1900

$T =$ 1900 August 3.199851 Gr.M.T.

$$\left.\begin{array}{rl}
\omega' = & 0^\circ\ 9'\ 41''.53 \\
\Omega' = & 331\ 43\ 52.98 \\
i' = & 82\ 53\ 13.27
\end{array}\right\}\text{Equator 1900.0}$$

$$\begin{array}{rl}
\log q = & 0.0063951 \\
e = & 1.0003293 \\
\log e = & 0.0001430
\end{array}$$

from which are found

$$\begin{array}{l}
x = r(9.9458067)\ \sin(\ 86^\circ\ 21'\ \ 7''.40 + v) \\
y = r(9.6866365)\ \sin(283\ \ \ 7\ 44.84 + v) \\
z = r(9.9966447)\ \sin(\ \ 0\ \ \ 9\ 41.53 + v)
\end{array}$$

and also

$$\left.\begin{array}{rl}
\omega = & 12^\circ\ 25'\ 40''.55 \\
\Omega = & 328\ \ \ 0\ 47.62 \\
i = & 62\ 31\ 16.38
\end{array}\right\}\text{Ecliptic of 1900.0}$$

As a final check on the accuracy of the computation the residuals due to elements and those due to equations were determined for each normal place. They are as follows:

	Elements		Equations	
	$\Delta\alpha \cos\delta$	$\Delta\delta$	$\Delta\alpha \cos\delta$	$\Delta\delta$
	''	''	''	''
July 26.7	+1.1	+0.3	+1.0	+0.3
" 31.7	−1.1	−0.3	−1.0	−0.3
Aug. 8.1	−0.4	−0.1	−0.5	−0.2
" 16.1	+1.1	−0.7	+0.8	−0.5
" 24.84	+0.8	+1.7	+0.7	+1.7
Sept. 8.62	−1.0	−0.4	−1.0	−0.4
" 22.0	−1.4	+0.2	−1.4	+0.3
Oct. 19.4	+1.1	+0.3	+1.1	+0.3

From the equations $[vv] = 11.2$

Collecting, the definitive elements referred to the ecliptic are

Epoch of osculation, July 29.0, 1900

$T =$ 1900 Aug. 3.199851 Gr.M.T.

$$\left.\begin{array}{rl}
\omega = & 12^\circ\ 25'\ 40''.55 \\
\Omega = & 328\ \ \ 0\ 47.62 \\
i = & 62\ 31\ 16.38
\end{array}\right\}\text{Ecliptic 1900.0}$$

$$\begin{array}{rl}
\log q = & 0.0063951 \\
\log e = & 0.0001430
\end{array}$$

6. Perturbations Before Discovery.

A comparison of these elements with those found by De Mello shows slight differences, but both computations show the important result that at the time of discovery the comet was describing a hyperbolic orbit, and this agreement is quite close notwithstanding the considerable omission of observations from this discussion. This fact tempted the writer to make a preliminary investigation of the perturbations during the year previous to the comet's discovery.

Accordingly $\omega^2 X$, $\omega^2 Y$, and $\omega^2 Z$ (WATSON, page 453) were computed for *Venus*, *Earth*, *Mars*, *Jupiter*, and *Saturn* at intervals of 40/3 days. These quantities were directly computed for *Mars* and *Saturn* at intervals of 40 days from June 4.0, 1899, to Aug. 18.0, 1900. Likewise they were computed for *Jupiter* until May 30, 1900, after which date they were directly computed at intervals of 40/3 days. All remaining intermediate values were found by interpolation. For *Venus* and the *Earth* the quantities were directly computed at intervals of 40/3 days throughout the whole period. The quantities

$$\Sigma(\omega^2 X),\ \Sigma(\omega^2 Y) \text{ and } \Sigma(\omega^2 Z)$$

were obtained from which mechanical quadrature gave the perturbations referred to the ecliptic in the table below. The computing machine was used whenever possible. In this table perturbations at intervals of 40 days only are included.

PERTURBATIONS. (Seventh decimal place).

Date	δx	δy	δz
1899 June 4	− 35	+291	+5413
July 14	−305	+244	+4259
Aug. 23	−483	+206	+3257
Oct. 2	−569	+173	+2401
Nov. 11	−575	+156	+1689
Dec. 21	−528	+154	+1116
1900 Jan. 30	−457	+151	+ 677
Mar. 11	−367	+126	+ 360
Apr. 20	−257	+ 75	+ 154
May 30	−129	+ 21	+ 41
July 9	− 21	+ 0	+ 1

The disturbances in the components of the velocity on June 4.0 were found to be $\delta\frac{dx}{dt} = -7.9$, $\delta\frac{dy}{dt} = -1.4$, $\delta\frac{dz}{dt} = -30.8$, in the same units.

Following the method of WATSON, Sec. 168, the definitive elements gave for this date the undisturbed coordinates

$x_0 = -3.1959772$; $y_0 = -0.6369125$; $z_0 = -4.2938069$;

and the velocities

$$\frac{dx_0}{dt} = +0.00857336\ ;\ \frac{dy_0}{dt} = -0.00182998\ ;$$

$$\frac{dz_0}{dt} = +0.00574773\ ;$$

from which, after corrections for perturbations had been applied, the following elements were deduced.

Epoch of osculation, June 4.0 1899

$$\begin{aligned} T &= 1900 \text{ August } 3.24627 \\ \omega &= 12^\circ\ 25'\ 42''.5 \\ \Omega &= 328\ \ 1\ 16.6 \\ i &= 62\ 31\ 19.2 \\ \log q &= 0.006438 \\ \log e &= 0.000058 \\ e &= 1.000133 \end{aligned}$$

(ω, Ω, i: Ecliptic of 1900.0)

According to this computation, therefore, the eccentricity had increased during the 400 days previous to the comet's discovery.

Among the disturbances caused by the planets during this period those of *Jupiter* were the most important during the greater part of the time, while those of *Saturn* and *Mars* were much less, as were also those of *Venus* and *Earth*, except near the time of discovery. Therefore the quantities, $\omega^2 X$, $\omega^2 Y$, $\omega^2 Z$ for *Jupiter* alone were computed for four dates at intervals of 100 days previous to June 4, 1899. When the position and motion of the comet during this time are considered, $\omega^2 X$, $\omega^2 Y$, $\omega^2 Z$, indicate that the effect of perturbations during this earlier period was acting in the same direction.

In the light of this preliminary study it seems not unreasonable therefore to account for this hyperbolic orbit at the time of discovery as the result of perturbations, and to conclude that a careful study sufficiently extended would show that this comet entered the solar system in a sensibly parabolic orbit.

7. CONCLUSION.

Defective though it is, this piece of work is now brought to a point where it is possible for the computer to lay it aside. The long computations involved would certainly never have reached even their present degree of perfection had it not been for the continued encouragement and many suggestions received from Prof. YOUNG of Princeton.

Besides being indebted to those already mentioned, I am also indebted to Prof. ANTONIO ABETTI of Arcetri for publications from that Observatory, to Prof. OTTO KNOPF of Jena for a correction to his observations as published, and to Prof. LOVETT and Mr. HILTEBEITEL of Princeton, for actually assisting in the computations.

Shattuck Observatory, Dartmouth College, Hanover, N.H.

OBSERVED MINIMA OF 4.1903 *DRACONIS*,

BY W. M. REED.

[Communicated by Prof. C. A. YOUNG.]

Seven minima of this star have recently been observed by Mr. ZACCHEUS DANIEL and the author with a photometer attached to the 23-inch equatorial of the Halsted Observatory. A part of the expense of the these observations is borne by the Carnegie Institution. An artificial star, caused by an electric light, was compared alternately with a neighboring comparison-star and with the variable. The light of the artificial star was diminished by means of

a "photometric wedge" photographically prepared, the gift of Prof. E. C. PICKERING. The author is indebted to Mr. E. S. KING for a provisional value of the scale of this wedge. It seems desirable to make a preliminary statement of these observations in order to call attention to certain unprecedented irregularities in this *Algol*-type variable. Although the minima occur with great regularity, and satisfy the elements of Mr. BLAJKO with a correction of only 1.3 minutes, yet the magnitude at minimum is far from constant. In the seven minima observed the magnitude at minimum has ranged from 12.7 to 13.6. Also the shape of the light-curve has undergone equally pronounced changes.

On July 7, the star diminished in brightness at the rate of about $0^{\mathrm{M}}.10$ in eight minutes, until within 30 minutes of minimum, when it decreased at the rate of $0^{\mathrm{M}}.10$ in two minutes. When the minimum was reached the increase began almost at once, and in a manner symmetrical with the decrease.

On July 15, the star decreased in light at the rate of about $0^{\mathrm{M}}.10$ in eight minutes, until about 20 minutes from minimum, when the rate increased to about $0^{\mathrm{M}}.10$ in one minute. It then staid constant in brightness at minimum light for 26 minutes. The increase was symmetrical with the decrease.

August 14, the third minimum was observed. The decrease was at a fairly uniform rate until the minimum light was reached, the rate being about $0^{\mathrm{M}}.10$ in 4 minutes. The variable staid at minimum brightness for about 20 minutes. On account of clouds only one magnitude of the increase was observed. The rate of the increase was about $0^{\mathrm{M}}.10$ in one minute, and therefore unsymmetrical with the decrease.

Setember 21, the variable began to decrease $2^{\mathrm{h}}\,12^{\mathrm{m}}$ before the time of minimum. At first the rate was about $0^{\mathrm{M}}.10$ in 12 minutes; gradually the rate was increased to 4 minutes, and during the last magnitude of decrease the rate was $0^{\mathrm{M}}.10$ in 2 minutes. The star remained at minimum light for 15 minutes. The curve of increase was unsymmetrical with the decrease. At first the rate was $0^{\mathrm{M}}.10$ in 4 minutes, then in 2 minutes, and finally $0^{\mathrm{M}}.10$ in 9 minutes. Observations were discontinued before the variable reached normal light.

September 25, the decrease was at a nearly uniform rate of $0^{\mathrm{M}}.10$ in 5 minutes. The star remained at minimum light for 32 minutes. During the first 40 minutes of the increase the star became brighter at the rate of $0^{\mathrm{M}}.10$ in 3 minutes, but during the next 40 minutes the increase was at the rate of only $0^{\mathrm{M}}.10$ in 13 minutes.

September 29, the increase and decrease were nearly symmetrical. The decrease occupied $1^{\mathrm{h}}\,45^{\mathrm{m}}$ in changing 2.2 magnitudes, while the increase occupied $1^{\mathrm{h}}\,34^{\mathrm{m}}$ for the same range. The light remained constant at minimum for 21 minutes.

Epoch	Observed Minima	O—C	Mag.	Stationary Period	No. Sett.
	d h m			m	
93	July 7 15 21.8	−0.9	13.5	4	375
99	15 18 50.2	−0.8	12.8	26	260
121	Aug. 14 15 11.2	+2.7	13.2	20	153
124	18 17 9.6	+1.1	13.6	–	213
149	Sept. 21 15 37.5	−9.3	13.2	15	392
152	25 17 31.8	+0.9	12.7	32	345
155	29 19 12.5	−2.6	13.0	21	249

The epochs are reckoned from the following elements recently published by Mr. S. BLAJKO in *A.N.* 3888:

Min. = 1903 Mar. $3^{\mathrm{d}}\,9^{\mathrm{h}}\,34^{\mathrm{m}}$ Gr. M.T. , $+1^{\mathrm{d}}\,8^{\mathrm{h}}\,34^{\mathrm{m}}\,43^{\mathrm{s}}$ E

The observed minima are given in Geocentric Gr. M.T. The moment midway in the "stationary period" has been chosen as the time of minimum. On August 18 the grouping of the observations was such that the exact length of the stationary period could not be determined. The increase and decrease were so rapid that a fairly accurate time of minimum was found. The column headed O—C gives the difference between the observed minima and those computed from the above elements. The fourth column gives the magnitude at minimum derived from a preliminary value of the magnitude of the comparison-star. While the relative value of these magnitudes are accurate the absolute value of the scale may be subject to correction. The fifth column, headed "stationary period," gives the number of minutes during which the variable was sensibly constant in brightness at minimum. The last column gives the number of settings of the instrument on both the variable and the comparison-star.

There seems to be a relation between the time during which the star is nearly constant in brightness and the magnitude at minimum. It is obvious from a comparison of the fourth and fifth columns that the fainter the magnitude at minimum the shorter is the stationary period.

On August 18, the faintest minimum so far observed, the duration of the "stationary period" must have been very brief, but the grouping of the observations was such as to give no positive information as to its actual length. In this case the increase and decrease were so rapid near the minimum light that the time of minimum could be fairly accurately determined.

It is interesting to note that if the eclipsing body were a double star, in which the components were close together, that the phenomena of these curves could be in part explained. Such a hypothesis would require that occasionally there should be a "standstill" on the curve, when first one body should enter upon the eclipse position followed after a short interval by the other. Such irregularities are suspected from the observations now accumulated. But a more thorough set of observations must be made before this interesting feature can be regarded as proved.

Princeton University, 1903 *Oct.* 12.

WOLF'S "NEW STAR" IN *CYGNUS*.

By E. E. BARNARD.

Telegraphic announcement was received on Oct. 5 of the discovery of a new star 11^{M}, by Dr. MAX WOLF, in the position,

$$1903.0 \quad 20^{h}\ 14^{m}\ 57^{s}.0 \quad +37°\ 9'\ 49''$$

Observations of this object were made here on the same evening with the large telescope. The star was estimated as $10\frac{1}{2}^{M}$, and was very red.

The following measures were made of its position with reference to Lund A.G.C. 9237 ($8^{M}.9$).

$\Delta\alpha$ 74″.99 (4 Obs.) = $0^{m}\ 6^{s}.27$, $\Delta\delta$ 13″.72 (4 obs.)

The new star was south following.

Following is the position of the comparison-star.

$$1903.0 \quad 20^{h}\ 14^{m}\ 50^{s}.75 \quad +37°\ 10'\ 1''.1$$

From this the position of WOLF's star is

$$1903.0 \quad 20^{h}\ 14^{m}\ 57^{s}.02 \quad +37°\ 9'\ 47''.4$$

It was also referred to the comparison-star by position angle and distance.

P.A. 100°.22 (6 obs.) , Dist. 76″.11 (5 obs. double dist.)

In the D.M. are two stars whose places for 1855.0 are

D.M.	+37°.3875	9^m.1	20^h 13^m 4^s.0	+37° 0′.4
D.M.	+37.3876	9.5	20 13 9.6	+37 0.4

The first of these is A.G.C. Lund 9237. The second occupies exactly the position of WOLF's object, and as there is no other star at this point, I assume that the "new star" is identical with D.M. +37°.3876 though it is at least one magnitude less than that given in D.M.

I have a trial plate, made May 8, 1902, at $11^{h}\ 15^{m}$ — $12^{h}\ 0^{m}$, with the 10-inch Brashear doublet for the Bruce telescope — then being tested here, that covers the region in question. WOLF's star is strongly shown on it. It appears to be roughly 2 magnitudes less than the Lund star preceding it. The magnitude is uncertain, but roughly it is what would be expected to result from the present red color and magnitude of the star.

The star was examined by Messrs. FROST, ADAMS and REESE through a small prism placed over the eyepiece of the micrometer. But they were unable to detect bright lines in its spectrum or to note any particular resemblance to the visual spectrum of *Nova Persei* when of about the same magnitude, although the dispersion employed renders definite statements difficult. The seeing was bad which made observations somewhat uncertain.

Mr. J. A. PARKHURST measured the brightness of the star with the photometer on the 12-inch and found it, at $9^{h}\ 15^{m}$ (Central Standard time), to be $10^{M}.6$ on the Harvard College Scale of magnitude.

Measures of the position of two small stars near WOLF's "Nova" = D.M. +37°.3876.

+37°.3876 and 13^{M} star n.f.
1903 Oct. 5 30°.11 (5 obs.) 49″.19 (8 obs.)

+37°.3876 and 14^{M} star s.p.
1903 Oct. 5 206°.52 (5 obs.) 66″.17 (8 obs.)

It has not been thought necessary to further measure these stars.

Yerkes Observatory, 1903 *Oct.* 6.

NOTE ON WOLF'S "NEW STAR" OF SEPTEMBER 21, 1903,

By HERBERT A. HOWE.

In a recent Astronomical Bulletin of the Harvard College Observatory, the declination of WOLF's new star is given as +37° 9′ 49″ (1903.0), as per telegram from Kiel. This object is of 0′.2 south of Lund A.G. Catal. 9237, by an observation made at the Chamberlin Observatory on Oct. 9. Unfortunately the catalogued declination of this star is 1′ too great; this fact has been determined at the Chamberlin Observatory by micrometrical comparison of the star with Nos. 9236, 9221, and 9211 of the same catalogue. The declination of the "new star," as determined by a rough measure, is +37° 8′ 49″ (1903.0). On Oct. 10 the "new star" seemed to be of 10^{M}, and to be quite red. It appears to be identical with one of the stars in the Birmingham Red Star Catalogue, which is called Es.-Birm. 662*a* on p. 104 of the revised edition of WEBB's *Celestial Objects for Common Telescopes*; the declination there given is 3′ too small.

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WEIGHTS AND SYSTEMATIC CORRECTIONS OF MERIDIAN OBSERVATIONS IN RIGHT-ASCENSION AND DECLINATION,

By LEWIS BOSS.

The systematic corrections published in this paper were, in all cases, determined from comparison with the finally computed positions of the Standard Stars (System B., *A.J.* 531–2). There were 699 of these stars, of which 265 are south of $-22°$. Sixty-six of the latter (with six others) the motions of which have been less accurately determined, have been excluded from publication for the present.

The adopted systematic corrections for right-ascensions are contained in Tables I, II and III; those for declinations, in Tables IV and V. They do not differ materially from the corrections which were employed in computing the positions for the Catalogue, except that corrections for magnitude-equation were not employed in deriving those positions.

If Table I be employed to free the individual catalogues from the effect of magnitude-equation, the Standard Catalogue itself can be made consistent with the right-ascensions thus corrected by application of the correction,

$$-0^s.0077\ (M-3.5).$$

In this way, also, the right-ascensions of the Standard Stars would be prepared for consistent use in the reduction of transits which have been properly corrected for magnitude-equation. The Standard Catalogue, corrected in this way, has already been employed with distinct advantage in the reduction of transits for the Albany Catalogue, 1896–1900.

The derivation of the magnitude-equation, $\varDelta\alpha_m$, Table I, has already been explained (*A.J.* 536). It only remains to add that the effect of the correction has been assumed to apply with sufficient accuracy down to the seventh magnitude. One may extend the corrections by extrapolation to still fainter magnitudes, but may anticipate that, in many instances, the quantities so determined may diverge materially from the truth. In some cases the material for determination of $\varDelta\alpha_m$ is quite scanty. For Piazzi, Greenwich 15, Dorpat 15, Königsberg 15, Cape 30, Armagh 40, Cape 50, Santiago 55 and 60, and Pulkowa 1892, it was deemed safest to assume the mean value of the magnitude-equation, $-0^s.0077\ (M-3.5)$, to be applicable to the right-ascensions of those catalogues. For nearly all catalogues of a date earlier than 1850, $\varDelta\alpha_m$ is quite uncertain, owing to small weight of material for its determination.

The adopted formula of correction in computing $\varDelta\alpha_a$, Table II, and $\varDelta\delta_a$, Table IV, is:

$$a \sin\alpha + b\cos\alpha + c\sin 2\alpha + d\cos 2\alpha$$

Terms in 2α have seldom been adopted, and in instances where they were employed the object is not so much to represent a known source of error (as in the case of terms of single period) as it is to use them in lieu of a graphic solution. It is especially desirable that the part of systematic correction of the form, $a\sin\alpha + b\cos\alpha$, at least, should be removed from the individual series of right-ascensions and declinations, since the equations which express the effects of various forms of sidereal rotation, of solar motion, and of correction for imperfect precession, contain terms of this form. A term of the form,

$$\varDelta'\alpha_a = (a'\sin\alpha + b'\cos\alpha)\tan\delta$$

is sometimes employed for the right-ascensions. In Table II the values of that part of $\varDelta'\alpha_a$ contained in parenthesis is given on the line below $\varDelta\alpha_a$ for the catalogue in question. Therefore, in order to form $\varDelta'\alpha_a$, the numbers on the second line for the respective catalogues (opposite "$\times (\tan\delta)$" in the margin) must be multiplied by $\tan\delta$ and then added to the corresponding numbers of the first line; so that the entire correction, having the argument, right-ascension in whole or in part, is $\varDelta\alpha_a + \varDelta'\alpha_a$.

The supposed equinox-corrections are always included in $\varDelta\alpha_a$, Table II; since it seems to be desirable that $\varDelta\alpha_a$ should exhibit clearly the discrepancies between the meridian of the standard and those of the individual catalogues.

In Table III, which exhibits values of $\varDelta\alpha_\delta$, the values of $\varDelta\alpha_\delta\cos\delta$ for 85° and 80° of declination are also given in order to facilitate interpolation. These can usually be extrapolated for declinations higher than 85°. The curves of correction were originally determined for $\varDelta\alpha_\delta\cos\delta$. In

many cases it would be mechanically impracticable to determine them otherwise.

It is scarcely necessary to remark that the determinateness of the several curves of correction varies greatly,— from the uncertainty which pertains to such catalogues as those of Piazzi, Madras, Armagh, Cape 50, and Santiago, to the definiteness which belongs to the better modern catalogues. What a simple matter the drawing of a curve of correction may become is illustrated by the following statement of observed values of $\Delta\alpha_s \cos\delta$, or $\Delta\delta_s$, for six catalogues, selected as fair representatives of a large number of the better class of modern catalogues.

Observed Systematic Corrections.

	$\Delta\alpha_s \cos\delta$			$\Delta\delta_s$		
δ	Wn. 75	Strass. 85	Lisb. 90	Paris 60	Grw. 80	Pulk. 85
°	s	s	s	″	″	″
+77	−.015	+.023	−.016	−.16	+.02	+.06
70	−.006	−.015	. .	−.03	+.26	−.03
65	−.004	+.025	−.002	+.19	+.16	+.13
60	−.010	+.013	+.001	−.01	+.25	+.01
55	−.019	+.016	−.001	+.25	+.12	+.05
50	−.023	+.011	−.006	+.22	+.09	+.01
45	−.024	+.012	−.006	+.06	+.12	+.05
40	−.005	+.003	−.008	−.08	−.05	+.09
35	+.010	−.001	−.004	−.25	−.06	+.14
30	+.007	−.002	−.002	−.16	−.06	+.19
25	+.011	−.007	−.005	−.14	+.26	+.21
20	+.008	−.002	−.001	−.25	+.15	+.25
15	+.011	−.008	+.003	−.27	+.29	+.25
10	+.006	−.008	+.006	−.31	+.40	+.24
+ 5	−.009	+.005	+.002	−.24	+.12	+.29
0	−.007	+.010	+.006	−.21	−.02	+.18
− 5	−.012	+.012	+.007	−.15	+.26	+.21
10	−.014	+.005	−.008	−.17	+.37	+.20
15	−.010	+.002	−.006	−.09	+.43	+.02
20	+.007	−.003	−.008	+.11	+.50	−.03
−25	+.003	−.003	−.006	+.51	+.59	. .

The preceding table shows how small are likely to be the remaining systematic uncertainties in reducing a single star-catalogue to a given system of Standard Stars. The consequent inference would be that, in any future investigations involving the motions derived from thousands of stars, the degree of systematic accuracy attainable would practically be measured by that of the Standard Catalogue itself. Thus, aside from certain incidental reservations, connected with the magnitude-equation and similar points, the question whether systematic errors can be avoided in researches upon precession, solar motion, and the like, resolves itself into the question, what degree of freedom from systematic error can be attained in the construction of extensive standard catalogues. The obvious advantage of this rests in the probability that a general discussion of stellar motions based upon the few large catalogues of observation would practically enjoy whatever of freedom from systematic error attaches to the mean of all the best observations ever made.

In drawing the curves it is of very great importance that due regard be had for the weights of the observed quantities, and that constant errors, pertaining to comparatively broad zones, be avoided. The latter requirement was kept constantly in mind. As an aid in the fulfillment of these requirements the means by weight of successive zones were formed in the combinations: +77°, 70°, 65°; +70°, 65°, 60°; +65°, 60°, 55°, etc. Thus, when these are plotted, we have a series of points, at intervals of 5°, each representing the mean observed correction for a zone 15° in breadth. As a matter of fact, for nearly all the modern catalogues (and for the better part of the older) these means defined the curves very closely; so that very little was left in doubt as to the true location of the curve at any point. Of course the maxima and minima points were less sharply indicated by the means of 15°-zones; yet it was rarely deemed best to follow up sharper inflections apparently indicated by the 5°-means. The entire process, in its practical working, tends to inspire confidence in the reality and substantial accuracy of the principal features of curves, as indicated in Tables III and V.

It may not be superfluous to call attention again to the object of systematic correction of meridian observations. The object of first importance is to obtain positions and motions of stars, in large numbers, which shall be, in the mean, as free as possible from the effects of systematic error. A secondary object is to secure greater accuracy in the computed position and motion of an individual star. Some computers seem to look upon this secondary purpose as the only one worth considering; and they are apparently disappointed if each adopted systematic correction does not manifestly improve the accordance of the various catalogues in each individual instance. From this point of view a systematic correction of 0″.1 is certainly of no importance. But when we are considering stellar motions in large numbers for determination of the apex of solar motion, for example, a systematic error of 0″.1 in the centennial motions might mean an error of more than one degree in the determination of the apex. In all such researches the casual errors of observation can be reduced to a role of minor importance by including a sufficient number of stars and by taking advantage of all the principal series of observations already on record.

The Tables of $\Delta\alpha_s$ and $\Delta\delta_s$ apply to the indiscriminate means of observations at upper and lower culminations, as printed in the respective catalogues. Doubtless, greater precison could have been reached through the separation of the two classes of observation. Except in a few cases, however, this separation would have been either impossible or impracticable. On the other hand, some effort has usually been exerted by the observers to bring observations at lower culmination into substantial harmony with those made at upper culmination; and even where this has not

been accomplished we may still obtain a mean systematic correction by the treatment of the results indiscriminately.

For Bradley 1755 (Auwers), St. Helena 1830, Cape 1833, and Madras 1875, it has been assumed that graduation errors form an important part of the systematic corrections required. The facts relative to Bradley's declinations have been set forth in *A.J.* 545; and relative to Cape 1833 (Henderson) in *A.J.* 541. The particulars regarding Madras 1875 are given further on, in Note 17.

Pulkowa 1855 is not included in Tables I, II and III, since it does not contain observed right-ascensions of the principal standard stars. Meanwhile, the correction of its right-ascensions may be assumed to be the mean between those applicable, respectively, to Pulkowa 1845 and Pulkowa 1865, upon which it was based; though this process may not yield a very accurate result.

In general, no attempt has been made to ascertain the systematic corrections applicable to the various zodiacal catalogues, to annual results not collected in the form of a catalogue, to various catalogues of limited extent, and to certain modern catalogues that contain very few observations of the principal stars. The requisite computations for ascertaining the systematic corrections of these various classes of catalogues can be attended to with much greater precision and economy of labor at a later stage of this investigation.

Tables VI and VII contain, respectively, the computed weights for the right-ascensions and declinations of the various catalogues. In great part they remain as they were adopted in the final approximation for determination of the positions contained in the catalogue. For many of the catalogues of smaller weight, and especially for the more extensive catalogues, the weights have been revised since the computations for the catalogue. In nearly all cases the differences from the weights previously assumed were comparatively unimportant.

The adopted unit of weight in right-ascension is supposed to correspond to a probable error of $\pm 0^s.020 \sec\delta$; and in declination, to a probable error of $\pm 0''.30$.

In the preliminary stages of the work in right-ascension I have been much indebted to the valuable tables by Dr. Auwers, *A.N.* 3615–16. In general, the weights for large numbers of observations are assumed to be less in the tables of this paper than in those of Auwers. The theoretical factors by which the weights of Auwers should be multiplied in order to reduce them to the units of Tables VI and VII are 0.694 and 1.414, respectively. It has been considered advisable to regard $\pm 0''.1$ as a sort of limit of precision attainable in the determination of either coordinate of an individual star. Within this limit it is assumed that minute sources of error, beyond the skill of the observer to evade, may be at work, tending to reduce all observations of high class to one level of precision, however much one may apparently excel another in recognized sources of excellence. For the catalogues of a date later than 1885 the weights of the individual catalogues are largely the result of certain approximate assumptions regarding the weights of the Standard Catalogue. To have made the computations rigorously would have cost what seemed to be an unjustifiable amount of labor.

The accuracy of the tables is greatest for the number of observations most frequently occurring. The most uncertain element is the rate of increase with number of observations. The adopted formulas for computing the weights were:

$$p_\alpha = \frac{(\pm 0^s.020)^2}{K^2r^2 + \frac{r^2}{n}} \text{ for right-ascensions, and}$$

$$p_\delta = \frac{(\pm 0''.30)^2}{K^2r^2 + \frac{r^2}{n}} \text{ for declinations.}$$

In these n represents the number of observations of a star, r represents the probable error of one observation, and Kr the probable error of an infinite number of observations upon any one star. Extensive tables, in which K has values ranging from one to 0.067, permit the values of K and r to be determined with comparative ease when the material is sufficient. In general, these formulas can be regarded merely as approximations. In catalogues where the observations have been made in a variety of conditions, over a considerable period, or by several observers, it is not practicable to represent the probable errors by any simple formula. On the whole, it is believed that the method adopted leads to results for weight which are, beyond question, superior to any which can be assigned without computation, on the basis of a general judgment alone. Furthermore, it should be remembered that these weights cannot be regarded as the weights of the catalogue positions as they stand; they are the weights of the corrected positions; and they take no account of the probable error of the corrections.

The extension of the weights in right-ascension outside the equatorial zone is a process not practically capable of a high degree of accuracy in the result. For observations of transit by "eye and ear" the polar right-ascensions are usually more precise than those of a corresponding number of observations in the equatorial zone. Sometimes this is the case when the transits have been registered on a chronograph. For catalogues in reference to which we may suppose that, either the collimation, or the polar deviation, have been badly determined, the weight may be decidedly less for stars of high declination, — as in the case of the two Madras Catalogues. In Table VI the weight is usually given for the equatorial zone, "Eq." Where it is also given for higher declinations it is intended

that interpolation shall be made from ±30° (or ±20°, if preferred) to the higher declination, without extrapolation from the highest declination to the pole. In the use of these weights it has been our practice to reduce them when observations of right-ascension have been made at a zenith-distance of 72°, or greater; or of declination, at a zenith-distance of 65°, or greater, according to the following table of arbitrary factors.

FACTORS FOR WEIGHTS.

ZD	R.A.	Decl.	*ZD*	R.A.	Decl.
65°	1.0	0.9	74°	0.8	0.5
66	1.0	0.9	75	0.8	0.5
67	1.0	0.8	76	0.7	0.4
68	1.0	0.8	77	0.7	0.4
69	1.0	0.8	78	0.6	0.3
70	1.0	0.7	79	0.5	0.2
71	1.0	0.7	80	0.4	0.1
72	0.9	0.6	81	0.25	0.0
73	0.9	0.6	82	0.1	0.0

Occasionally this amount of diminution has been inferred from statements of probable error contained in the introduction to the respective catalogues.

Some references to the peculiarities of individual catalogues are contained in the series of articles upon the Standard Catalogue. Others will be found in the notes hereto appended, to which reference is made by numbers prefixed to the designation of individual star-catalogues in the tables.

TABLE I. MAGNITUDE-EQUATION.

Magnitude	2M.0	3M.0	4M.0	5M.0	6M.0	7M.0
Br. 1755	+.017	+.006	−.006	−.017	−.028	−.040
Pi. 1800	+.012	+.004	−.004	−.012	−.019	−.027
Grw. 15	+.012	+.004	−.004	−.012	−.019	−.027
Dpt. 15	+.012	+.004	−.004	−.012	−.019	−.027
Kgb. 15	+.012	+.004	−.004	−.012	−.019	−.027
Kgb. 25	+.018	+.006	−.006	−.018	−.029	−.041
Dpt. 30	+.017	+.006	−.006	−.017	−.028	−.040
Cape 30	+.012	+.004	−.004	−.012	−.019	−.027
St. H. 30	+.012	+.004	−.004	−.012	−.019	−.027
Åbo 30	+.017	+.006	−.006	−.017	−.029	−.041
Grw. 30	+.016	+.005	−.005	−.016	−.026	−.036
Camb. 30	+.008	+.003	−.003	−.008	−.014	−.020
Cape 33	+.014	+.005	−.005	−.014	−.024	−.033
1) Madr. 35	+.024	+.008	−.008	−.024	−.039	−.055
Arm. 40	+.012	+.004	−.004	−.012	−.019	−.027

TABLE II. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\delta$. RIGHT-ASCENSION.

R.A.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
	s	s	s	s	s	s	s	s	s	s	s	s	s
Br. 1755	−.075	−.078	−.081	−.083	−.086	−.088	−.089	−.090	−.090	−.089	−.087	−.085	−.083
{ Pi. 1800	+.100	+.101	+.103	+.107	+.111	+.116	+.120	+.125	+.129	+.132	+.135	+.136	+.136
{ ×(tan δ)	+.048	+.024	−.002	−.028	−.051	−.072	−.087	−.096	−.099	−.095	−.085	−.069	−.048
Grw. 15	−.025	−.022	−.018	−.015	−.011	−.007	−.004	−.002	.000	.000	−.001	−.002	−.005
4) Dpt. 15	+.034	+.034	+.032	+.027	+.021	+.013	+.004	−.005	−.014	−.021	−.027	−.032	−.034
Kgb. 15	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082
5) Kgb. 20	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034
Kgb. 25	+.019	+.022	+.025	+.030	+.033	+.036	+.038	+.039	+.040	+.039	+.037	+.035	+.032
Dpt. 30	−.016	−.014	−.011	−.010	−.009	−.009	−.010	−.011	−.013	−.016	−.019	−.022	−.025
Cape 30	+.027	+.035	+.041	+.046	+.048	+.047	+.045	+.040	+.033	+.025	+.016	+.007	−.001
St. H. 30	−.065	−.064	−.062	−.059	−.056	−.051	−.046	−.041	−.036	−.033	−.030	−.028	−.027
Åbo 30	+.014	+.014	+.015	+.016	+.017	+.017	+.018	+.018	+.018	+.018	+.018	+.017	+.016
Grw. 30	−.061	−.063	−.065	−.067	−.069	−.070	−.071	−.071	−.070	−.069	−.068	−.066	−.063
Camb. 30	−.019	−.020	−.022	−.023	−.025	−.027	−.028	−.030	−.030	−.031	−.031	−.030	−.029
Cape 33	+.006	+.006	+.007	+.008	+.011	+.014	+.017	+.021	+.025	+.028	+.031	+.034	+.035
Madr. 35	−.062	−.057	−.052	−.047	−.042	−.038	−.035	−.033	−.032	−.033	−.035	−.038	−.042
Arm. 40	+.045	+.046	+.047	+.049	+.050	+.052	+.053	+.054	+.055	+.056	+.056	+.056	+.055
Cape 40	−.001	−.005	−.007	−.010	−.012	−.014	−.015	−.015	−.015	−.013	−.011	−.009	−.006
Grw. 40	+.094	+.086	+.078	+.070	+.063	+.058	+.054	+.052	+.052	+.054	+.058	+.064	+.071
Grw. 45	+.040	+.035	+.029	+.022	+.016	+.010	+.005	+.001	−.001	−.002	−.001	+.002	+.006
Rad. 45	+.023	+.015	+.005	−.007	−.020	−.034	−.046	−.057	−.066	−.072	−.074	−.073	−.068
Pulk. 45	+.019	+.019	+.019	+.020	+.020	+.021	+.022	+.023	+.024	+.025	+.026	+.026	+.027
Paris 45	+.027	+.025	+.023	+.021	+.019	+.018	+.017	+.017	+.017	+.018	+.019	+.021	+.023
Stgo. 50	+.012	+.009	+.007	+.004	+.001	−.001	−.004	−.006	−.007	−.007	−.007	−.005	−.004
Grw. 50	+.011	+.007	+.002	−.003	−.007	−.011	−.015	−.017	−.018	−.018	−.017	−.015	−.011

TABLE I. MAGNITUDE-EQUATION. — *Cont.*

Magnitude		$2^{\mathrm{M}}.0$	$3^{\mathrm{M}}.0$	$4^{\mathrm{M}}.0$	$5^{\mathrm{M}}.0$	$6^{\mathrm{M}}.0$	$7^{\mathrm{M}}.0$
Cape	40	+.012	+.004	–.004	–.012	–.020	–.027
Grw.	40	–.001	.000	.000	+.001	+.002	+.003
Grw.	45	+.001	.000	.000	–.001	–.001	–.002
Rad.	45	+.024	+.008	–.008	–.024	–.040	–.056
Pulk.	45	+.008	+.003	–.003	–.008	–.014	–.019
Paris	45	+.007	+.002	–.002	–.007	–.012	–.017
Stgo.	50	+.018	+.006	–.006	–.018	–.030	–.042
Grw.	50	+.007	+.002	–.002	–.007	–.012	–.016
Cape	50	+.012	+.004	–.004	–.012	–.019	–.027
Stgo.	55	+.012	+.004	–.004	–.012	–.019	–.027
Cape	60	+.016	+.005	–.005	–.016	–.027	–.038
Wn.	60	+.009	+.003	–.003	–.009	–.015	–.021
Grw.	60	+.010	+.003	–.003	–.010	–.017	–.024
Rad.	60	+.019	+.006	–.006	–.019	–.032	–.044
Stgo.	60	+.012	+.004	–.004	–.012	–.019	–.027
Melb.	60	+.014	+.005	–.005	–.014	–.023	–.032
Paris	60	+.005	+.002	–.002	–.005	–.008	–.012
Grw.	64	+.013	+.004	–.004	–.013	–.022	–.030
Cape	65	+.020	+.007	–.007	–.020	–.034	–.047
Brs.	65	+.006	+.002	–.002	–.006	–.010	–.014
Harv.	65	+.020	+.007	–.007	–.020	–.033	–.046
Pulk.	65	+.014	+.005	–.005	–.014	–.024	–.034
Melb.	70	+.011	+.004	–.004	–.011	–.018	–.026

TABLE I. MAGNITUDE-EQUATION. — *Cont.*

Magnitude		$2^{\mathrm{M}}.0$	$3^{\mathrm{M}}.0$	$4^{\mathrm{M}}.0$	$5^{\mathrm{M}}.0$	$6^{\mathrm{M}}.0$	$7^{\mathrm{M}}.0$
Grw.	72	+.014	+.005	–.005	–.014	–.024	–.033
1) Madr.	75	+.015	+.005	–.005	–.015	–.024	–.034
Wn.	75	+.013	+.004	–.004	–.013	–.021	–.029
Pulk.	75	+.008	+.002	–.002	–.008	–.012	–.018
Harv.	75	+.010	+.003	–.003	–.010	–.017	–.024
Cord.	75	+.015	+.005	–.005	–.015	–.024	–.034
Paris	75	+.007	+.002	–.002	–.007	–.011	–.016
Cape	80	+.017	+.006	–.006	–.017	–.028	–.039
Melb.	80	+.007	+.002	–.002	–.007	–.011	–.016
Grw.	80	+.010	+.003	–.003	–.010	–.016	–.023
Pulk.	85	+.012	+.004	–.004	–.012	–.019	–.027
Cape	85	+.012	+.004	–.004	–.012	–.020	–.027
Stbg.	85	+.009	+.003	–.003	–.009	–.016	–.022
Rad.	90	+.011	+.004	–.004	–.011	–.019	–.026
Cape	90	+.019	+.006	–.006	–.019	–.031	–.044
Mdn.	90	+.019	+.006	–.006	–.019	–.032	–.045
Ber.	90	+.005	+.002	–.002	–.005	–.008	–.012
Lisb.	90	+.012	+.004	–.004	–.012	–.020	–.029
Grw.	90	+.009	+.003	–.003	–.009	–.015	–.021
2) Pulk.	92	+.012	+.004	–.004	–.012	–.019	–.027
Mt. H.	95	+.016	+.005	–.005	–.016	–.026	–.037
Ber.	95	+.011	+.004	–.004	–.011	–.019	–.027
3) Alb.	98	.000	.000	.000	.000	.000	.000

TABLE II. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_{\alpha}$. RIGHT-ASCENSION.

R.A.		12^{h}	13^{h}	14^{h}	15^{h}	16^{h}	17^{h}	18^{h}	19^{h}	20^{h}	21^{h}	22^{h}	23^{h}	0^{h}
Br.	1755	$-^{\mathrm{s}}.083$	$-^{\mathrm{s}}.080$	$-^{\mathrm{s}}.077$	$-^{\mathrm{s}}.075$	$-^{\mathrm{s}}.072$	$-^{\mathrm{s}}.070$	$-^{\mathrm{s}}.069$	$-^{\mathrm{s}}.068$	$-^{\mathrm{s}}.068$	$-^{\mathrm{s}}.069$	$-^{\mathrm{s}}.071$	$-^{\mathrm{s}}.073$	$-^{\mathrm{s}}.075$
{ Pi.	1800	+.136	+.135	+.133	+.129	+.125	+.120	+.116	+.111	+.107	+.104	+.101	+.100	+.100
{ ×(tan δ)		–.048	–.024	+.002	+.028	+.051	+.072	+.087	+.096	+.099	+.095	+.085	+.069	+.048
Grw.	15	–.005	–.008	–.012	–.015	–.019	–.023	–.026	–.028	–.030	–.030	–.029	–.028	–.025
4) Dpt.	15	–.034	–.034	–.032	–.027	–.021	–.013	–.004	+.005	+.014	+.021	+.027	+.032	+.034
Kgb.	15	–.082	–.082	–.082	–.082	–.082	–.082	–.082	–.082	–.082	–.082	–.082	–.082	–.082
5) Kgb.	20	–.034	–.034	–.034	–.034	–.034	–.034	–.034	–.034	–.034	–.034	–.034	–.034	–.034
Kgb.	25	+.032	+.028	+.025	+.021	+.018	+.015	+.013	+.012	+.011	+.012	+.013	+.016	+.019
Dpt.	30	–.025	–.027	–.030	–.031	–.032	–.032	–.031	–.030	–.028	–.025	–.022	–.019	–.016
Cape	30	–.001	–.009	–.015	–.020	–.022	–.021	–.019	–.014	–.007	+.001	+.010	+.019	+.027
St. H.	30	–.027	–.028	–.030	–.033	–.036	–.041	–.046	–.051	–.056	–.059	–.062	–.064	–.065
Åbo	30	+.016	+.015	+.014	+.013	+.013	+.012	+.012	+.011	+.011	+.012	+.012	+.013	+.014
Grw.	30	–.063	–.061	–.059	–.057	–.055	–.054	–.053	–.053	–.054	–.055	–.056	–.058	–.061
Camb.	30	–.029	–.028	–.027	–.025	–.023	–.022	–.020	–.019	–.018	–.018	–.018	–.018	–.019
Cape	33	+.035	+.035	+.035	+.033	+.031	+.028	+.024	+.020	+.016	+.013	+.010	+.008	+.006
Madr.	35	–.042	–.047	–.052	–.057	–.062	–.066	–.069	–.071	–.072	–.071	–.069	–.066	–.062
Arm.	40	+.055	+.054	+.053	+.051	+.050	+.048	+.047	+.046	+.045	+.044	+.044	+.044	+.045
Cape	40	–.006	–.003	.000	+.003	+.005	+.007	+.008	+.008	+.007	+.006	+.004	+.002	–.001
Grw.	40	+.071	+.079	+.087	+.094	+.101	+.107	+.111	+.113	+.113	+.111	+.107	+.101	+.094
Grw.	45	+.006	+.011	+.017	+.024	+.030	+.036	+.041	+.045	+.047	+.048	+.047	+.044	+.040
Rad.	45	–.068	–.060	–.050	–.038	–.025	–.012	+.001	+.012	+.021	+.026	+.029	+.028	+.023
Pulk.	45	+.027	+.027	+.026	+.026	+.025	+.024	+.023	+.022	+.021	+.020	+.019	+.019	+.019
Paris	45	+.023	+.025	+.028	+.029	+.031	+.031	+.033	+.034	+.033	+.032	+.031	+.029	+.027
Stgo.	50	–.004	–.001	+.001	+.004	+.007	+.010	+.012	+.014	+.015	+.015	+.015	+.014	+.012
Grw.	50	–.011	–.007	–.003	+.002	+.007	+.011	+.014	+.017	+.018	+.018	+.017	+.014	+.011

TABLE II. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_{\alpha}$. RIGHT-ASCENSION.—Cont.

R.A.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
	s	s	s	s	s	s	s	s	s	s	s	s	s
Cape 50	+.027	+.026	+.025	+.023	+.021	+.019	+.016	+.014	+.012	+.010	+.009	+.009	+.009
Stgo. 55	+.065	+.057	+.048	+.039	+.030	+.023	+.016	+.012	+.009	+.009	+.012	+.016	+.023
Cape 60	+.035	+.031	+.027	+.023	+.019	+.016	+.013	+.010	+.009	+.009	+.009	+.011	+.014
Wn. 60	+.033	+.026	+.018	+.011	+.003	−.003	−.009	−.013	−.014	−.014	−.012	−.008	−.002
Grw. 60	+.034	+.032	+.028	+.024	+.019	+.014	+.010	+.006	+.003	+.002	+.001	+.001	+.003
{ Rad. 60	+.033	+.029	+.024	+.021	+.017	+.015	+.014	+.014	+.016	+.018	+.021	+.025	+.030
{ ×(tan δ)	+.024	+.021	+.017	+.012	+.006	−.001	−.007	−.013	−.018	−.022	−.024	−.025	−.024
Stgo. 60	+.065	+.057	+.048	+.039	+.030	+.023	+.016	+.012	+.009	+.009	+.012	+.016	+.023
{ Melb. 60	+.062	+.053	+.045	+.037	+.029	+.022	+.018	+.015	+.014	+.016	+.020	+.025	+.032
{ ×(tan δ)	+.023	+.028	+.031	+.032	+.031	+.027	+.022	+.015	+.008	−.001	−.009	−.016	−.023
Paris 60	+.050	+.045	+.039	+.034	+.028	+.024	+.020	+.017	+.016	+.016	+.018	+.021	+.025
Grw. 64	+.042	+.039	+.036	+.033	+.030	+.026	+.024	+.022	+.020	+.020	+.020	+.021	+.023
Cape 65	−.013	−.013	−.013	−.012	−.012	−.012	−.012	−.012	−.012	−.012	−.011	−.011	−.011
{ Brs. 65	+.068	+.058	+.047	+.036	+.027	+.018	+.012	+.009	+.008	+.011	+.015	+.023	+.032
{ ×(tan δ)	−.003	+.006	+.015	+.023	+.029	+.033	+.035	+.035	+.032	+.027	+.020	+.012	+.003
Harv. 65	−.033	−.037	−.040	−.043	−.045	−.046	−.045	−.043	−.040	−.037	−.033	−.028	−.024
Pulk. 65	−.008	−.008	−.007	−.006	−.005	−.004	−.003	−.002	−.002	−.001	−.001	−.001	−.002
Melb. 70	+.050	+.041	+.032	+.023	+.014	+.007	+.002	−.001	−.001	+.001	+.005	+.012	+.020
Grw. 72	+.039	+.037	+.034	+.031	+.028	+.025	+.022	+.020	+.018	+.017	+.017	+.017	+.019
Madr. 75	+.042	+.043	+.044	+.046	+.047	+.049	+.050	+.051	+.052	+.053	+.053	+.052	+.052
Wn. 75	−.002	−.003	−.004	−.004	−.004	−.004	−.004	−.003	−.002	−.002	−.001	.000	+.001
Pulk 75	−.003	−.001	.000	+.002	+.004	+.006	+.008	+.009	+.010	+.010	+.010	+.009	+.008
Harv. 75	−.013	−.012	−.011	−.010	−.008	−.006	−.004	−.003	−.001	.000	+.001	+.002	+.002
6) Cord. 75	+.021	+.012	+.002	−.008	−.018	−.026	−.033	−.037	−.039	−.039	−.035	−.030	−.022
Paris 75	+.059	+.052	+.044	+.037	+.029	+.022	+.017	+.013	+.011	+.011	+.013	+.017	+.022
Cape 80	+.046	+.044	+.042	+.039	+.036	+.033	+.031	+.028	+.027	+.026	+.026	+.026	+.027
Melb. 80	+.055	+.052	+.047	+.042	+.036	+.030	+.024	+.019	+.015	+.013	+.012	+.012	+.014
Grw. 80	+.038	+.037	+.036	+.034	+.033	+.032	+.032	+.031	+.031	+.031	+.031	+.032	+.033
Pulk. 85	+.014	+.015	+.016	+.018	+.019	+.021	+.023	+.024	+.025	+.026	+.026	+.026	+.025
Cape 85	+.023	+.020	+.017	+.014	+.011	+.009	+.008	+.008	+.008	+.009	+.010	+.013	+.015
Stbg. 85	+.014	+.014	+.014	+.014	+.014	+.014	+.014	+.014	+.015	+.016	+.016	+.017	+.017
Rad. 90	+.025	+.024	+.023	+.021	+.019	+.017	+.014	+.012	+.010	+.009	+.008	+.008	+.008
Cape 90	+.024	+.023	+.021	+.020	+.019	+.018	+.017	+.016	+.016	+.017	+.017	+.018	+.019
Mdn. 90	+.007	+.007	+.006	+.006	+.006	+.005	+.005	+.005	+.006	+.006	+.007	+.007	+.008
Ber. 90	+.021	+.020	+.019	+.018	+.017	+.017	+.016	+.016	+.016	+.017	+.017	+.018	+.019
Lisb. 90	+.015	+.014	+.014	+.015	+.015	+0.15	+.015	+.016	+.016	+.017	+.017	+.017	+.017
Grw. 90	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045
2) Pulk. 92	+.023	+.022	+.021	+.020	+.019	+.018	+.017	+.017	+.017	+.017	+.018	+.019	+.020
Mt. H. 95	+.024	+.024	+.023	+.023	+.023	+.024	+.024	+.025	+.027	+.028	+.029	+.031	+.032
Ber. 95	+.022	+.022	+.021	+.020	+.020	+.019	+.018	+.018	+.017	+.017	+.017	+.017	+.018
3) Alb. 98	.000	.000	.000	.000	.000	.000	.000	.000.	.000	.000	.000	.000	.000

TABLE III. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_{\delta}$. RIGHT-ASCENSION.

	$\Delta\alpha_{\delta} \cos\delta$		$\Delta\alpha_{\delta}$										
	+85°	+80°	+80°	+75°	+70°	+65°	+60°	+55°	+50°	+45°	+40°	+35°	+30°
	s	s	s	s	s	s	s	s	s	s	s	s	s
Br. 1755	−.009	−.017	−.099	−.089	−.075	−.059	−.037	−.009	+.006	+.012	+.014	+.013	+.012
Pi. 1800	+.038	+.040	+.230	+.175	+.161	+.178	+.187	+.167	+.132	+.100	+.093	+.087	+.079
Grw. 15	.000	.000	.000	.000	.000	−.001	−.005	−.012	−.019	−.027	−.034	−.041	−.043
4) Dpt. 15	.000	+.003	+.017	+.031	+.035	+.028	+.016	+.003	−.009	−.019	. . .	. . .	. . .
Kgb. 15	.000	−.002	−.012	−.019	−.021	−.019	−.015	−.007	+.002	+.009	+.011	+.008	+.001
5) Kgb. 25	.000	.000	.000	.000	.000	.000	−.003	−.010	−.022	−.024	−.016	−.004	+.002
Dpt. 30	+.001	+.002	+.011	+.016	+.023	+.033	+.037	+.036	+.033	+.031	+.029	+.025	+.021
Cape 30	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
St. H. 30	. . .	. . .	. . .	. . .	. . .	. . .	+.017	+.014	+.012	+.010	+.009	+.007	+.006
Åbo 30	.000	+.003	+.020	+.046	+.049	+.043	+.035	+.028	+.023	+.018	+.012	+.004	−.004

Table II. Systematic Corrections of the Form, $\Delta\alpha_\alpha$. Right-Ascension. — Cont.

R.A.	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	0^h
	s	s	s	s	s	s	s	s	s	s	s	s	s
Cape 50	+.009	+.010	+.011	+.013	+.015	+.017	+.020	+.022	+.024	+.026	+.027	+.027	+.027
Stgo. 55	+.023	+.031	+.040	+.049	+.058	+.065	+.072	+.076	+.079	+.079	+.076	+.072	+.065
Cape 60	+.014	+.017	+.021	+.025	+.029	+.033	+.036	+.038	+.039	+.040	+.039	+.037	+.035
Wn. 60	−.002	+.005	+.012	+.020	+.028	+.034	+.040	+.043	+.045	+.045	+.043	+.039	+.033
Grw. 60	+.003	+.006	+.010	+.014	+.019	+.023	+.028	+.031	+.034	+.036	+.037	+.036	+.034
Rad. 60	+.030	+.034	+.039	+.043	+.046	+.048	+.049	+.049	+.047	+.045	+.042	+.038	+.033
× (tan δ)	−.024	−.021	−.017	−.012	−.006	+.001	+.007	+.013	+.018	+.022	+.024	+.025	+.024
Stgo. 60	+.023	+.031	+.040	+.049	+.058	+.065	+.072	+.076	+.079	+.079	+.076	+.072	+.065
Melb. 60	+.032	+.041	+.049	+.058	+.065	+.072	+.076	+.079	+.080	+.078	+.074	+.069	+.062
× (tan δ)	−.023	−.028	−.031	−.032	−.031	−.027	−.022	−.015	−.008	+.001	+.009	+.016	+.023
Paris 60	+.025	+.030	+.036	+.041	+.047	+.051	+.055	+.058	+.059	+.059	+.057	+.054	+.050
Grw. 64	+.023	+.026	+.029	+.032	+.035	+.039	+.041	+.043	+.045	+.045	+.045	+.044	+.042
Cape 65	−.011	−.011	−.011	−.012	−.012	−.012	−.012	−.012	−.012	−.012	−.013	−.013	−.013
Brs. 65	+.032	+.042	+.053	+.063	+.073	+.081	+.087	+.090	+.091	+.089	+.084	+.077	+.068
× (tan δ)	+.003	−.006	−.015	−.023	−.029	−.033	−.035	−.035	−.032	−.027	−.020	−.012	−.003
Harv. 65	−.024	−.020	−.016	−.013	−.012	−.011	−.012	−.013	−.016	−.020	−.024	−.028	−.033
Pulk. 65	−.002	−.002	−.003	−.004	−.005	−.006	−.007	−.008	−.009	−.009	−.009	−.009	−.008
Melb. 70	+.020	+.029	+.038	+.047	+.056	+.063	+.068	+.071	+.071	+.069	+.065	+.058	+.050
Grw. 72	+.019	+.021	+.023	+.026	+.030	+.033	+.035	+.038	+.040	+.041	+.041	+.040	+.039
Madr. 75	+.052	+.051	+.049	+.048	+.046	+.045	+.043	+.042	+.041	+.041	+.041	+.041	+.042
Wn. 75	+.001	+.001	+.002	+.002	+.002	+.002	+.002	+.001	+.001	.000	−.001	−.002	−.002
Pulk. 75	+.008	+.006	+.005	+.003	+.001	−.001	−.003	−.004	−.005	−.005	−.005	−.004	−.003
Harv. 75	+.002	+.001	.000	−.001	−.003	−.005	−.007	−.008	−.010	−.011	−.012	−.013	−.013
6) Cord. 75	−.022	−.013	−.003	+.007	+.017	+.025	+.032	+.036	+.038	+.038	+.035	+.029	+.021
Paris 75	+.022	+.029	+.037	+.044	+.052	+.059	+.064	+.068	+.070	+.070	+.068	+.064	+.059
Cape 80	+.027	+.029	+.032	+.034	+.037	+.040	+.043	+.045	+.047	+.048	+.048	+.047	+.046
Melb. 80	+.014	+.017	+.021	+.027	+.033	+.039	+.044	+.049	+.053	+.056	+.057	+.057	+.055
Grw. 80	+.033	+.034	+.035	+.036	+.037	+.038	+.039	+.040	+.040	+.040	+.039	+.039	+.038
Pulk. 85	+.025	+.024	+.023	+.022	+.020	+.018	+.017	+.016	+.014	+.014	+.013	+.014	+.014
Cape 85	+.015	+.018	+.021	+.024	+.027	+.029	+.030	+.030	+.030	+.029	+.028	+.025	+.023
Stbg. 85	+.017	+.017	+.018	+.018	+.018	+.017	+.017	+.017	+.016	+.016	+.015	+.015	+.014
Rad. 90	+.008	+.009	+.010	+.012	+.014	+.017	+.019	+.021	+.023	+.024	+.025	+.025	+.025
Cape 90	+.019	+.021	+.022	+.023	+.025	+.026	+.027	+.027	+.027	+.027	+.026	+.025	+.024
Mdn. 90	+.008	+.008	+.009	+.009	+.009	+.010	+.010	+.010	+.009	+.009	+.008	+.008	+.007
Ber. 90	+.019	+.020	+.021	+.022	+.023	+.023	+.024	+.024	+.024	+.023	+.023	+.022	+.021
Lisb. 90	+.017	+.018	+.018	+.017	+.017	+.017	+.017	+.016	+.016	+.015	+.015	+.015	+.015
Grw. 90	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045
2) Pulk. 92	+.020	+.021	+.022	+.023	+.024	+.025	+.026	+.026	+.026	+.026	+.025	+.024	+.023
Mt. H. 95	+.032	+.033	+.033	+.033	+.033	+.033	+.032	+.031	+.030	+.028	+.027	+.026	+.024
Ber. 95	+.018	+.018	+.019	+.020	+.020	+.021	+.022	+.022	+.023	+.023	+.023	+.023	+.022
3) Alb. 98	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

Table III. Systematic Corrections of the Form, $\Delta\alpha_\delta$. Right-Ascension.

	$\Delta\alpha_\delta$											
	+25°	+20°	+15°	+10°	+5°	0	−5°	−10°	−15°	−20°	−25°	−30°
	s	s	s	s	s	s	s	s	s	s	s	s
Br. 1755	+.009	+.005	+.001	−.002	−.003	−.001	.000	−.004	−.009	−.018	−.028	−.038
Pi. 1800	+.069	+.057	+.036	+.008	−.015	−.027	−.036	−.047	−.061	−.076	−.095	−.118
Grw. 15	−.040	−.034	−.022	−.006	+.009	+.021	+.028	+.031	+.034	+.039	+.046	+.058
4) Dpt. 15	...	...	...	...	...	...	...	...	...	...	...	...
Kgb. 15	−.007	−.013	−.014	−.010	−.002	+.009	+.014	+.012	+.005	−.004	−.017	...
5) Kgb. 25	+.005	+.005	+.003	.000	−.003	−.006	−.009	−.012	−.015	−.019	−.023	...
Dpt. 30	+.013	+.007	+.003	.000	−.003	−.007	−.010	−.013	−.016	−.017	...	...
Cape 30	...	...	.000	−.002	−.008	−.017	−.017	−.010	−.001	+.007	+.015	+.021
St. H. 30	+.006	+.004	+.004	+.003	.000	−.006	−.009	−.008	−.005	−.009	−.020	−.036
Åbo 30	−.008	−.007	−.006	−.005	−.002	+.002	+.006	+.010	+.013	+.015	...	...

Table III. Systematic Corrections of the Form, $\Delta\alpha_\delta$. Right-Ascension. — Cont.

	$\Delta\alpha_\delta \cos\delta$		$\Delta\alpha_\delta$										
	+85°	+80°	+80°	+75°	+70°	+65°	+60°	+55°	+50°	+45°	+40°	+35°	+30°
Grw. 30	.000	.000	.000	+.003	+.018	+.037	+.054	+.052	+.040	+.027	+.018	+.013	+.014
Camb. 30	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	+.004
Cape 33	...	...	...	...	...	...	...	...	...	+.089	+.079	+.066	+.052
Madr. 35	...	+.060	+.343	+.230	+.173	+.140	+.117	+.102	+.090	+.079	+.067	+.054	+.041
Arm. 40	...	.000	.000	−.017	−.033	−.040	−.040	−.033	−.021	−.014	−.010	−.010	−.009
Cape 40	...	...	...	...	...	...	...	...	...	+.059	+.046	+.035	+.024
Grw. 40	−.025	−.032	−.187	−.151	−.120	−.094	−.063	−.033	−.014	−.008	−.008	−.007	−.007
Grw. 45	−.006	−.014	−.081	−.093	−.088	−.076	−.060	−.042	−.025	−.011	−.006	−.006	−.009
Rad. 45	−.010	−.010	−.058	−.036	−.016	+.006	+.027	+.035	+.031	+.021	+.013	+.011	+.017
Pulk. 45	+.001	+.003	+.018	+.023	+.018	+.011	+.002	−.004	−.007	−.011	−.012	−.010	−.008
Paris 45	+.008	+.008	+.046	+.031	+.023	+.026	+.027	+.022	+.015	+.004	−.003	−.007	−.008
Stgo. 50	...	...	...	...	...	...	...	...	...	+.024	+.023	+.023	+.022
Grw. 50	−.020	−.025	−.142	−.108	−.083	−.058	−.034	−.012	.000	+.002	−.003	−.010	−.013
Cape 50	...	...	...	...	...	...	...	...	...	+.056	+.032	+.015	+.003
Stgo. 55	...	...	...	...	...	...	...	...	...	...	...	...	+.035
Cape 60	...	...	...	...	...	...	...	...	...	...	+.021	+.015	+.009
Wn. 60	.000	.000	.000	−.003	−.007	−.014	−.017	−.016	−.010	+.001	+.010	+.013	+.010
Grw. 60	−.006	−.007	−.039	−.036	−.037	−.041	−.046	−.046	−.044	−.040	−.037	−.032	−.026
Rad. 60	−.007	−.007	−.042	−.036	−.039	−.045	−.040	−.025	−.005	+.004	+.002	−.003	−.005
Stgo. 60	...	...	...	...	...	...	...	...	...	...	...	...	+.035
Melb. 60	...	...	...	...	...	...	...	...	...	...	−.026	−.024	−.020
Paris 60	.000	.000	.000	+.012	+.023	+.024	+.017	+.008	+.007	+.010	+.014	+.013	+.008
Grw. 64	.000	.000	.000	.000	.000	+.005	+.008	+.006	−.004	−.013	−.020	−.026	−.024
Cape 65	...	...	...	...	...	...	...	...	...	...	...	...	+.019
Brs. 65	+.005	−.010	−.055	−.093	−.117	−.114	−.095	−.067	−.034	−.015	−.008	−.007	−.009
Harv. 65	−.008	−.007	−.041	−.004	+.030	+.045	+.045	+.039	+.031	+.022	+.018	+.016	+.016
Pulk. 65	−.005	−.007	−.039	−.032	−.029	−.024	−.020	−.017	−.018	−.022	−.022	−.019	−.012
Melb. 70	...	...	...	...	...	...	...	...	...	...	+.011	+.010	+.009
Grw. 72	+.010	+.010	+.058	+.039	+.028	+.016	+.008	+.007	+.005	−.003	−.016	−.020	−.018
Madr. 75	...	−.047	−.273	−.141	−.075	−.035	−.007	+.013	+.024	+.027	+.022	+.014	+.006
Wn. 75	+.001	−.003	−.019	−.023	−.021	−.018	−.023	−.034	−.035	−.026	−.010	+.005	+.011
Pulk. 75	−.003	−.009	−.052	−.051	−.039	−.024	−.015	−.014	−.014	−.013	−.012	−.012	−.012
Harv. 75	−.002	−.002	−.014	−.013	−.011	−.004	+.005	+.007	+.003	+.003	+.003	+.005	+.007
6) Cord. 75	...	...	...	...	...	...	...	...	...	...	...	...	−.012
Paris 75	...	...	...	...	...	...	...	...	.000	+.017	+.022	+.017	+.009
7) Cape 80	...	...	...	...	...	...	...	...	...	...	+.010	+.006	+.003
Melb. 80	...	...	...	...	...	...	...	...	...	...	+.003	+.005	+.007
Grw. 80	+.009	+.009	+.051	+.032	+.023	+.019	+.011	+.003	−.003	−.006	−.009	−.009	−.008
Pulk. 85	.000	.000	.000	−.016	−.019	−.015	−.007	+.002	+.003	−.002	−.004	−.001	−.001
Cape 85	...	...	...	...	...	...	...	...	...	−.016	−.016	−.016	−.015
Stbg. 85	+.001	+.004	+.024	+.038	+.042	+.038	+.032	+.025	+.017	+.010	+.005	.000	−.003
Rad. 90	...	+.039	+.225	+.144	+.095	+.053	+.022	+.005	−.005	−.008	−.013	−.022	−.026
Cape 90	...	...	...	...	...	...	...	...	...	+.020	+.016	+.013	+.009
Mdn. 90	+.001	+.003	+.019	+.022	+.029	+.031	+.026	+.019	+.011	+.007	+.006	+.008	+.009
Ber. 90	−.006	−.006	−.036	−.027	−.023	−.022	−.022	−.023	−.025	−.026	−.026	−.023	−.018
Lisb. 90	...	...	...	...	...	.000	−.004	−.006	−.009	−.009	−.008	−.007	−.005
Grw. 90	−.002	+.004	+.023	+.030	+.027	+.014	−.004	−.015	−.015	−.008	−.007	−.006	−.006
2) Pulk. 92	.000	.000	.000	−.002	−.006	−.011	−.015	−.017	−.018	−.018	−.018	−.018	−.016
Mt.H. 95	+.013	+.015	+.086	+.064	+.054	...	...	...	...	...	−.008	.000	+.005
Ber. 95	...	+.012	+.069	+.041	+.020	+.005	−.004	−.010	−.016	−.017	−.015	−.013	−.011
3) Alb. 98	...	...	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE III. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\delta$. RIGHT-ASCENSION. — Cont.

	$\Delta\alpha_\delta$ +25°	+20°	+15°	+10°	+5°	0	−5°	−10°	−15°	−20°	−25°	−30°
Grw. 30	+.020	+.018	+.013	+.006	−.002	−.009	−.017	−.021	−.026	−.029	−.031	−.035
Camb. 30	+.009	+.013	+.008	−.002	−.007	−.006	.000	+.001	−.001	−.006	−.013	−.019
Cape 33	+.034	+.015	+.003	−.005	−.009	−.012	−.014	−.013	−.011	−.005	+.001	+.004
Madr. 35	+.031	+.023	+.014	+.006	−.001	−.009	−.018	−.027	−.038	−.049	−.057	−.062
Arm. 40	−.009	−.008	−.007	.000	+.005	+.001	−.002	−.001	+.005	+.010	+.012	+.014
Cape 40	+.012	−.001	−.016	−.019	−.014	−.004	+.003	+.007	+.009	+.009	+.009	+.008
Grw. 40	−.008	−.008	−.008	−.005	+.002	+.008	+.012	+.014	+.015	+.011	−.002	−.020
Grw. 45	−.008	−.004	+.002	+.005	+.006	+.003	+.002	+.002	+.002	−.001	−.009	−.021
Rad. 45	+.022	+.024	+.019	+.004	−.017	−.023	−.020	−.015	−.015	−.021	−.029	. . .
Pulk. 45	−.006	−.003	.000	+.003	+.005	+.005	+.003	.000	+.002	+.008	+.018	. . .
Paris 45	−.006	−.001	+.003	+.004	+.003	+.001	+.001	+.002	+.001	−.004	−.004	.000
Stgo. 50	+.022	+.019	+.013	+.005	−.004	−.012	−.018	−.021	−.023	−.025	−.027	−.033
Grw. 50	−.012	−.008	−.002	+.001	+.002	+.004	+.007	+.008	+.008	+.006	+.005	+.004
Cape 50	−.006	−.010	−.008	−.003	+.006	+.008	+.007	+.003	.000	.000	.000	−.006
Stgo. 55	+.025	+.016	+.008	+.002	−.005	−.010	−.015	−.018	−.020	−.021	−.028	−.044
Cape 60	+.002	−.006	−.008	−.004	.000	+.004	+.008	+.006	−.003	−.003	+.003	+.017
Wn. 60	+.006	+.003	+.001	.000	−.002	−.003	−.004	−.003	−.002	+.002	+.007	+.012
Grw. 60	−.020	−.013	−.006	.000	+.006	+.011	+.013	+.014	+.012	+.007	+.004	+.002
Rad. 60	−.004	+.002	+.011	+.017	+.018	+.013	+.001	−.008	−.017	−.028	−.040	−.052
Stgo. 60	+.025	+.016	+.008	+.002	−.005	−.010	−.015	−.018	−.020	−.021	−.028	−.044
Melb. 60	−.013	−.005	−.002	.000	.000	.000	+.004	+.008	+.008	+.005	.000	−.005
Paris 60	+.003	+.003	+.003	+.003	+.001	−.003	−.006	−.007	−.008	−.012	−.019	−.023
Grw. 64	−.018	−.008	−.001	+.003	+.008	+.011	+.011	+.010	+.004	.000	.000	.000
Cape 65	+.009	−.002	−.008	−.013	−.014	−.012	−.007	.000	+.012	+.028	+.044	+.048
Brs. 65	−.009	−.008	−.008	−.005	−.002	+.001	+.004	+.007	+.015	+.021	+.021	+.012
Harv. 65	+.014	+.012	+.009	+.004	−.001	−.007	−.010	−.009	−.012	−.021	−.033	−.046
Pulk. 65	−.004	.000	+.003	+.005	+.007	+.011	+.008	+.001	−.006	−.010	. . .	. . .
Melb. 70	+.009	+.008	+.007	+.002	−.004	−.008	−.010	−.010	−.009	−.008	−.008	−.007
Grw. 72	−.012	−.006	−.001	+.002	+.006	+.009	+.008	+.003	.000	.000	.000	.000
Madr. 75	+.002	+.002	−.001	−.006	−.010	−.010	−.004	+.001	+.004	+.006	+.013	+.024
Wn. 75	+.012	+.010	+.007	+.002	−.005	−.010	−.012	−.011	−.006	−.003	−.004	−.009
Pulk. 75	−.011	−.005	.000	+.004	+.009	+.012	+.010	+.004	.000	.000	. . .	. . .
Harv. 75	+.006	+.004	+.002	.000	.000	−.001	−.005	−.008	−.006	−.001	+.007	+.017
6) Cord. 75	−.003	+.002	+.003	+.002	−.003	−.004	+.001	+.009	+.013	+.015	+.012	+.004
Paris 75	+.006	+.005	+.005	+.004	.000	−.004	−.009	−.010	−.007	−.002	.000	. . .
7) Cape 80	+.001	−.001	−.003	−.003	−.003	.000	+.004	+.010	+.017	+.026	+.034	+.040
Melb. 80	+.009	+.008	+.002	−.006	−.011	−.012	−.008	.000	+.014	+.029	+.040	+.049
Grw. 80	−.006	−.003	.000	+.002	+.004	+.005	+.005	+.003	+.001	+.001	+.001	+.001
Pulk. 85	−.006	−.008	−.006	−.003	+.001	+.004	+.005	+.004	+.004	+.004	. . .	. . .
Cape 85	−.012	−.008	−.003	−.002	−.003	−.003	+.001	+.007	+.016	+.024	+.031	+.033
Stbg. 85	−.005	−.006	−.006	−.003	+.003	+.009	+.008	+.005	+.002	−.002	−.007	−.010
Rad. 90	−.026	−.023	−.019	−.013	−.006	+.004	+.019	+.040	+.058	+.076	+.096	+.118
Cape 90	+.004	+.001	+.001	.000	−.004	−.006	−.006	−.005	.000	+.008	+.019	+.028
Mdn. 90	+.008	+.006	+.002	.000	−.002	−.004	−.007	−.009	−.011	. . .	. . .	. . .
Ber. 90	−.013	−.008	−.004	.000	+.004	+.008	+.012	+.015	. . .	. . .	. . .	. . .
Lisb. 90	−.002	.000	+.002	+.004	+.005	+.004	+.001	−.003	−.006	−.007	−.007	−.007
Grw. 90	−.006	−.004	−.002	.000	+.001	+.002	+.004	+.007	+.009	+.010	+.010	+.007
2) Pulk. 92	−.011	−.004	.000	+.004	+.005	+.005	+.004	+.002	−.002	−.009	. . .	. . .
Mt. H. 95	+.007	+.008	+.007	+.004	−.001	−.005	−.008	−.010	−.012	−.014	−.012	−.009
Ber. 95	−.009	−.007	−.004	+.002	+.006	+.008	+.009	+.011	+.013	+.016	+.017	. . .
3) Alb. 98	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE III. $\Delta\alpha_s$ SOUTH OF −30° OF DECLINATION. RIGHT-ASCENSION. — Cont.

	$\Delta\alpha_s$ −30°	−35°	−40°	−45°	−50°	−55°	−60°	−65°	−70°	−75°	−80°	$\Delta\alpha_s \cos\delta$ −80°	−85°
	s	s	s	s	s	s	s	s	s	s	s	s	s
Pi. 1900	−.118	−.151	−.192	−.241	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Cape 30	+.021	+.027	+.033	+.043	+.059	+.077	+.094	+.111	+.137	+.181	+.271	+.047	. . .
St.H. 30	−.036	−.045	−.052	−.060	−.070	−.082	−.098	−.116	−.143	−.189	−.282	−.049	. . .
Cape 33	+.004	+.004	000	−.005	−.009	−.010	−.008	−.003	000	000	000	.000	.000
Madr. 35	−.062	−.066	−.071	−.076	−.084	−.094	−.108	−.128	. . .	. . .	. . .	. . .	. . .
Cape 40	+.008	+.007	+.009	+.011	+.016	+.021	+.026	+.031	+.038	+.050	+.075	+.013	+.013
Stgo. 50	−.033	−.044	−.052	−.056	−.054	−.042	−.032	−.018	+.002	+.026	+.059	+.010	. . .
Cape 50	−.006	−.026	−.061	−.075	−.072	−.040	−.012	000	000	000	000	.000	.000
Stgo. 55	−.044	−.073	−.093	−.100	−.093	−.046	−.018	−.005	000	000	000	.000	.000
Cape 60	+.017	+.030	+.039	+.044	+.051	+.059	+.060	+.046	+.030	+.018	+.010	+.002	+.001
Wn. 60	+.012	+.018	+.024	000	000	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Stgo. 60	−.044	−.073	−.093	−.100	−.093	−.046	−.018	−.005	000	000	000	.000	.000
Melb. 60	−.005	−.010	−.016	−.024	−.032	−.039	−.039	−.027	+.008	+.043	+.071	+.012	+.007
Cape 65	+.048	+.046	+.030	+.021	+.025	+.046	+.082	+.109	+.123	+.113	+.102	+.018	+.010

TABLE IV. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\delta_\alpha$. DECLINATION.

			0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
			″	″	″	″	″	″	″	″	″	″	″	″	″
8)	Br.	1755 N.	−.30	−.27	−.22	−.16	−.08	00	+.08	+.16	+.22	+.27	+.30	+.32	+.30
		S.	+.02	−.08	−.14	−.13	−.09	+.01	+.13	+.22	+.27	+.26	+.17	+.03	−.12
	Pi.	1900	−.11	+.05	+.20	+.33	+.45	+.53	+.58	+.59	+.55	+.49	+.39	+.25	+.11
	Grw.	15	−.09	−.07	−.04	−.02	+.02	+.04	+.07	+.09	+.11	+.11	+.11	+.10	+.09
	Kgb.	20	+.05	−.02	−.09	−.16	−.21	−.25	−.27	−.27	−.26	−.23	−.18	−.12	−.05
	Bb.	25	−.08	+.07	+.22	+.34	+.45	+.53	+.57	+.57	+.53	+.46	+.35	+.22	+.08
9)	Dpt.	30	−.02	+.02	+.06	+.09	+.12	+.14	+.15	+.15	+.14	+.12	+.09	+.06	+.02
	Åbo	30	−.12	−.05	+.02	+.09	+.16	+.21	+.25	+.27	+.28	+.26	+.23	+.18	+.12
	Cape	30	+.06	+.12	+.17	+.21	+.24	+.25	+.24	+.22	+.18	+.13	+.07	00	−.06
	Grw.	30	+.08	+.12	+.15	+.17	+.18	+.18	+.16	+.13	+.10	+.06	+.01	−.04	−.08
	St. H.	30	+.50	+.46	+.39	+.30	+.18	+.05	−.08	−.20	−.32	−.41	−.47	−.50	−.50
	Cape	33	−.16	−.17	−.17	−.16	−.13	−.10	−.06	−.02	+.03	+.07	+.11	+.14	+.16
	Camb.	30	−.19	−.17	−.14	−.11	−.06	−.01	+.04	+.09	+.13	+.16	+.19	+.19	+.19
10)	Madr.	35	−.08	−.11	−.16	−.20	−.24	−.25	−.21	−.13	−.02	+.11	+.23	+.32	+.36
	Cape	40	+.01	00	00	−.01	−.01	−.02	−.02	−.02	−.02	−.02	−.02	−.02	−.01
	Grw.	40	−.17	−.18	−.17	−.16	−.13	−.09	−.05	00	+.04	+.08	+.12	+.15	+.17
	Grw.	45	−.06	−.05	−.04	−.03	−.01	00	+.02	+.04	+.04	+.06	+.06	+.06	+.06
	Arm.	40	−.07	−.02	+.03	+.08	+.12	+.16	+.19	+.20	+.20	+.18	+.16	+.12	+.07
11)	Rad.	45	+.05	+.03	00	−.02	−.04	−.06	−.08	−.09	−.09	−.09	−.08	−.07	−.05
	Pulk.	45	+.07	+.06	+.05	+.03	+.01	−.01	−.03	−.05	−.06	−.07	−.08	−.08	−.07
	Paris	45	−.07	−.07	−.07	−.07	−.06	−.05	−.03	−.01	+.01	+.03	+.05	+.06	+.07
	Stgo.	50	−.21	−.15	−.08	−.01	+.07	+.14	+.20	+.25	+.28	+.29	+.28	+.26	+.21
	Grw.	50	−.07	−.08	−.08	−.07	−.06	−.05	−.03	−.01	+.01	+.03	+.05	+.06	+.07
12)	Cape	50	−.03	−.08	−.12	−.15	−.17	−.18	−.18	−.17	−.14	−.11	−.06	−.02	+.03
	Stgo.	55	−.04	−.10	−.15	−.19	−.22	−.24	−.24	−.22	−.18	−.14	−.09	−.03	+.04
	Pulk.	55	+.03	+.03	+.04	+.04	+.03	+.03	+.02	+.01	00	−.01	−.02	−.02	−.03
13)	Wn.	60	−.07	−.10	−.12	−.13	−.14	−.13	−.12	−.10	−.07	−.04	00	+.04	+.07
		−20°	.00	00	00	00	00	00	00	00	00	00	00	00	00
	Grw.	60	+.01	+.02	+.03	+.04	+.05	+.05	+.05	+.04	+.04	+.03	+.02	00	−.01
	Rad.	60	+.08	+.05	+.02	−.01	−.05	−.08	−.10	−.12	−.13	−.13	−.12	−.10	−.08
	Cape	60	+.03	+.06	+.08	+.10	+.11	+.11	+.11	+.10	+.08	+.06	+.03	00	−.03
	Paris	60	−.08	−.04	00	+.04	+.08	+.11	+.14	+.16	+.16	+.16	+.14	+.11	+.08
	Stgo.	60	−.24	−.24	−.22	−.19	−.15	−.10	−.04	+.03	+.09	+.14	+.18	+.22	+.24
	Melb.	60	−.09	−.04	−.04	−.07	−.13	−.17	−.17	−.11	00	+.16	+.33	+.47	+.55
	Grw.	64	+.04	+.03	+.02	+.01	00	−.01	−.02	−.03	−.04	−.04	−.04	−.04	−.04

Table III. $\Delta\alpha_s$ South of −30° of Declination. Right-Ascension. — Cont.

	$\Delta\alpha_s$ −30°	−35°	−40°	−45°	−50°	−55°	−60°	−65°	−70°	−75°	−80°	$\Delta\alpha_s \cos\delta$ −80°	−85°
	s	s	s	s	s	s	s	s	s	s	s	s	s
Harv. 65	−.046	−.062	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Melb. 70	−.007	−.005	−.004	−.006	−.011	−.019	−.032	−.044	−.057	−.071	−.089	−.015	−.011
Madr. 75	+.024	+.034	+.039	+.038	+.031	+.017	−.007	−.038	. . .	. . .	. . .	. . .	. . .
Wn. 75	−.009	−.016	−.023	(−.030)	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Harv. 75	+.017	+.026	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
6) Cord. 75	+.004	−.010	−.020	−.024	−.025	−.026	−.027	−.028	−.023	−.010	000	.000	.000
7) Cape 80 {	+.040	+.045 +.014	+.024	+.034 +.053	+.058	+.065 −.029	−.025	−.018 −.003	−.003	−.003 −.018	−.026	−.005	−.005
Melb. 80	+.049	+.054	+.056	+.053	+.047	+.029	+.008	−.011	−.025	−.035	−.048	−.008	−.008
Cape 85	+.033	+.030	+.028	+.025	+.023	+.018	+.014	+.009	+.003	000	000	.000	.000
Strb. 85	−.010	−.017	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Cape 90	+.028	+.034	+.038	+.039	+.039	+.038	+.037	+.037	+.035	+.036	+.040	+.007	+.006
Mt.H. 95	−.009	−.004	+.006	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
3) Alb. 98	000	000	000	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

Table IV. Systematic Corrections of the Form, $\Delta\delta_\alpha$. Declination.

			12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	0h
			″	″	″	″	″	″	″	″	″	″	″	″	″
8)	Br.	1755 N.	+.30	+.27	+.22	+.16	+.08	.00	−.08	−.16	−.22	−.27	−.30	−.32	−.30
		S.	−.12	−.25	−.34	−.35	−.29	−.18	−.03	+.11	+.21	+.24	+.21	+.13	+.02
	Pi.	1800	+.11	−.05	−.20	−.33	−.45	−.53	−.58	−.59	−.55	−.49	−.39	−.25	−.11
	Grw.	15	+.09	+.07	+.04	+.02	−.02	−.04	−.07	−.09	−.11	−.11	−.11	−.10	−.09
	Kgb.	20	−.05	+.02	+.09	+.16	+.21	+.25	+.27	+.27	+.26	+.23	+.18	+.12	+.05
	Bb.	25	+.08	−.07	−.22	−.34	−.45	−.53	−.57	−.57	−.53	−.46	−.35	−.22	−.08
9)	Dpt.	30	+.02	−.02	−.06	−.09	−.12	−.14	−.15	−.15	−.14	−.12	−.09	−.06	−.02
	Åbo	30	+.12	+.05	−.02	−.09	−.16	−.21	−.25	−.27	−.28	−.26	−.23	−.18	−.12
	Cape	30	−.06	−.12	−.17	−.21	−.24	−.25	−.24	−.22	−.18	−.13	−.07	.00	+.06
	Grw.	30	−.08	−.12	−.15	−.17	−.18	−.18	−.16	−.13	−.10	−.06	−.01	+.04	+.08
	St. H.	30	−.50	−.46	−.39	−.30	−.18	−.05	+.08	+.21	+.32	+.41	+.47	+.50	+.50
	Cape	33	+.16	+.17	+.17	+.16	+.13	+.10	+.06	+.02	−.03	−.07	−.11	−.14	−.16
	Camb.	30	+.19	+.17	+.14	+.11	+.06	+.01	−.04	−.09	−.13	−.16	−.18	−.19	−.19
10)	Madr.	35	+.36	+.35	+.30	+.20	+.10	.00	−.07	−.11	−.12	−.11	−.09	−.07	−.08
	Cape	40	−.01	.00	.00	+.01	+.01	+.02	+.02	+.02	+.02	+.02	+.02	+.02	+.01
	Grw.	40	+.17	+.18	+.17	+.16	+.13	+.09	+.05	.00	−.04	−.08	−.12	−.15	−.17
	Grw.	45	+.06	+.05	+.04	+.03	+.01	.00	−.02	−.04	−.05	−.06	−.06	−.06	−.06
	Arm.	40	+.07	+.02	−.03	−.08	−.12	−.16	−.19	−.20	−.20	−.18	−.16	−.12	−.07
11)	Rad.	45	−.05	−.03	.00	+.02	+.04	+.06	+.08	+.09	+.09	+.09	+.08	+.07	+.05
	Pulk.	45	−.07	−.06	−.05	−.03	−.01	+.01	+.03	+.05	+.06	+.07	+.08	+.08	+.07
	Paris	45	+.07	+.08	+.08	+.07	+.06	+.05	+.03	+.01	−.01	−.03	−.05	−.06	−.07
	Stgo.	50	+.21	+.15	+.08	+.01	−.07	−.14	−.20	−.25	−.28	−.29	−.28	−.26	−.21
	Grw.	50	+.07	+.08	+.08	+.07	+.06	+.05	+.03	+.01	−.01	−.03	−.05	−.06	−.07
12)	Cape	50	+.03	+.08	+.12	+.15	+.17	+.18	+.18	+.17	+.14	+.11	+.06	+.02	−.03
	Stgo.	55	+.04	+.10	+.15	+.19	+.22	+.24	+.24	+.22	+.18	+.14	+.09	+.03	−.04
	Pulk.	55	−.03	−.03	−.04	−.04	−.03	−.03	−.02	−.01	.00	+.01	+.02	+.02	+.03
13)	Wn.	60 }	+.07	+.10	+.12	+.13	+.14	+.13	+.12	+.10	+.07	+.04	.00	−.04	−.07
		−20° }	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
	Grw.	60	−.01	−.02	−.03	−.04	−.05	−.05	−.05	−.04	−.04	−.03	−.02	.00	+.01
	Rad.	60	−.08	−.05	−.02	+.01	+.05	+.08	+.10	+.12	+.13	+.13	+.12	+.10	+.08
	Cape	60	−.03	−.06	−.08	−.10	−.11	−.11	−.11	−.10	−.08	−.06	−.03	.00	+.03
	Paris	60	+.08	+.04	.00	−.04	−.08	−.11	−.14	−.16	−.16	−.16	−.14	−.11	−.08
	Stgo.	60	+.24	+.24	+.22	+.19	+.15	+.10	+.04	−.03	−.09	−.14	−.18	−.22	−.24
	Melb.	60	+.55	+.55	+.46	+.29	+.09	−.12	−.29	−.40	−.42	−.38	−.29	−.18	−.09
	Grw.	64	−.04	−.03	−.02	−.01	.00	+.01	+.02	+.03	+.04	+.04	+.04	+.04	+.04

TABLE IV. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\delta_\alpha$. DECLINATION. — Cont.

			0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
			′	′	′	′	′	′	′	′	′	′	′	′	′
	Cape	65	−.02	−.02	−.02	−.01	−.01	−.01	00	00	+.01	+.01	+.02	+.02	+.02
	Brs.	65	−.05	−.03	00	+.02	+.04	+.06	+.08	+.08	+.09	+.09	+.08	+.07	+.05
	Pulk.	65	+.04	+.04	+.04	+.03	+.02	+.01	00	−.01	−.02	−.03	−.04	−.04	−.04
	Leid.	67	−.02	−.02	−.03	−.03	−.03	−.02	−.02	−.01	−.01	00	+.01	+.01	+.02
	Melb.	70	−.11	−.10	−.09	−.07	−.05	−.02	+.01	+.04	+.06	+.08	+.10	+.11	+.11
	Grw.	72	00	00	00	−.01	−.01	−.01	−.01	−.01	−.01	−.01	00	00	00
	Madr.	75	+.17	+.25	+.31	+.35	+.36	+.35	+.32	+.26	+.19	+.11	+.01	−.08	−.17
	Wn.	75	−.02	−.01	00	+.01	+.02	+.02	+.03	+.03	+.04	+.04	+.03	+.03	+.02
	Pulk.	75	+.06	+.07	+.07	+.06	+.06	+.04	+.03	+.01	00	−.02	−.04	−.05	−.06
	Harv.	75	00	00	00	00	00	00	00	00	00	00	00	00	00
18)	Cord.	75	00	−.02	−.04	−.05	−.06	−.07	−.07	−.07	−.06	−.05	−.04	−.02	00
	Paris	75	−.13	−.06	+.01	+.08	+.15	+.21	+.25	+.28	+.28	+.27	+.24	+.19	+.13
14)	Cape	80	−.10	−.12	−.12	−.12	−.11	−.09	−.07	−.04	−.01	+.02	+.05	+.08	+.10
		−25°	+.05	+.05	+.04	+.04	+.02	+.01	00	−.01	−.02	−.04	−.04	−.05	−.05
		−35	−.23	−.21	−.17	−.13	−.07	−.01	+.05	+.11	+.16	+.20	+.22	+.24	+.23
		−45	−.16	−.16	−.14	−.12	−.09	−.05	−.01	+.03	+.07	+.11	+.13	+.15	+.16
	Melb.	80	−.07	−.08	−.09	−.09	−.09	−.08	−.06	−.04	−.02	+.01	+.03	+.05	+.07
	Grw.	80	+.06	+.04	+.02	+.02	+.02	+.04	+.06	+.07	+.07	+.04	00	−.05	−.10
	Pulk.	85	+.02	+.01	+.01	00	−.01	−.01	−.02	−.02	−.03	−.03	−.03	−.02	−.02
	Cape	85	+.12	+.11	+.08	+.06	+.02	−.01	−.04	−.07	−.10	−.11	−.12	−.13	−.12
	Stbg.	85	+.03	+.03	+.02	+.01	+.01	00	−.01	−.02	−.02	−.03	−.03	−.03	−.03
	Rad.	90	+.07	+.10	+.12	+.13	+.14	+.13	+.12	+.10	+.07	+.04	00	−.04	−.07
	Cape	90	−.06	−.06	−.06	−.05	−.04	−.03	−.01	+.01	+.02	+.04	+.05	+.06	+.06
	Grw.	90	00	+.01	+.02	+.02	+.02	+.03	+.03	+.02	+.02	+.02	+.01	00	00
	Madn.	90	−.21	−.12	−.04	+.03	+.08	+.10	+.11	+.11	+.11	+.13	+.15	+.18	+.21
	Ber.	90	+.06	+.05	+.04	+.03	+.01	00	−.02	−.04	−.05	−.06	−.06	−.06	−.06
	Mun.	92	+.01	+.01	00	00	00	−.01	−.01	−.01	−.01	−.01	−.01	−.01	−.01
	Mt. H.	95	+.05	+.06	+.07	+.07	+.07	+.06	+.05	+.04	+.02	00	−.02	−.04	−.05
	Ber.	95	+.08	+.07	+.06	+.04	+.02	00	−.03	−.05	−.06	−.08	−.08	−.09	−.08
15)	W.-Ott.	97	+.07	+.13	+.18	+.21	+.23	+.24	+.23	+.20	+.16	+.11	+.05	−.01	−.07
	Alb.	98	−.06	−.07	−.07	−.07	−.06	−.06	−.04	−.02	00	+.01	+.03	+.05	+.06

TABLE V. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\delta_\delta$. DECLINATION.

			+90°	+85°	+80°	+75°	+70°	+65°	+60°	+55°	+50°	+45°	+40°	+35°	+30°
			′	′	′	′	′	′	′	′	′	′	′	′	′
8)	Br.	1755	See *A.J.* 545.												
	Pi.	1800	.00	+.04	+.08	+.16	+.24	+.23	+.14	− .06	− .38	− .64	−.85	−1.08	−1.29
	Grw.	15	.00	−.05	−.14	−.26	−.37	−.47	−.56	− .63	− .68	− .74	−.78	− .82	− .81
	Kgb.	20	...	...	.00	.00	+.03	+.08	+.08	+ .04	− .03	− .04	.00	.00	.00
	Bb.	25	...	...	...	...	...	...	...	...	...	...	...	...	...
9)	Dpt.	30	.00	−.04	−.09	−.14	−.20	−.26	−.33	− .41	− .43	− .38	−.32	− .27	− .24
	Åbo	30	.00	−.04	−.08	−.09	−.06	−.05	−.05	− .06	− .09	− .13	−.18	− .24	− .31
	Cape	30	...	...	...	...	...	...	...	...	...	...	...	...	...
	Grw.	30	+.17	+.15	+.12	+.07	+.02	−.05	−.12	− .23	− .39	− .62	−.92	−1.04	−1.09
	St. H.	30	...	...	...	...	...	...	...	+1.64	+1.26	+1.06	+.98	+ .99	+1.09
	Cape	33	...	...	...	...	...	...	...	...	...	...	−.08	+ .14	+ .29
	Camb.	30	+.11	+.03	−.06	−.16	−.28	−.35	−.40	− .46	− .54	− .62	−.63	− .59	− .48
10)	Madr.	35	...	...	.00	+.07	+.18	+.34	+.51	+ .61	+ .66	+ .66	+.62	+ .53	+ .36
	Cape	40	...	...	...	...	...	...	...	...	...	− .74	−.49	− .31	− .22
	Grw.	40	.00	−.03	−.05	−.06	−.06	−.05	−.04	− .02	+ .01	+ .04	+.06	+ .07	+ .07

Table IV. Systematic Corrections of the Form, $\Delta\delta_\alpha$. Declination. — Cont.

		12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	0h
		′	′	′	′	′	′	′	′	′	′	′	′	′
Cape	65	+.02	+.02	+.02	+.01	+.01	.00	.00	.00	−.01	−.01	−.02	−.02	−.02
Brs.	65	+.05	+.03	.00	−.02	−.04	−.06	−.08	−.08	−.09	−.09	−.08	−.07	−.05
Pulk.	65	−.04	−.04	−.04	−.03	−.02	−.01	.00	+.01	+.02	+.03	+.04	+.04	+.04
Leid.	67	+.02	+.02	+.03	+.03	+.03	+.02	+.02	+.01	+.01	.00	−.01	−.01	−.02
Melb.	70	+.11	+.10	+.09	+.07	+.05	+.02	−.01	−.04	−.06	−.08	−.10	−.11	−.11
Grw.	72	.00	.00	.00	+.01	+.01	+.01	+.01	+.01	+.01	+.01	.00	.00	.00
Madr.	75	−.17	−.25	−.31	−.35	−.36	−.35	−.32	−.26	−.19	−.11	−.01	+.08	+.17
Wn.	75	+.02	+.01	.00	−.01	−.02	−.02	−.03	−.03	−.04	−.04	−.03	−.03	−.02
Pulk.	75	−.06	−.07	−.07	−.06	−.06	−.04	−.03	−.01	.00	+.02	+.04	+.05	+.06
Harv.	75	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
18) Cord.	75	.00	+.02	+.04	+.05	+.06	+.07	+.07	+.07	+.06	+.05	+.04	+.02	.00
Paris	75	+.13	+.06	−.01	−.08	−.15	−.21	−.25	−.28	−.28	−.27	−.24	−.19	−.13
14) Cape	80	+.10	+.12	+.12	+.12	+.11	+.09	+.07	+.04	+.01	−.02	−.05	−.08	−.10
	−25°	−.05	−.05	−.04	−.04	−.02	−.01	.00	+.01	+.02	+.04	+.04	+.05	+.05
	−35	+.23	+.21	+.17	+.13	+.07	+.01	−.05	−.11	−.16	−.20	−.22	−.24	−.23
	−45	+.16	+.16	+.14	+.12	+.09	+.05	+.01	−.03	−.07	−.11	−.13	−.15	−.16
Melb.	80	+.07	+.08	+.09	+.09	+.09	+.08	+.06	+.04	+.02	−.01	−.03	−.05	−.07
Grw.	80	−.10	−.14	−.16	−.16	−.13	−.08	−.02	+.03	+.08	+.10	+.10	+.08	+.06
Pulk.	85	−.02	−.01	−.01	.00	+.01	+.01	+.02	+.02	+.03	+.03	+.03	+.02	+.02
Cape	85	−.12	−.11	−.08	−.06	−.02	+.01	+.04	+.07	+.10	+.11	+.12	+.13	+.12
Stbg.	85	−.03	−.03	−.02	−.01	−.01	.00	+.01	+.02	+.02	+.03	+.03	+.03	+.03
Rad.	90	−.07	−.10	−.12	−.13	−.14	−.13	−.12	−.10	−.07	−.04	.00	+.04	+.07
Cape	90	+.06	+.06	+.06	+.05	+.04	+.03	+.01	−.01	−.02	−.04	−.05	−.06	−.06
Grw.	90	.00	−.01	−.02	−.02	−.02	−.03	−.03	−.02	−.02	−.02	−.01	.00	.00
Madn.	90	+.21	+.22	+.21	+.17	+.10	.00	−.11	−.21	−.29	−.33	−.32	−.28	−.21
Ber.	90	−.06	−.05	−.04	−.03	−.01	.00	+.02	+.04	+.05	+.06	+.06	+.06	+.06
Mun.	92	−.01	−.01	.00	.00	.00	+.01	+.01	+.01	+.01	+.01	+.01	+.01	+.01
Mt. H.	95	−.05	−.06	−.07	−.07	−.07	−.06	−.05	−.04	−.02	.00	+.02	+.04	+.05
Ber.	95	−.08	−.07	−.06	−.04	−.02	.00	+.03	+.05	+.06	+.08	+.08	+.09	+.08
15) W.-Ott.	97	−.07	−.13	−.18	−.21	−.23	−.24	−.23	−.20	−.16	−.11	−.05	+.01	+.07
Alb.	98	+.06	+.07	+.07	+.07	+.06	+.06	+.04	+.02	.00	−.01	−.03	−.05	−.06

Table V. Systematic Corrections of the Form, $\Delta\delta_\delta$. Declination.

		+25°	+20°	+15°	+10°	+5°	0	−5°	−10°	−15°	−20°	−25°	−30°
		′	′	′	′	′	′	′	′	′	′	′	′
8) Br.	1755	See *A.J.* 545.											
Pi.	1800	−1.53	−1.86	−2.06	−2.10	−1.99	−1.96	−2.12	−2.33	−2.26	−1.79	−1.26	−1.16
Grw.	15	−.77	−.72	−.70	−.71	−.73	−.78	−.84	−.92	−1.01	−1.10	−1.20	−1.30
Kgb.	20	.00	−.05	−.12	−.09	−.01	+.05	+.08	+.05	+.01	. . .	. . .	. . .
Bb.	25	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.29	+.17
9) Dpt.	30	−.22	−.25	−.34	−.51	−.68	−.77	−.83	−.88	−.92	−.97	. . .	. . .
Åbo	30	−.38	−.43	−.47	−.50	−.53	−.55	−.59	−.67	−.77	−.91	. . .	. . .
Cape	30	. . .	. . .	. . .	−.20	−.14	−.06	+.02	+.10	+.18	+.24	+.28	+.30
Grw.	30	−1.16	−1.33	−1.51	−1.60	−1.59	−1.45	−1.39	−1.56	−2.07	−2.53	−2.86	. . .
St. H.	30	+1.32	+1.37	+1.26	+1.02	+.91	+.92	+1.01	+1.01	+.72	−.03	−.35	−.18
Cape	33	+.40	+.38	−.32	−.22	−.12	−.11	−.21	−.14	+.20	+.53	+.72	+.79
Camb.	30	−.30	−.16	−.14	−.16	−.23	−.42	−.69	−.95	−1.22	−1.49	−1.76	. . .
10) Madr.	35	+.07	−.17	−.30	−.36	−.36	−.33	−.28	−.20	−.10	+.05	+.25	+.47
Cape	40	−.21	−.23	−.20	−.12	+.04	+.38	+.58	+.56	+.40	+.31	+.22	+.10
Grw.	40	+.08	+.10	+.14	+.18	+.24	+.30	+.37	+.42	+.46	+.48	+.49	. . .

TABLE V. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\delta_\delta$. DECLINATION. — Cont.

			+90°	+85°	+80°	+75°	+70°	+65°	+60°	+55°	+50°	+45°	+40°	+35°	+30°
	Grw.	45	.00	+.02	+.04	+.06	+.09	+.12	+.12	+.08	.00	−.08	−.08	+.03	+.14
	Arm.	40	.00	.00	.00	.00	.00	.00	.00	−.14	−.49	−.67	−.62	−.56	−.55
11)	Rad.	45	.00	+.16	+.50	+.74	+.67	+.32	+.15	+.08	+.01	−.12	−.31	−.53	−.70
	Pulk.	45	.00	.00	.00	.00	+.04	+.10	+.17	+.21	+.23	+.26	+.32	+.36	+.36
	Paris	45	.00	.00	.00	.00	−.08	−.15	−.17	−.13	−.04	+.11	+.18	+.12	+.03
	Stgo.	50	...	...	...	...	...	...	...	...	...	...	+.17	+.39	+.56
	Grw.	50	−.12	−.13	−.14	−.15	−.16	−.17	−.18	−.18	−.17	−.15	−.12	−.07	−.01
12)	Cape	50	...	...	...	...	...	...	...	...	...	...	...	...	...
	Stgo.	55	...	...	...	...	...	...	...	...	...	...	...	...	...
	Pulk.	55	−.26	−.22	−.16	−.05	+.04	+.08	+.09	+.09	+.12	+.21	+.30	+.35	+.35
	Wn.	60	.00	+.16	+.36	+.43	+.41	+.35	+.37	+.40	+.38	+.32	+.25	+.14	+.06
	Grw.	60	.00	+.06	+.13	+.19	+.25	+.29	+.31	+.30	+.26	+.18	+.08	+.06	+.14
	Rad.	60	.00	+.27	+.68	+.90	+.86	+.68	+.49	+.38	+.29	+.06	−.30	−.70	−.90
	Cape	60	...	...	...	...	...	...	...	...	...	−.02	−.06	−.11	−.16
	Paris	60	.00	−.03	−.07	−.06	−.03	+.02	+.08	+.15	+.16	+.07	−.07	−.14	−.17
	Stgo.	60	...	...	...	...	...	...	...	...	...	...	...	...	.00
16)	Melb.	60	...	...	...	...	...	...	...	...	...	...	+.38	+.34	+.30
	Grw.	64	.00	.00	.00	.00	−.02	−.04	−.04	.00	+.06	+.06	+.06	+.06	+.10
	Cape	65	...	...	...	...	...	...	...	...	...	...	+.20	+.13	+.05
	Brs.	65	+.38	+.41	+.44	+.47	+.50	+.52	+.43	+.19	−.03	−.14	−.16	−.09	.00
	Pulk.	65	.00	.00	.00	.00	.00	.00	+.04	+.11	+.21	+.26	+.27	+.24	+.23
	Leid.	67	.00	+.03	+.07	+.10	+.05	−.09	−.19	−.20	−.18	−.15	−.10	−.08	−.15
	Melb.	70	...	...	...	...	...	...	...	...	...	...	+.76	+.48	+.19
	Grw.	72	.00	−.04	−.09	−.14	−.20	−.28	−.36	−.44	−.49	−.53	−.54	−.54	−.52
17)	Madr.	75	...	...	+.26	−.04	−.64	−.70	+.60	−.13	−.23	−.22	−.26	−.17	+.54
	Wn.	75	.00	+.01	+.03	+.06	+.09	+.12	+.17	+.21	+.18	+.06	−.10	−.24	−.30
	Pulk.	75	.00	.00	.00	+.02	+.04	+.06	+.08	+.10	+.12	+.12	+.10	+.06	+.02
	Harv.	75	.00	+.02	+.06	+.10	+.09	+.05	−.05	−.14	−.11	+.10	+.26	+.28	+.24
18)	Cord.	75	...	...	...	...	...	...	...	...	...	...	...	...	−.94
	Paris	75	.00	.00	.00	−.04	−.10	−.17	−.21	−.23	−.26	−.29	−.33	−.34	−.32
14)	Cape	80	...	...	...	...	...	...	...	...	...	+.06	−.02	−.07	−.11
	Melb.	80	...	...	...	...	...	...	...	...	...	+1.00	+.72	+.41	+.11
	Grw.	80	.00	+.02	+.05	+.11	+.17	+.21	+.21	+.18	+.10	+.01	−.04	−.06	−.01
	Pulk.	85	.00	.00	.00	+.02	+.04	+.05	+.06	+.04	+.04	+.05	+.09	+.14	+.18
	Cape	85	...	...	...	...	...	...	...	...	...	−1.03	−.81	−.62	−.47
	Stbg.	85	.00	.00	.00	−.02	−.06	−.07	−.07	−.06	−.07	−.09	−.08	−.06	−.05
	Rad.	90	.00	.00	.00	−.04	−.13	−.32	−.54	−.58	−.49	−.45	−.50	−.62	−.72
	Cape	90	...	...	...	...	...	...	...	...	...	−.30	−.07	+.02	.00
	Grw.	90	−.10	−.10	−.10	−.09	−.06	−.01	.00	−.03	−.09	−.10	−.08	−.06	−.05
	Madn.	90	.00	−.06	−.10	−.13	−.14	−.14	−.14	−.12	−.08	−.02	+.10	+.26	+.32
	Ber.	90	.00	+.03	+.07	+.13	+.16	+.15	+.13	+.09	+.04	−.01	−.06	−.12	−.14
	Mun.	92	.00	−.05	−.14	−.24	−.28	−.25	−.24	−.32	−.44	−.63	−.78	...	...
	Mt. H.	95	.00	+.05	+.08	+.07	+.05	.00	...	...	...	+.09	.00	−.17	−.25
	Ber.	95	+.06	+.06	+.06	+.06	+.07	+.11	+.19	+.18	+.07	−.03	−.10	−.14	−.16
15)	W.-Ott.	97	.00	.00	.00	.00	.00	+.02	+.07	+.06	−.03	−.19	...	...	−.85
	Alb.	98	−.08	+.02	+.18	+.31	+.37	+.36	+.29	+.17	+.10	+.06	+.06	+.04	−.01

Table V. Systematic Corrections of the Form, $\Delta\delta_\delta$. Declination. — Cont.

			+25°	+20°	+15°	+10°	+5°	0	−5°	−10°	−15°	−20°	−25°	−30°
			″	″	″	″	″	″	″	″	″	″	″	″
	Grw.	45	+ .17	+ .12	.00	− .04	.00	+ .09	+ .10	+ .07	.00	− .07	− .13	− .21
	Arm.	40	− .53	− .52	− .55	− .71	− .86	− .88	− .83	− .76	− .69	− .58	− .31	+ .22
11)	Rad.	45	− .77	− .72	− .58	− .32	− .08	+ .17	+ .41	+ .56	+ .54	+ .28	− .32	. . .
	Pulk.	45	+ .35	+ .34	+ .33	+ .34	+ .35	+ .38	+ .42	+ .47	+ .53	+ .60	. . .	. . .
	Paris	45	.00	.00	+ .01	− .07	− .15	− .19	− .19	− .15	− .05	+ .11	+ .34	+ .60
	Stgo.	50	+ .69	+ .78	+ .84	+ .82	+ .85	+ .93	+ .99	+1.02	+1.00	+ .95	+ .82	+ .56
	Grw.	50	+ .05	+ .13	+ .15	+ .11	− .01	− .13	− .14	− .13	− .11	− .07	− .03	+ .01
12)	Cape	50	. . .	. . .	. . .	. . .	. . .	− .40	− .34	− .19	+ .02	+ .14	+ .14	+ .07
	Stgo.	55	+ .26	+ .20	+ .14	+ .10	+ .06	+ .04	+ .03	− .01	− .11	− .25	− .35	− .34
	Pulk.	55	+ .32	+ .33	+ .37	+ .39	+ .40	+ .40	+ .42	+ .50	+ .71	+ .99	. . .	. . .
	Wn.	60	+ .02	− .02	− .07	− .17	− .19	− .14	− .04	− .01	.00	+ .03	+ .12	+ .31
	Grw.	60	+ .26	+ .29	+ .28	+ .23	+ .12	+ .04	+ .02	+ .04	+ .10	+ .20	+ .32	+ .46
	Rad.	60	− .87	− .67	− .45	− .22	+ .01	+ .26	+ .51	+ .74	+ .82	+ .66	+ .36	+ .13
	Cape	60	− .21	− .26	− .28	− .31	− .33	− .34	− .34	− .31	− .22	− .08	− .01	+ .02
	Paris	60	− .18	− .22	− .26	− .28	− .26	− .20	− .18	− .15	− .05	+ .14	+ .43	+ .78
	Stgo.	60	− .06	− .14	− .28	− .46	− .56	− .52	− .30	− .12	− .14	− .20	− .12	+ .21
16)	Melb.	60	+ .28	+ .28	+ .35	+ .50	+ .78	+ .90	+ .90	+ .76	+ .55	+ .44	+ .41	+ .40
	Grw.	64	+ .12	+ .11	+ .09	+ .05	.00	− .04	− .05	.00	+ .11	+ .26	+ .46	+ .66
	Cape	65	− .02	− .09	− .16	− .20	− .24	− .25	− .24	− .19	− .11	.00	+ .04	− .03
	Brs.	65	+ .04	+ .03	.00	− .03	− .04	− .04	− .02	+ .02	+ .08	+ .16	+ .27	+ .47
	Pulk.	65	+ .24	+ .31	+ .36	+ .35	+ .33	+ .35	+ .40	+ .46	+ .54	+ .64	. . .	. . .
	Leid.	67	− .29	− .36	− .37	− .32	− .22	− .05	+ .04	+ .06	+ .01	− .13	. . .	. . .
	Melb.	70	− .09	− .35	− .40	− .26	− .17	− .18	− .25	− .35	− .40	− .42	− .41	− .41
	Grw.	72	− .50	− .52	− .59	− .74	− .95	−1.15	−1.26	−1.34	−1.46	−1.70	−2.09	−2.53
17)	Madr.	75	+ .62	+ .79	+ .33	− .41	− .58	+ .66	− .05	− .09	+ .01	− .02	+ .10	+ .86
	Wn.	75	− .31	− .29	− .27	− .26	− .25	− .24	− .26	− .30	− .31	− .29	− .22	− .08
	Pulk.	75	+ .02	+ .05	+ .09	+ .11	+ .12	+ .12	+ .17	+ .27	+ .42	+ .60	. . .	. . .
	Harv.	75	+ .23	+ .31	+ .29	+ .06	.00	+ .12	+ .25	+ .31	+ .32	+ .33	+ .36	+ .42
18)	Cord.	75	− .82	− .70	− .60	− .54	− .49	− .48	− .49	− .46	− .37	− .31	− .28	− .28
	Paris	75	− .28	− .26	− .25	− .24	− .23	− .22	− .22	− .21	− .16	− .02	+ .23	+ .49
14)	Cape	80	− .14	− .16	− .17	− .18	− .15	− .07	− .03	− .02	− .03	− .07	− .12	− .21
	Melb.	80	− .20	− .41	− .41	− .32	− .28	− .32	− .45	− .55	− .60	− .61	− .65	− .67
	Grw.	80	+ .11	+ .23	+ .28	+ .26	+ .15	+ .11	+ .22	+ .34	+ .44	+ .51	+ .61	+ .79
	Pulk.	85	+ .22	+ .24	+ .25	+ .25	+ .24	+ .22	+ .19	+ .15	+ .06	− .14	. . .	. . .
	Cape	85	− .37	− .31	− .30	− .29	− .28	− .27	− .26	− .26	− .27	− .28	− .28	− .24
	Stbg.	85	− .08	− .10	− .08	− .02	.00	.00	.00	.00	− .03	− .08	− .13	− .18
	Rad.	90	− .76	− .77	− .77	− .77	− .76	− .74	− .68	− .61	− .54	− .47	− .39	− .32
	Cape	90	− .05	− .06	− .04	− .01	− .02	− .05	− .09	− .11	− .10	− .08	− .01	+ .04
	Grw.	90	− .04	− .02	.00	+ .02	+ .03	+ .04	+ .06	+ .12	+ .21	+ .34	+ .50	+ .70
	Madn.	90	+ .31	+ .28	+ .29	+ .34	+ .38	+ .38	+ .34	+ .29	+ .23	+ .18	. . .	. . .
	Ber.	90	− .11	− .08	− .06	− .03	.00	+ .03	+ .05	+ .08	+ .10	. . .	. . .	. . .
	Mun.	92	. . .	. . .	− .70	− .74	− .79	− .84	− .89	− .97	−1.05	. . .	. . .	. . .
	Mt. H.	95	− .23	− .14	− .10	− .09	− .10	− .05	+ .04	+ .14	+ .21	+ .26	+ .32	+ .43
	Ber.	95	− .14	− .12	− .12	− .12	− .11	− .09	− .02	+ .08	+ .22	+ .42	+ .62	. . .
15)	W.-Ott.	97	−1.01	−1.07	−1.05	− .98	− .85	− .72	− .64	− .59	− .56	− .52	− .46	− .35
	Alb.	98	− .06	− .09	− .10	− .09	− .05	+ .01	.00	− .05	− .08	− .06	− .02	+ .06

Table V. Systematic Corrections, $\Delta\delta_s$, South of $-30°$.

	$-30°$	$-35°$	$-40°$	$-45°$	$-50°$	$-55°$	$-60°$	$-65°$	$-70°$	$-75°$	$-80°$	$-85°$	$-90°$
	″	″	″	″	″	″	″	″	″	″	″	″	″
Pi. 1800	−1.16	−1.36	−1.98	−2.63									
Bb. 25	+ .17	+ .08	+ .06	+ .16	+ .34	+ .56	+ .97	+1.20	+1.03	+.58	+.32	+.13	.00
Cape 30	+ .30	+ .26	+ .18	+ .04	− .14	− .30	− .38	− .40	− .32	−.16	.00	.00	.00
St. H. 30	− .18	+ .33	+ .52	+ .48	+ .27	+ .13	+ .10	+ .22	+ .55	+.75	+.73		
Cape 33	+ .79	+ .79	+ .51	− .06	− .25	− .23	− .25	− .30	− .16	−.02	.00	.00	.00
10) Madr. 35	+ .47	+ .75	+1.00	+1.15	+1.23	+1.24	+1.20	+1.13					
Cape 40	+ .10	− .04	− .22	− .38	− .52	− .64	− .74	− .79	− .71	−.42	−.21	−.07	.00
Stgo. 50	+ .56	+ .25	+ .13	+ .27	+ .36	+ .36	+ .19	.00	.00	.00	.00	.00	.00
12) Cape 50	+ .07	− .01	− .08	− .17	− .36	− .32	− .17	− .23	− .37	−.31	−.12	−.03	.00
Stgo. 55	− .34	− .19	− .07	− .04	− .07	− .18	− .35				−.50	−.36	−.17
Wn. 60	+ .31	+ .64	+ .97										
Cape 60	+ .02	.00	− .04	− .06	− .06	− .04	.00	+ .05	+ .08	+.10	+.10	+.09	+.08
Stgo. 60	+ .21	+ .63	+ .83	+ .88	+ .76	+ .55	+ .37	+ .25	+ .15	+.08	+.03	.00	.00
Melb. 60	+ .40	+ .38	+ .30	+ .16	− .05	− .29	− .48	− .54	− .52	−.45	−.33	−.17	.00

Table VI. Weights in Right-Ascension for the Principal Catalogues.

Catal.	Wts.	.05	.1	.15	.2	.25	.3	.35	.4	.5	.6	.7	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Bradley 1755	Eq.	2	4	6	8	10	13	15	17																
19)	+60°	1	2	3	4	5	6	7	8																
20) Piazzi 1800		8	27																						
Greenw. 15			1	2	3		4	5	6	8	10	13	17												
Dorpat 15				1			2		3	4	5	6	7	16											
Königsb. 15		1	2	3	4	6	7	8	10	13	16	21	26												
Königsb. 25			1		2	3	4		5	6	7	9	10	19	31	50	78								
Dorpat 30			1	2		3	4	5	6	7	9	10	12	26	51										
Cape 30		1	2	4	5	7	9	12	15	21	38	75													
St. H. 30			1	2	3	4		6	7	8	11	15	20												
Åbo 30				1			2			3	4	5	6	10	14	20	27	35	44	63					
Greenw. 30			2	5	7	9	12	16	21	31	36														
Cambr. 30			1	2	3	4	5	6	8	11	15	25	65												
Cape 33			1			2		3	4	5	6	7	9	20	45										
21) Madras 35		1	2	4	6	8	11	14	19	28															
Armagh 40		1	2	4	6	9	13	18	27																
Cape 40		1	2	3	4	5	6	7	8	9	12	14	19	29	44										
Greenw. 40			1	2	3		4	5	6	8	10	13	15	43											
Greenw. 45			1		2	3		4	5	6	8	9	11	23	40	75									
Radcl. 45	Eq.	1		2	3	4	5	6	7	10	13	18	25												
	+60°			1			2		3	5	7	10	13												
22) Pulkowa 45	S.					1				2		3	4	6	9	13	16	21	27	37	58				
	N.										1		2		3	5	6	7	8	10	14	19	25	35	48
Paris 45	Eq.		1	2		3	4	5	6	7	8	10	13	23	39	64	110								
	+60°			1			2		3		4	5	7	11	17	25	39	58	95						
Santiago 50			1	2		3	4		5	6	8	10	12	26	55										
Greenw. 50			1		2		3		4	5	7	8	9	17	27	41	61	91							
Cape 50			1	2	3		4		6	10	15	25													
Santiago 55			1	2	3		4		6	10	15	25													
Cape 60				1			2		3		4	5	6	11	17	25	38	57							
Wash'n 60			1		2		3		4	5	7	8	10	17	28	41	61	91							
Greenw. 60					1			2		3		4	5	8	13	18	24	32	42	65					
Radcl. 60			1	2	3	4		5	6	8	10	12	17												
Santiago 60			1	2	3		4		6	10	15	25													
Melb. 60				1			2			3	4	5	6	10	14	20	26	34	44						
Paris 60	Eq.	1		2	3	4	5		6	8	9	11	14	23	52	71				150	250				
	+55°		1		2	3		4	5	6	8	9	11	22	52	71				150	250				
	+80°			1		2		3		4	5	6	8	15	52	71				150	250				
Greenw. 64					1				2		3		4	7	10	13	17	22	26	35	50	75			

Table V. Systematic Corrections, $\Delta\delta_s$, South of −30°. — Cont.

	−30°	−35°	−40°	−45°	−50°	−55°	−60°	−65°	−70°	−75°	−80°	−85°	−90°
	″	″	″	″	″	″	″	″	″	″	″	″	″
Cape 65	−.03	−.03	+.02	+.09	+.13	+.14	+.14	+.16	+.17	+.19	+.21	+.23	+.25
Melb. 70	−.41	−.47	−.63	−.63	−.51	−.45	−.46	−.46	−.42	−.31	−.18	−.08	.00
17) Madr. 75	+.86	+.95	+1.11	+.75	+.13	+.16	+1.60	+1.20	+2.00	. .	. .	. .	. .
Wn. 75	−.08	+.22	+.71	. . .	. . .	. . .	. . .	. . .	. . .	. .	. .	. .	. .
Harv. 75	+.42	+.51	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. .	. .	. .	. .
18) Cord. 75	−.28	−.26	−.23	−.18	−.14	−.11	−.08	−.06	−.04	−.03	−.02	−.01	.00
14) Cape 80	−.21	−.29	−.26	−.10	+.04	+.12	+.16	+.17	+.25	+.43	+.44	+.26	+.06
Melb. 80	−.67	−.69	−.71	−.71	−.67	−.60	−.45	−.28	−.17	−.10	−.05	−.02	.00
Cape 85	−.24	−.19	−.16	−.17	−.24	−.30	−.29	−.20	−.12	−.06	−.03	−.01	.00
Cape 90	+.04	+.05	+.02	−.02	−.03	.00	+.02	+.04	+.04	+.03	+.01	.00	.00
Mt.H. 95	+.43	+.65	+1.12	. . .	. . .	. . .	. . .	. . .	. . .	. .	. .	. .	. .
15) W-Ott. 97	−.35	−.22	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. .	. .	. .	. .
Alb. 98	+.06	+.15	+.25	. . .	. . .	. . .	. . .	. . .	. . .	. .	. .	. .	. .

Table VI. Weights in Right-Ascension for the Principal Catalogues. — Cont.

	Catal.		Wts.	.05	.1	.15	.2	.25	.3	.35	.4	.5	.6	.7	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
	Cape	65				1			2		3	4	5	6	7	12	20	32	50								
	Brussels	65			1		2		3		4		5	7	8	28											
	Harvard	65				1		2			3	4	5	6	7	15											
22)	Pulkowa	65	S.									1			2	3	4	6	7	9	10	14	18	28	35	56	
			N.											1		2	3	4	5	6	7	9	12	14	18	23	28
	Melb.	70					1			2		3		4	5	8	12	17	22	28	35	50	78				
	Greenw.	72	Eq.					1				2		3	4	6	9	11	14	18	22	30	39	54	74		
			+60°									1			2	3	5	6	7	9	11	14	20	28	37	53	77
	Madras	75	Eq.	1	3	4	6	8	9	11	13	16	20	24	34	61											
			+60°	1	2	4	5	7	10	13	17	23	46														
			−50°	2	4	7	11	17	25	61																	
	Wash'n	75				1			2			3	4	5	6	9	13	17	22	27	32	41	55	73	94		
	Pulkowa	75	Eq.		1		2		3		4	5	6	7	8	13	18	24	30	36	43	53	69	86			
			+60°				1			2		3		4	5	8	11	14	18	21	25	30	42	54	67	84	
	Harvard	75	Eq.				1			2		3	4	5	6	9	12	16	20	25	30	38	50	65	83		
			+70°							1				2	3	4	6	8	10	12	14	18	25	34	45	61	83
23)	Cordoba	75					1		2	3	4	5	6	8	10	28											
	Paris	75			1		2		3		4	5	6	7	10	17	29	47	80								
	Cape	80			1			2		3		4	5	6	8	12	18	26	34	45	57	83					
	Melb.	80			1		2		3		4	5	6	7	9	15	23	35	50	72							
	Greenw.	80						1				2			3	6	8	11	14	18	23	30	45	64			
22)	Pulkowa	85	S.									1			2	3	4	6	7	9	10	14	18	28	35	56	
		92	N.											1		2	3	4	5	6	7	9	12	14	18	23	28
	Cape	85	Eq.			1		2			3	4	5	6	7	10	15	19	23	28	33	40	51	63	76		
			−60°					1				2		3	4	6	8	11	14	17	20	27	37	50	68		
	Strassb.	85									1				2	4	5	7	9	11	13	17	25	36	51	80	
	Radcl.	90				1		2			3	4		5	7	10	15	20	25	31	38	49	66				
	Cape	90				1			2			3	4	5	6	9	13	17	22	27	32	40	54	71	91		
	Madison	90												1		2	3	4	5	6	8	10	15	24	38	70	
24)	Berlin	90													1		2		3	4	5	6	9	12	17	24	40
	Lisbon	90													1		2	3		4	5	7	9	12	16	22	32
	Greenw.	90							1			2		3	4	5	8	10	12	15	18	24	32	43	57	77	
	Mt. H.	95																			10						
	Berlin	95													1		2		3	4	5	6	9	12	17	24	40
25)	Albany	98										1			2	3	4	6	7	9	11	14	20				

TABLE VII. WEIGHTS IN DECLINATION FOR THE PRINCIPAL CATALOGUES.

Catal.	Wts.	.05	0.1	.15	.2	.25	.3	.35	.4	.5	.6	.7	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
26) Bradley	1755	1	2	4	5	8	11	17	28																
Piazzi	1800	1	4	9	18																				
Greenwich	15			1		2			3	4	5	6	8	12	18	25	35								
Königsberg	20	Argument, probable error.																							
Brisbane	25	2	11																						
Dorpat	30								1			2	3	4	7	11	17	30	69						
Åbo	30								1			2	3	4	7	11	17	30	69						
Cape	30		1	2	4	7	10																		
Greenwich	30	1		2		3	4	5	6	7	10	14	26												
St. Helena	30	1	2	3	5	6	8	11	15	23															
Cape	33			1			2			3	4	5	6	13											
Cambridge	30				1		2			3	4	5	8	19											
Madras	35		1	2	3	4	6	8	12	19															
Cape	40				1			2		3	4	5	7	16											
Greenwich	40				1		2			3	4	5	7	12	22	46									
Greenwich	45				1				2		3	4	5	9	15	27	51								
Armagh	40		1	2	3	4	5	6	7	9	13	18													
Radcliffe	45				1		2		4	6	11	24													
27) Pulkowa	45												1		2	3	4	6	7	12					
Paris	45			1			2		3	4	5	6	8	13	23	38	70								
Santiago	50	1		2		3	4	5	6	7	10	14	26												
Greenwich	50				1		2			3	4	5	7	12	22	46									
Cape	50				1				2	3	4	5	6	15											
Santiago	55	1		2	3		4	5	7	10	13	18	25												
28) Pulkowa	55				1				2		3	4	5	7	11	17	25	36	61						
Washington	60		1		2		3	4	5	6	7	9	13	28	80										
Greenwich	60								1			2	3	5	7	11	15	23	38						
Radcliffe	60		1	2	3	4	5	6	8	12															
Cape	60					1			2		3	4	5	7	11	17	25	39	61						
Paris	60			1			2		3	4	5	6	8	13	23	38	70								
Santiago	60	1		2	3		4	5	7	10	13	18	25												
Melbourne	60			1			2		3	4	5	6	8	13	23	38	70								

NOTE 1. The magnitude-equations for the two Madras Catalogues are very uncertain. There is reason to believe that they increase more rapidly than the proportion of numerical magnitude. The ground for this suspicion is given in the chapter on Magnitude-Equation, *A.J.* 536.

NOTE 2. The corrections for Pulkowa 92 (right-ascensions of secondary standards determined with the Transit) rest on the comparison of only 148 stars with the Standard Catalogue. Many of these stars depend upon a comparatively small number of observations. For observed $\Delta\alpha_m$ we have

Mag.	p_α	$\Delta\alpha_m$
1.9	8	$+0^s.011$
3.1	28	-0.005
4.0	94	$+0.005$
4.9	80	$+0.006$
6.0	17	$+0.006$

All the corrections for this catalogue in the present tables must be regarded as approximate only.

NOTE 3. The corrections for Albany 98 are intended to apply, not to the positions obtained in the preliminary reductions on quasi fundamental principles, but to the positions to be published in the final catalogue for 1900.

NOTE 4. STRUVE's right-ascensions as published in "Catalogue I," Part II, Volume I, of the Dorpat observations, were corrected (by means of the equations given for each star of the Catalogue) to the aberration, $20''.50$, and nutation, $9''.224$. Large corrections were also applied to the catalogue positions on account of the adopted corrections to the assumed right-ascensions of STRUVE's six standard stars, as follows :

I	*Capella*,	$+0^s.17$	IV	*α Persei*	$+0^s.21$
II	*α Lyrae*,	$+0.17$	V	*δ Cassiopeae*,	$+0.24$
III	*α Cygni*,	$+0.18$	VI	*ε Urs. Maj.*,	$+0.18$

The corrections given in Tables II and III apply to STRUVE's right-ascensions so treated. The right-ascensions of "Catalogue II," 1814, were reduced to Catalogue I by the application of the following correction,

$$+0^s.18 - 0^s.0442\,\nu + [+^s.019 + ^s.031 \sin(268^\circ + \alpha)] \sec\delta$$

ν is taken from Catalogue II.

Table VII. Weights in Declination for the Principal Catalogues. — Cont.

	Catal.	Wts.	.05	.1	.15	.2	.25	.3	.35	.4	.5	.6	.7	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
	Greenwich	64								1			2	3	5	7	11	15	23	38						
29)	Cape	65					1			2		3	4	5	7	11	17	25	39	61						
	Brussels	65						1			2		3	6												
	Pulkowa	65												1		2		3	4	5	6	9	11	15	19	27
30)	Leiden	67																		16	30					
	Melbourne	70						1			2		3	4	7	13	28									
	Greenwich	72					1			2		3	4	5	7	11	17	25	38	61						
	Madras	75	1	2	3	4	5	7	8	10	14	20	30													
	Washington	75				1				2		3	4	5	8	11	15	19	24	30	41	61	91			
	Pulkowa	75								1			2	3	4	5	7	9	11	13	18	24	34	46	66	
	Harvard	75		1		2	3		4	5	6	7	9	12	21	36	61									
31)	Cordoba	75				1				2	3	5	7	11												
	Paris	75			1			2		3	4	5	6	8	13	23	38	70								
	Cape	80						1				2		3	5	7	10	14	18	25						
	Melbourne	80					1			2		3	4	5	8	13	22	36								
	Greenwich	80						1			2		3	4	6	8	11	15	18	23	33	52	88			
	Pulkowa	85												1		2		3	4	5	6	9	11	15	19	27
	Cape	85						1				2	3	4	5	8	12	16	22	30	50					
	Strassburg	85						1				2		3	5	7	9	12	15	19	25	38	57	91		
	Radcliffe	90					1			2		3	4	5	7	11	16	22	30	42	76					
	Cape	90					1				2		3	4	6	9	12	16	21	27	40	68				
	Greenwich	90						1				2		3	5	7	9	12	15	19	25	38	57	91		
	Madison	90									1			2	3	5	6	8	9	11	15	20	28	38	54	78
24)	Berlin	90												1		2		3		4	6	8	12	19	36	
	Munich	92												1				15								
	Mt. Hamilton	95																	10							
	Berlin	95												1		2		3		4	6	8	12	19	36	
	Wien-Ottak.	97				1				2		3	4	5	8	12	17	23	30	39	60					
32)	Albany	98										1		2	3	4	5	7	10	14						

Note 5. Under Königsberg 20 are designated the right-ascensions of circumpolar stars observed by Bessel, and reduced by Argelander, Volume VI, p. XV, of the Königsberg observations. These correspond to Bessel's equinox of 1815; but are, otherwise, supposed to be homogeneous with Bessel's right-ascensions of time stars with the Reichenbach circle.

Note 6. The corrections in right-ascension for the General Catalogue, Cordoba 1875, apply to the mean for the years, 1872–1880, inclusive. The following corrections are adopted in order to reduce the right-ascensions, clamp east, to the mean. For right-ascensions, clamp west, the opposite sign must be employed.

Corrections, Clamp East.

δ (°)	Corr. (s)	δ (°)	Corr. (s)
0	−.006	−45	+.029
−5	−.001	50	+.030
10	+.004	55	+.030
15	+.009	60	+.030
20	+.014	65	+.030
25	+.018	70	+.030
30	+.023	75	+.030
35	+.027	80	+.030
40	+.028	85	+.030

The clamp was east in 1872, 1875.67 to 1877.0, 1878, 1880 to 1884.

Note 7. The values of $\Delta\alpha_\delta$ for the Cape Catalogue for 1880, are, in part, treated by zones. Thus at −35°, −45°, −55°, −65° and −75° in Table III, two values of $\Delta\alpha_\delta$ are given. The uppermost in each case pertains to the zone of lesser declination, the lowermost to that of greater declination. The great leap at −55° seems to be well established. Since, for nearly all the stars compared with the Standard Catalogue the number of observations is only three, the present results for the zones can only be regarded as approximate.

Note 8. The line in Table IV referring to Bradley, 1755, and marked "N," is applicable to declinations from Quadrant North; the lower line to those from Quadrant South. The curve of $\Delta\delta_\delta$ for Bradley is too irregular to admit of accurate interpolation from a table given for intervals of 5° only. A table for Bradley's declinations will be found in *A.J.* 545. ΔA_δ is applicable to the declinations formed (without the correction, "Δz") from zenith-distances; $\Delta\delta_\delta$ applies to declinations from the Catalogue.

Note 9. The corrections for Dorpat 30 (*Positiones Mediæ*) are applicable to the catalogue declinations revised by the addition of "*Correctiones Ultimæ*," p. 357, P.M.

NOTE 10. Madras 35 refers to DOWNING's new edition of TAYLOR's Catalogue for 1835.

NOTE 11. Radcliffe 1845. The corrections in Tables IV and V correspond to the catalogue declinations corrected for the quantities given in the table, p. viii, of the introduction to the Catalogue.

NOTE 12. The corrections in Tables IV and V for Cape 50 are very uncertain, owing to the very small number of observations of the principal stars contained in the Catalogue.

NOTE 13. As might naturally be expected from the long period embraced in YARNALL's Catalogue there is great uncertainty in $\Delta\delta_\alpha$. South of —20° it is assumed to be zero.

NOTE 14. The values of $\Delta\delta_\alpha$ for Cape 80, adopted in Table IV, are determined from comparison with the Standard Catalogue in zones. The second line applies to the zone —25° to —35°; the third line to —35° to —45°; and the fourth line, to all declinations south of —45°.

NOTE 15. "W.-Ott. 97" indicates GROSSMAN's Catalogue of declinations observed at Von Kuffner's Observatory in Wien-Ottakring, and published in *Abh., Kön. Sächs. Ges. der Wiss.*, Band XXVII, Leipzig.

NOTE 16. To the declinations of the Williamstown observations (Melb. 60) are first applied the corrections on account of latitude, graduation error, and flexure, contained in the table at p. xxi of the introduction to the Catalogue (*Melb. Obs.*, Vol. I). Table V refers to the declinations thus corrected.

NOTE 17. The process by which $\Delta\delta_\delta$ for Madras 1875 was obtained assumes that a large part of the error of that Catalogue is due to the faulty application of division-correction. Shortly after the publication of that Catalogue, I compared it with the system B_δ (*Am. Ephem.* 1881–1899) and its southward extension, B_δ (*A. J.* 448–50). This comparison not only indicated recurrence of systematic errors at intervals of 60°, but also that these errors would have been far less striking if the correction for error of graduation had been applied in the reductions with the opposite sign. The following exhibit contains, in the first column, the effect of division correction, D, as adopted in the reductions for the Catalogue; in the second column, the division correction, D_0, as it results from comparison of the Catalogue declinations with the Standard Catalogue; and in the third column that part of adopted $\Delta\delta_\delta$ which is found to recur at intervals of 60°.

ANALYSIS OF MADRAS DECLINATIONS (1875).

δ	D	D_0	D_c	δ	D	D_0	D_c
°	″	″	″	°	″	″	″
+60	—.35	+.32	+.68	+30	—.22	+.36	+.57
57	—.16	+.34	+.47	27	—.40	+.19	+.61
54	+.20	—.13	—.25	24	—.63	+.15	+.76
51	+.27	+.01	—.27	21	—.51	+.31	+.81
48	+.13	—.02	—.08	18	—.22	+.23	+.49
45	+.01	—.02	—.18	15	—.07	+.23	+.28
42	+.03	—.56	—.39	12	+.08	+.04	—.04
39	+.27	+.04	—.27	9	+.31	—.28	—.59
36	+.20	—.10	—.28	6	+.11	—.56	—.64
33	—.05	+.25	+.30	3	—.29	—.18	+.06

D_c represents the smoothed discrepancies which actually exist between the declinations of the Standard Catalogues, B_δ and B, and those of Madras 75; $D_c + 2D$ represents the discrepancies which would have existed if the correction for graduation error had been applied in the reductions with the contrary sign. Taking the discrepancies without regard to sign we have:

$$\Sigma\,[D_c] = 8''.02 \quad ; \quad \Sigma\,[D_c + 2D] = 4''.04$$

Therefore the agreement with the Standard Catalogue would have been twice as good if the correction for error of graduation had been applied with the opposite sign. The new system of declinations, B, differs so little in 1875 from B_δ and B, that I have not thought it worth while to repeat this investigation.

The general curve of $\Delta\delta_\delta$ for Madras 75 has been slightly modified in places to conform better with the system, B.

ADOPTED VALUES OF $\Delta\delta_\delta$ FOR MADRAS 75.

δ	$\Delta\delta_\delta$	δ	$\Delta\delta_\delta$	δ	$\Delta\delta_\delta$	δ	$\Delta\delta_\delta$
°	″	°	″	°	″	°	″
+80	+.26	+42	—.45	+ 4	—.34	—34	+0.91
79	+.17	41	—.49	3	.00	35	+ .95
78	+.07	40	—.26	2	+.36	36	+1.09
77	—.01	39	—.27	+ 1	+.46	37	+1.24
76	—.07	38	—.38	0	+.66	38	+1.23
75	—.04	37	—.40	— 1	+.69	39	+1.16
74	—.06	36	—.36	2	+.66	40	+1.11
73	—.14	35	—.17	3	+.48	41	+ .99
72	—.31	34	+.09	4	+.24	42	+ .89
71	—.47	33	+.29	5	—.05	43	+ .80
70	—.64	32	+.42	6	—.24	44	+ .73
69	—.81	31	+.50	7	—.35	45	+ .75
68	—.97	30	+.54	8	—.29	46	+ .72
67	—.93	29	+.56	9	—.19	47	+ .64
66	—.84	28	+.58	10	—.09	48	+ .46
65	—.70	27	+.60	11	+.01	49	+ .30
64	—.45	26	+.60	12	+.07	50	+ .13
63	—.09	25	+.62	13	+.09	51	— .04
62	+.27	24	+.76	14	+.05	52	— .18
61	+.49	23	+.92	15	+.01	53	— .12
60	+.60	22	+.92	16	—.05	54	.00
59	+.63	21	+.85	17	—.11	55	+ .16
58	+.59	20	+.79	18	—.22	56	+ .44
57	+.41	19	+.67	19	—.25	57	+ .82
56	+.16	18	+.55	20	—.02	58	+1.21
55	—.13	17	+.44	21	—.02	59	+1.45
54	—.34	16	+.34	22	—.12	60	+1.60
53	—.45	15	+.33	23	—.15	61	+1.7
52	—.39	14	+.28	24	—.10	62	+1.7
51	—.31	13	+.16	25	+ 10	63	+1.6
50	—.23	12	—.02	26	+.37	64	+1.4
49	—.15	11	—.22	27	+.59	65	+1.2
48	—.10	10	—.41	28	+.71	66	+1.1
47	—.11	9	—.62	29	+.81	67	+1.2
46	—.16	8	—.79	30	+.86	68	+1.4
45	—.22	7	—.78	31	+.89	69	+1.7
44	—.27	6	—.70	32	+.91	—70	+2.0
+43	—.35	+ 5	—.58	—33	+.93		

NOTE 18. As the result of an analysis of GOULD's General Catalogue made some years ago, but not published, combined with the studies for the present Standard Catalogue, we have the following table of corrections, which are intended to serve as the means for reducing the declinations obtained in each position of the circle to the mean of eight positions, 1872 to 1880. Having reduced the declinations of separate years to the mean, the corrections given in Tables IV and V are still to be applied. As in the case of Madras 35, Cape 50, Cape 80 and the Santiago catalogues, the determination of accurate tables of $\Delta\delta_\alpha$ and $\Delta\delta_\delta$ for the separate years of GOULD's Catalogue is a problem for the future, to be solved by the use of a great number of additional secondary standards.

CORDOBA 75. $\Delta\delta_\delta$, TO REDUCE SEPARATE YEARS TO CATALOGUE MEAN.

Year / δ	72	73	74.0 to 75.67	75.67 to 77.0	77	78	79	80	δ
°	″	″	″	″	″	″	″	″	°
0	+.72	—.09	—.10	+.33	—.29	—.54	—.63	+.59	0
— 5	+.78	—.10	—.17	+.26	—.32	—.38	—.62	+.54	— 5
10	+.83	—.11	—.20	+.18	—.38	—.22	—.59	+.48	10
15	+.87	—.14	—.13	+.10	—.44	—.15	—.53	+.42	15
20	+.89	—.17	—.06	+.02	—.49	—.11	—.43	+.34	20
25	+.88	—.19	.00	—.02	—.52	—.12	—.28	+.28	25
30	+.72	—.19	+.01	+.06	—.51	—.14	—.25	+.26	30
35	+.54	—.17	+.01	+.19	—.40	—.18	—.23	+.23	35
40	+.47	—.17	+.01	+.27	—.30	—.24	—.20	+.19	40
45	+.41	—.17	+.06	+.25	—.25	—.32	—.13	+.15	45
50	+.35	—.15	+.13	+.22	—.18	—.38	—.04	+.12	50
55	+.28	—.15	+.16	+.20	—.13	—.41	.00	+.07	55
60	+.23	—.13	+.15	+.18	—.07	—.41	.00	+.05	60
65	+.18	—.11	+.12	+.16	—.04	—.36	.00	+.03	65
70	+.13	—.09	+.09	+.12	.00	—.29	.00	.00	70
75	+.10	—.06	+.07	+.09	.00	—.22	.00	.00	75
80	+.06	—.04	+.04	+.06	.00	—.15	.00	.00	80
85	+.03	—.02	+.02	+.03	.00	—.07	.00	.00	85
—90	.00	.00	.00	.00	.00	.00	.00	.00	—90

$\Delta\alpha_\alpha$, TO REDUCE SEPARATE YEARS TO CATALOGUE MEAN.

R. A.									R. A.
h	″	″	″	″	″	″	″	″	h
0	—.02	+.09	+.14	—.16	—.08	—.19	—.01	+.37	12
1	+.01	+.05	+.20	—.12	—.09	—.24	—.02	+.30	13
2	+.04	+.01	+.24	—.08	—.09	—.28	—.02	+.21	14
3	+.06	—.05	+.26	—.03	—.10	—.30	—.03	+.09	15
4	+.08	—.09	+.27	+.02	—.09	—.30	—.04	—.02	16
5	+.10	—.12	+.26	+.08	—.08	—.27	—.03	—.13	17
6	+.11	—.15	+.23	+.13	—.06	—.23	—.03	—.23	18
7	+.11	—.17	+.19	+.16	—.04	—.17	—.03	—.32	19
8	+.10	—.18	+.12	+.18	—.02	—.11	—.02	—.40	20
9	+.09	—.17	+.06	+.20	+.01	—.03	—.01	—.43	21
10	+.07	—.16	—.01	+.20	+.03	+.05	.00	—.44	22
11	+.05	—.13	—.08	+.19	+.06	+.13	+.01	—.42	23
12	+.02	—.09	—.14	+.16	+.08	+.19	+.01	—.37	24

When the argument for $\Delta\delta_\alpha$ is on the right, employ the opposite sign.

NOTE 19. The weight in right-ascension of BRADLEY is computed on the supposition that the equatorial probable error is $\pm0^s.13 \sec\delta$; and the circumpolar probable error, $\pm0^s.087 \sec\delta$ (corresponding to 60° of declination); and that the minimum possible probable error is nominally $\pm0^s.0087$. But since there are elements of systematic error which it would be difficult to determine (especially as to $\Delta\delta_m$) it is thought best not to assign any weight exceeding 0.4. Between +30° and +60° the weights may be interpolated. For equatorial stars we have:

Obs.	p_α
1	0.025
2	0.05
3	0.07

NOTE 20. In greater detail we have for the weight of PIAZZI's right-ascensions:

Obs.	p_α	Obs.	p_α
5–7	0.03	20–26	0.07
8–9	0.04	27–34	0.08
10–14	0.05	35+	0.1
15–19	0.06		

NOTE 21. For Madras 35, DOWNING's new edition of TAYLOR's Catalogue, it has been our practice to assign half weight to the catalogue positions north of +65° and south of —30°. There appears to be a relatively large number of anomalous residuals.

NOTE 22. The weights for the right-ascensions at Pulkowa, determined by means of the great transit, appear to be larger for high declinations in all the series: and in a greater ratio, of course, for Pulkowa 1845. In Table VI, the assumption is that for the right-ascensions of all stars north of the Pulkowa zenith the weights shall be taken from the line marked "N" for the Pulkowa Catalogues; between +20° and +60° the weights should be interpolated between "S," as for +20°, and "N," as for +60°. The differential weights for Pulkowa 92 appear to be somewhat larger than those assigned in the table, but it seemed best to await more evidence before accepting this larger weight.

NOTE 23. The weights in right-ascension for Cordoba 75 (GOULD's General Catalogue) correspond to a single clamp.

NOTE 24. Dr. AUWERS has pointed out that the weights for the stars used as standard by KÜSTNER are greater than for the others (*A.N.*, Bd. 151, s. 227). No distinction has been made in the present discussion; though it is undoubtedly real. The lower grade of weights has been adopted.

NOTE 25. The weights for Albany in right-ascension are for a single clamp.

NOTE 26. In substantial conformity with Table VII the weight in declination for BRADLEY can be computed from the table, p. 21, of the introduction to the catalogue of AUWERS, through multiplication of the numbers in that table by 0.075; but this factor should be very decidedly decreased for zenith distances greater than 75°. For stars observed in two positions of the quadrant, or both below and above pole, the adopted weight may be the sum of the weights for the separate positions.

NOTE 27. For the Pulkowa vertical circle the weights between declinations, +55° and +65°, where the method of reversal is not followed, should be reduced to two-thirds of the values given in Table VII. Since the probable error of a single observation is very small the diminution of weight with zenith distance is more rapid. The following schedule of factors has been adopted:

δ	Factor
°	
+10	0.9
0	0.6
—10	0.4
—15	0.3
—20	0.1

NOTE 28. The number of observations most frequently occurring in Pulkowa 1855 is four. The weight in declination corresponding to this is 0.8.

NOTE 29. The weight in declination for Cape 65 within 10° or 15° from the pole seems to be decidedly greater than for zenithal stars.

NOTE 30. Weight, 4, has been assigned to declinations taken from the *Leiden Annals*, Volume II, and weight, 5, to the declinations of 57 fundamental stars, *A.N.*, Bd 80, s. 93.

NOTE 31. The weights for the Cordoba declinations, as given in Table VII, refer to the result for a single position of the circle. The weight assignable to the declination deduced from a combination of the several positions is the sum of the separate weights.

NOTE 32. The weights in declination assigned to Albany 1898 are due to one of the four positions: Circle AE, AW, BE, BW. The adopted weight is the sum of these separate weights.

ON THE APPARENT EXTENT OF THE ILLUMINATION SURROUNDING A NEW STAR ON THE HYPOTHESIS THAT IT IS REFLECTED LIGHT,

By SIMON NEWCOMB.

Mr. OTTO C. LUYTIES, of Baltimore, has called my attention to a lack of rigor in the method heretofore adopted of estimating the apparent magnitude of the illuminated nebula surrounding a new star, on the supposition that it shines by reflecting the light of the star. The method as hitherto applied rests on the supposition that the apparent radius of illumination at any moment is determined by a tangent drawn from the earth to the surface of the sphere of illumination. The fault consists in leaving out of consideration the fact that the light sent out by the star in directions near that of the earth will reach us at an earlier moment than it will if sent out at right-angles to that direction. The rigorous method of treatment is this: Let S be the position of the star, E that of the earth, and P that of a particle in the neighborhood of the star. Put

ρ, the distance SP of the nebulous particle from the star.

θ, the angle ESP made by the direction of the ray with that of the earth.

τ, the interval between the time at which outburst of star is seen from the earth, and time of observation.

v, the speed of light.

π, the parallax of the star.

σ, the angular radius of illumination, as seen from earth.

We shall proceed on the supposition that the outburst was a momentary one, which immediately subsided. Then, at the time τ after the outburst is seen, the reflected light visible from the earth will be that from all the particles which fulfil the condition:

$$EP + PS - ES = v\tau \tag{1}$$

Owing to the minuteness of the ratio PS to EP and ES we may treat the lines EP and ES as parallel, so that

$$ES - EP = \rho \cos\theta$$

The equation (1) therefore gives us $\rho(1-\cos\theta) = v\tau$,

$$\text{or} \qquad \rho = \frac{v\tau}{2\sin^2\frac{1}{2}\theta} \tag{2}$$

which is the equation of the required surface.

The apparent radius of the surface, as projected upon the sphere, will be

$$\sigma = \rho \sin\theta \sin\pi = \frac{v\tau \sin\pi}{\tan\frac{1}{2}\theta}$$

or, if we express σ and π in seconds of arc,

$$\sigma = \frac{v\tau\pi}{\tan\frac{1}{2}\theta}$$

This equation implies that we take the earth's mean distance from the sun as the unit of length. Taking also the day as the unit of time we shall have $v = 174$, and the radius of the apparent illumination will become

$$\sigma = \frac{174\,\tau\pi}{\tan\frac{1}{2}\theta} \tag{3}$$

This value increases indefinitely with smaller values of θ. There is, therefore, no well-defined limit to the apparent radius of illumination. The practical limit will depend on the distance to which the nebula extends in the direction of the earth, and upon its density. If we put a for this distance, or for the maximum value of ρ, the minimum value of θ will be given by the equation (2) in the form

$$2\sin^2\tfrac{1}{2}\theta = \frac{v\tau}{a} \tag{4}$$

By substituting this expression in (3) we have for the limitized value of σ

$$\theta_1 = \sqrt{2av\tau}\ .\ \pi \quad \cos\tfrac{1}{2}\theta \tag{5}$$

It will be seen that the visible extent of the illumination at any moment is in form a function of two completely unknown quantities, a and π. In addition to this we have another unknown element in the distance at which the light would cease to affect the photographic plate, owing to the faintness of the reflection. I conceive that it would be unprofitable to make hypotheses as to the magnitudes of these uncertain quantities.

The tacit assumption on which KAPTEYN based his estimate is that of $\theta = 90°$. Then, σ being determined from the Lick photographs, the value of π was derived. His result was 0″.02. I regard so large a parallax as this as altogether without the bounds of reasonable probability, believing that π is more likely to be less than one-tenth of this quantity than greater. Assuming it as great as one-tenth, the illumination would have had to expand with ten times the speed of light in order to make its apparent speed that seen on the Lick photographs. The enigma involved in this conclusion seems to be solved by the considerations just set forth. The principal difficulty which is still left is the faintness of the reflection at the great distances from the star at which the reflecting particles must have been found. The more likely hypothesis seems to be that we have to do with corpuscles thrown out from the star with a speed approximating to that of light.

CONTENTS

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NOMENCLATURE OF NEWLY DISCOVERED VARIABLE STARS,

Provis. Notation *A.N.*	Name	Position of 1900.0 R.A.	Position of 1900.0 Decl.	Prec. 1900 R.A.	Prec. 1900 Decl.	Chart-Place R.A.	Chart-Place Decl.	Magnitude Max.	Magnitude Min.	
		h m s	° '	s	'	h m s	° '			
11.1903	*RU Andromedae*	1 32 47	+38 9.5	+3.49	+0.31	1 30 11	+37 55.6	9	13	*ph*
15.1903	*Z Cephei*	2 12 48	+81 13	+7.81	+0.28	2 7 6	+81 0	9.10	<13	*ph*
56.1903	*RR Cephei*	2 29 23	+80 42.3	+8.03	+0.27	2 24 15	+80 30.2	9	<13	*ph*
14.1902	*Z Persei*	2 33 40	+41 46.1	+3.81	+0.26	2 30 50	+41 34.3	9	12	*v*
22.1903	*X Camelopardalis*	4 32 36	+74 56	+7.68	+0.12	4 26 48	+74 50	9	13	*ph*
5.1903	*RS Tauri*	5 46 3	+15 51.3	+3.45	+0.02	5 43 28	+15 50.3	8.9	10 11	*v*
1.1903	*Z Aurigae*	5 53 39	+53 18.0	+4.86	+0.01	5 50 3	+53 16.9	9	11	*v*
20.1903	*W Camelopardalis*	6 12 0	+75 32	+8.25	−0.02	6 5 48	+75 32	10.11	12	*ph*
14.1903	*RS Geminorum*	6 55 14	+30 39.8	+3.84	−0.08	6 52 21	+30 43.3	9.10	11.12	*ph*
9.1903	*Z Geminorum*	7 1 36	+22 41.0	+3.61	−0.09	6 58 53	+22 44.9	9.10	<12	*v*
16.1903	*RR Monocerotis*	7 12 27	+ 1 16.6	+3.10	−0.10	7 10 7	+ 1 21.2	9	<13	*ph*
13.1903	*RR Geminorum*	7 15 11	+31 4.2	+3.83	−0.10	7 12 18	+31 9.0	10	11.12	*ph*
21.1903	*Y Camelopardalis*	7 27 39	+76 16.9	+8.15	−0.12	7 21 30	+76 22.3	9.10	<11.12	*ph*
4.1902	*Y Geminorum*	7 35 16	+20 39.6	+3.53	−0.13	7 32 37	+20 45.3	8.9	–	*ph*
2.1903	*Y Draconis*	9 31 5	+78 18.2	+6.98	−0.27	9 25 47	+78 30.1	9	13	*ph*
3.1903	*W Ursae Maj.*	9 36 44	+56 24.6	+4.25	−0.27	9 33 32	+56 36.7	8	9	*v*
4.1903	*Z Draconis*	11 39 49	+72 49.0	+3.45	−0.33	11 37 12	+73 4.0	9.10	12.13	*ph*
57.1903	*T Ursae min.*	13 32 38	+73 56.4	+1.25	−0.31	13 31 42	+74 10.2	9	<13	*ph*
29.1903	*ST Herculis*	15 47 47	+48 47.1	+1.79	−0.18	15 46 27	+48 55.4	7.8	8.9	*v*
18.1902	*W Coronae*	16 11 50	+38 2.7	+2.14	−0.15	16 10 14	+38 9.6	7.8	13	*v*
31.1903	*SU Herculis*	17 44 42	+22 34	+2.52	−0.02	17 42 48	+22 35	10	<12	*ph*
76.1901	*RT Ophiuchi*	17 51 51	+11 10.9	+2.81	−0.01	17 49 45	+11 11.5	9	<10	*v*
19.1903	*RZ Lyrae*	18 39 54	+32 41.7	+2.23	+0.06	18 38 14	+32 39.1	10	11.12	*ph*
17.1903	*RY Lyrae*	18 41 15	+34 34.0	+2.17	+0.06	18 39 38	+34 31.4	10	12	*ph*
17.1902	*RW Lyrae*	18 42 7	+43 31.9	+1.82	+0.06	18 40 45	+43 29.2	9	<12	*ph*
10.1903	*RX Lyrae*	18 50 27	+32 42.3	+2.23	+0.07	18 48 46	+32 39.0	11	<15	*ph*
55.1903	*VW Cygni*	20 11 21	+34 11.8	+2.31	+0.18	20 9 37	+34 3.7	9.10	11.12	*ph*
21.1902	*V Sagittae*	20 15 46	+20 47.3	+2.65	+0.19	20 13 47	+20 39.0	9.10	13	*ph*
16.1902	*Z Delphini*	20 28 3	+17 6.2	+2.74	+0.20	20 26 0	+16 57.2	9	<11	*ph*
15.1902	*Y Delphini*	20 36 52	+11 30.9	+2.86	+0.21	20 34 43	+11 21.5	9.10	<13	*v*
58.1903	*VX Cygni*	20 53 34	+39 47.5	+2.26	+0.23	20 51 52	+39 37.2	9	9.10	*ph*
20.1902	*VV Cygni*	21 2 20	+45 22.6	+2.12	+0.24	21 0 45	+45 11.9	11	<12	*ph*
19.1902	*RT Pegasi*	21 59 49	+34 38.2	+2.61	+0.29	21 57 51	+34 25.3	9.10	13.14	*v*
. .	*RS Andromedae*	23 50 19	+48 4.9	+3.01	+0.33	23 48 4	+47 49.5	7.8	8.9	*ph*
12.1903	*Nova Geminorum*	6 37 49	+30 2.6	+3.83	−0.05	6 34 56	+30 5.0	5	–	–

The Committee for the A.G. Catalogue of Variable Stars:
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ON THE LIGHT-VARIATIONS OF 320 *U CEPHEI*,

By PAUL S. YENDELL.

The announcement of the variability of *U Cephei*, by CERASKI, in 1880 (*A.N.*, Vol. 97, s. 319), attracted much attention among the observers of variable stars, it being the first of its type, of which at that time but five were known, that had been found since WINNECKE's discovery of *U Coronae* in 1869.

Many observers at once turned their attention to the star. GLASENAPP, WILSING, KNOTT, SCHMIDT, PICKERING, the BAXENDELLS, father and son, published numerous observations during the early years.

Although the main peculiarity of its light-curve was at once noticed, the work of these observers was for the most part directed to the investigation of the star's elements of variation, rather than to the course and character of its light-changes. KNOTT's numerous observations, extending from the time of the star's discovery until 1889, are mostly confined to the three hours on either side of the minimum, and very few of them were made during the time of the star's normal brightness.

The earliest mean light-curve which is known to me is by PICKERING, and was formed from about three hundred photometric observations made at the Harvard Observatory. It was published in 1881, in the Proceedings of the American Academy of Arts and Sciences, Vol. XVI. A discussion of the star variations, including a mean light-curve, was published by WILSING in 1884 (*A.N.*, 2596). In 1889, CHANDLER (*A.J.*, Vol. IX, p. 49), published a discussion of the star's elements of variation, with "Spring" and "Autumn" light-curves, showing the course of the light-changes, with a comparison of the same with the above mentioned curves of WILSING and PICKERING. Since the date of CHANDLER's paper, the only mean light-curve of *U Cephei* that has come to my knowledge is one by BOHLIN, of Upsala, from observations made in 1896, and published in the *A.N.*, 3762.

I began work on *U Cephei* in 1888, and since that time no year has passed without my securing more or less observations of it, although there have been several years during which I have observed no minimum.

Early in the nineties I began to observe the star with a view to the accumulation of material for a mean light-curve of a more or less definitive character, setting my minimum number at a thousand observations, and keeping in view the purpose of securing as nearly as possible as many of the Spring as of the Autumn curve. The latter aim, however, has not been accomplished, mostly from conditions dependent on the weather, and when in 1902 the desired number of observations had been secured, the Autumn observations were so far in the majority that when the list was closed only about one-fourth of the whole number of observations represented the Spring curve.

In the Spring of 1902 I began to collect material from other observers, with the idea of forming a general mean curve from as many different series of observations as I could get together. I already had the published work of KNOTT. BAXENDELL kindly sent me in manuscript his own and his father's observations. CHANDLER, PLASSMAN, SPERRA and SCHWAB all transmitted theirs, so that, including my own material, I had nearly three thousand observations available for my purpose.

Upon further consideration of this plan, the probability suggested itself that the personal differences in the work of the various observers would be likely so far to affect the general result as to detract largely from its value; and after carefully weighing the matter, I abandoned the idea, and each observer's work has been treated by itself. Each series, including my own, was divided into convenient groups in the order of time, and separate mean curves made from the various groups, giving a sequence of curves, representing as many mean epochs, from 1880 till 1903. It was thought that in this way, any progressive change in the course of the light-variations might be brought out.

At the outset, I intended to make use of the Harvard Photometry scale of magnitudes, as being the only one available for stars of all the magnitudes included in the light-range of the variable. Magnitudes for the comparison-stars used were kindly furnished for the purpose by Prof.

Pickering. But the values for the stars b and d were discordant with their relative values, according to both my own and Knott's light-scales, and after reducing a number of observations, I found that the form of the light-curve near the minimum was seriously distorted by this discrepancy. Light-scales were then formed from the other series of observations, and with one exception the same discordance was found to exist in each case. The use of the Harvard magnitudes was thereupon abandoned.

It so happened, however, that very shortly after the reductions had been discontinued for this reason, and when I had almost decided to make use of the provisional magnitude scale formed from my own step-scale, which I had previously used, Dr. Müller, of the Potsdam Astrophysical Observatory, most kindly offered to make photometric measures of these comparison-stars for the purpose of this work, which offer I very thankfully accepted. His results, coming to hand in August, 1902, proved to be accordant in relative values with all the light-scales excepting the one already alluded to, so that simple relations between them were readily established, and the work was resumed. The subjoined table includes the comparison-stars used in all the series under consideration. The first column gives the letters by which they are designated; the second their DM. numbers; the third, headed HP., the Harvard Photometry magnitudes; and the fourth, headed P, the Potsdam measurements.

	DM.	HP	P
k	81 13	6.40	6.58
e	18	7.54	7.43
f	30	7.89	8.04
p	80 34	7.72	7.76
m	81 34	8.56	8.52
g	81 27	8.46	8.53
h	29	8.58	8.57
a	80 21	8.82	8.93
b	22	9.42	9.17
d	81 22	9.00	9.29
c	80 83	–	9.44

The elements on which the present curves were based were suggested by Chandler. They satisfy the whole of the observations at my disposal, at least as well as any yet proposed. The elements of 1897 had ceased to represent the star's variations; the departure from them began about 1894, and has gone on increasing, until in the autumn of 1902 the minima were nearly three hours late by them. (See also Hartwig's VJS Ephemerides, for 1902, p. 269, and 1903, p. 285).

It was therefore necessary to find elements which would fairly represent the observed dates, and the following suggested as above, satisfy the list of minima hereinafter to be given with an average departure of about eight minutes, the algebraic mean of the O—C being $-0^{m}.48$, corresponding to Epoch 1533.

Chandler's Elements.

1880 June $23^{d}\ 7^{h}\ 43^{m}.5$ G.M.T. $+2^{d}\ 11^{h}\ 49^{m}\ 44^{s}.7$ E

In forming the curves, the first question which presented itself, was the division of the observations for the Spring and Autumn curves, as done by Wilsing and Chandler, and which is necessary in any discussion of observations of this star made by the Argelander method.

As the difference between the two curves is undoubtedly the result of subjective causes, due to the varying presentation of the group formed by the star and its comparison-stars at different hour-angles, it appears that the dividing line should be drawn at the angle at which these disturbances disappear. The principal comparison-stars are assembled in two definite groups, the brighter ones north preceding the star, and the fainter ones south following it, and a line drawn through the approximate centers of these groups, and passing very near the variable, becomes horizontal and parallel to the normal position of the axis of the eyes at hour-angles $2^{h}\ 24^{m}$ west and $9^{h}\ 36^{m}$ east. The line joining these hour-angles was accordingly taken as the critical line, and all observations taken at angles east of it were used in forming the East or Autumn curve, and all taken west of it for the West or Spring curve.

The observations of each observer having been divided, as mentioned above, into convenient groups in the order of time, mean curves were formed from the several groups, the normals for these curves being generally formed from five observations each, excepting in the more sparsely observed times at the beginning and end of the period of change; in my own group for 1898–1902, the observations being numerous, the normals were formed from ten observations each.

The observations of the late Mr. Knott were published in book form in 1899, under the editing of Prof. Turner of Oxford. I am indebted to the kindness of Mrs. Knott for a copy of the volume. Knott's observations of *U Cephei* occupy thirty-three pages of the book, twenty-six of which are filled by the observations themselves, which extend from 1880 to 1897, with a few observations on a single date in 1889. There are in all about 850 observations, of which about 660 are available for my purpose, being made by the Argelander method, so that they can be reduced homogeneously with the work of the other observers. The remaining observations are noted as "gauged," the gauging having been done as a check, by the method of limiting apertures. The observations were grouped as follows:

	Spring	Autumn
1880–81	116	136
1882–83	208	61
1884–87	367	20 (no curve)

The observations in 1889 were only ten in number, and have been grouped with those of 1884–87.

The readings from these curves are given in Table I.

The observations of the BAXENDELLS, senior and junior, were kindly forwarded to me in manuscript by the latter. The resulting magnitudes only are given, but as the comparison-stars employed were the same as those used by KNOTT, and as the latter gives BAXENDELL's estimated magnitudes for the purpose of comparison, they were easily reduced to the Potsdam scale by graphic process.

The observations of BAXENDELL, Sr., are 174 in number, extending from 1880 to 1887, and are distributed as follows: Autumn 89, Spring 85.

The readings from the mean curves are in Table II.

The younger BAXENDELL's observations cover a space from 1884 to 1887, and number 117, all except 25 belonging to the Spring curve, The Autumn curve being a single curve, is omitted.

The readings are in Table II.

SPERRA's observations were transmitted in manuscript. There are about 180 of them, of which about one-third are of the star at its normal light, the rest divided between the Spring and Autumn curves. His mean value for the normal light is $6^{M}.94$.

The readings from the mean curves are contained in Table III.

CHANDLER's observations, to the number of 215, were handed to me also in manuscript. They were made in 1887 and 1888. There are 56 which belong to the Spring curve, and 153 to the Autumn; the remainder are at the normal light, and give a mean value of $7^{M}.21$.

The readings are given in Table IV.

The observations of PLASSMANN have been sent to me partly in pamphlet form, and partly in manuscript, by himself. They are about three hundred in number, and form two groups, one in 1894, and the other in 1901–02. Those of 1894 all belong to the Spring curve, while those of 1901–02 are divided between the two.

The readings are given in Table V.

SCHWAB's observations were also forwarded in manuscript. There are in all about 250 of them, of which about 50 are of the star at its normal light, and the rest divided pretty evenly between the Spring and Autumn curves, of neither of which, however, are both branches represented. The observations cover a period from December, 1900, to November, 1902. His mean value for the normal light is $6^{M}.96$.

The readings are given in Table VI.

My own observations of the star were begun 1888 May 16, and when on 1902 Oct. 21, the series for that year came to a close, I found the total number since the beginning to be 1175. Twenty-three of these, depending on comparisons with the stars *m* and *p*, whose values on my light-scale were discordant with the photometric magnitudes, were rejected. Of the 1152 remaining available, 866 belonged to the East or Autumn curve, and 286 to the West or Spring curve. Of these, 103 were observations of the star at its normal brightness, made for the purpose of ascertaining whether the light at this phase were constant. Eighty-five were in East hour-angle, belonging to the Autumn curve, and 18 were of the West or Spring group; it was found, however, that there was no sensible difference between the normals formed from the two groups.

The comparison-stars and light-scale used were as follows: the first column gives the notation used; the second, headed DM., their *Durchmusterung* numbers; the third, P, their Potsdam magnitudes; and the fourth, Lt., my own step-scale, formed from all my observations to 1900 July 4, which was retained in the reductions, as in my judgement the later observations would not have sensibly changed it.

	DM	P	Lt.
k	81 13	6.58	31.9
e	18	7.43	24.4
f	30	8.04	19.3
g	27	8.53	14.8
h	29	8.57	14.0
a	80 21	8.93	9.2
b	22	9.17	4.9
d	81 22	9.29	1.9
c	80 23	9.44	0.0

By this light-scale, the value of a step is $0^{M}.114$ from *k* to *g*, and $0^{M}.062$ from *g* to *c*, being nearly twice as great among the brighter stars as among the fainter ones, but pretty constant in each group.

The observations were divided into four groups in the order of time, and each group subdivided into the West (Spring) and the East (Autumn) groups, as follows:

	Spring	Autumn
1888–1890	15 obs	98 obs.
1891–1894	177	339
1895–1898	30	189
1899–1902	122	400

The observations of the Spring curve for the groups 1888–1890 and 1895–1898 were so few, that no use was made of them excepting in the general mean curves.

The observations were assembled in groups of five, excepting in the Autumn group of 1899–1902, where, being numerous, they were grouped in tens.

The readings for these curves are given in Table VII.

In forming the general mean curve, 781 observations were found available for the Autumn curve, and for the Spring, 268. At the normal light, there were, as mentioned above, 85 observations in East hour-angle, and 18 in West, in all 103; 23 normals were formed from these.

In forming the normals for the Autumn curve as far as practicable 20 observations were used for each normal, so as to give nearly equal weights, but at the beginning and end of the period of change, the observations were less numerous, and the normals are therefore formed from smaller groups. The corresponding normals in the Spring table were formed from 10 observations each.

Table VIII contains these normals. The column $T-t$ gives the interval from the computed time of minimum; M the magnitude; Obs. the number of observations which make up each normal; and v the departure of each normal from the curve as drawn.

The last 15 normals in the Autumn table, and the last 8 in the Spring one fall in the time of the star's normal brightness. They give no indication of any real fluctuation in brightness during that part of the period, the average departure from a mean of $7^{M}.09$ being $0^{M}.03$, and the probable error of a single normal $\pm 0^{M}.02$, the residuals being pretty impartially distributed over the whole forty-eight hours of this portion of the star's period.

The mean minimum light shown is $9^{M}.18$ for the Autumn curve, and $9^{M}.06$ for the Spring curve. Neither curve shows any correction to the time of minimum.

The readings from the general mean curves are given in Table IX.

The duration of the light-changes shown by these curves is longer than it has hitherto been stated. It is from $-5^{h}\,40^{m}$ to $+5^{h}\,40^{m}$, occupying therefore 11 hours and 20 minutes. These limits are well-defined in the Autumn curve, but the beginning of the Spring curve is less satisfactory, being much distorted to about $-2^{h}\,30^{m}$. With this exception, the difference between the two curves are: the general comparative flatness of the Spring curve, its brighter minimum, and its greater asymmetry, as compared with the Autumn curve.

The latter is so far the more fully observed and better made out curve, made from observations taken at far the more favorable season of the year, and the precautions taken to avoid subjective errors have been so unremitting, that it seems to me to be probably a very good approximation to the star's real light-curve. Its departures from actual symmetry are very slight up to $7^{M}.5$, and at $9^{M}.00$, $8^{M}.65$, and $7^{M}.73$ they disappear.

Assuming the curve to be symmetrized by averaging the values of each pair of readings, the probable error of one of these readings is $\pm 0^{M}.025$, while their mean departure from the symmetrized curve is $0^{M}.027$.

Assuming again that the minimum light is constant from -1^{h} to $+1^{h}$, the probable error of one of the nine normals is $\pm 0^{M}.018$, while their mean departure from their mean value is, as in the other part of the curve examined, $0^{M}.027$.

The impression remaining on my mind after fifteen years' constant and careful study of the star, is that the course of the light-changes is really that which would result from an annular eclipse; a symmetrical curve, with inflection increasing with its proximity to the minimum, and an interval of constant minimum light, central at the moment of minimum.

In these series of observations there is no evidence of any progressive change in the star's light-curve. My own series is the only one of those examined in which there is any approach to sufficient observation of the beginning and end of the period occupied by the star's light-changes to furnish any evidence as to possible change in its length, and in this series these points are not made out with enough precision to give any valuable indications in either direction. Examination and comparison of the times at which $8^{M}.00$, at which point the more rapid change is well under way, is passed on the decrease and increase show very considerable differences in the curves of different observers, but no evidence of progressive change.

The difference between the Spring and Autumn values of the observed minimum light varies all the way from almost nothing to fully four-tenths of a magnitude. And whereas in all but one of the other series the Spring curves show the fainter minima, in my own and SCHWAB's the reverse is the case, pointing strongly to the subjective nature of the difference. This curious discrepancy is possibly due to the use of different comparison-stars at the minimum phase, and perhaps also to the fact that, especially of later years, I have been very solicitous to eliminate the hour-angle disturbance from my observations as far as possible. To this practice also I ascribe the fact that in my last four years' group the difference in the two curves at the minimum is very slight, though the Spring curve is much the flatter on both sides of the minimum. In the earliest of my own curves the similarity to SCHOENFELD's curve of *S Cancri*, in the rise from an hour before the minimum point to an hour after is very marked. This entirely disappears in the next group (1891–1894), and does not reappear later. I suspect it to be due to the influence of a knowledge of CHANDLER's 1889 curve to which reference has already been made, which has this peculiarity strongly marked, and whose authority could hardly fail to influence an observer new to the work, when in doubt between two estimates. This may or may not have been the case, but with the gain in experience and confidence the phenomenon vanishes from my results.

A comparison of the minimum brightness found by the various observers gives some apparent sign of change. Of these series, KNOTT's indicates a slow decrease of the minimum light; the others, to SPERRA's in 1894–1897, show a small increase; PLASSMANN's and SCHWAB's are discordant; my own from 1891 to 1902 show an increase of $0^{M}.20$ in the Autumn curves, which is offset by the constancy of the Spring ones, the mean of both being about $0^{M}.09$. The means of KNOTT's and my own series indicate an increase of about a tenth of a magnitude, and this is also the general drift of all the groups.

This change, if real, is very slow, and can only be verified by long observation.

Table X shows the ninety-six minima deduced from the several series by the method of equal brightnesses. In the last column, headed Obs., the letters signify: K, KNOTT; B, BAXENDELL, SR.; b, BAXENDELL, JR.; C, CHANDLER; Y, YENDELL; S, SPERRA; P, PLASSMANN; and Sch., SCHWAB.

The weights in the table are on a scale of five.

Table I. Knott.

Time from Minimum	1880–1881 Spring	1880–1881 Aut'n	1882–1883 Spring	1882–1883 Aut'n	1884–1887 Spring
h m	M	M	M	M	M
−3 40	. .	. .	7.44	. .	7.59
20	. .	. .	7.56	. .	7.69
3 0	7.55	. .	7.68	. .	7.79
2 40	7.70	. .	7.81	. .	7.92
20	7.90	8.10	7.95	. .	8.08
2 0	8.13	8.37	8.09	8.26	8.28
1 40	8.40	8.67	8.34	8.50	8.54
20	8.69	8.86	8.74	8.74	8.98
1 0	9.08	8.95	9.29	8.99	9.20
0 40	9.31	9.00	9.35	9.06	9.28
−0 20	9.33	9.03	9.34	9.08	9.31
0 0	9.33	9.06	9.34	9.08	9.34
+0 20	9.32	9.04	9.34	9.08	9.35
40	9.30	9.00	9.34	9.05	9.35
1 0	9.30	8.95	9.33	8.95	9.33
20	9.20	8.86	9.31	8.73	9.26
40	8.65	8.62	8.70	8.50	8.78
2 0	8.33	8.29	8.20	8.28	8.14
20	8.10	8.04	. .	8.09	7.89
+2 40	7.92	7.90	. .	7.94	. .

Table II.

Time from Minimum	Baxendell, Sr. 1880–1887 Spring	Baxendell, Sr. 1880–1887 Aut'n	Baxendell, Jr. 1884–1887 Spring
h m	M	M	M
−2 40	. .	7.87	7.97
20	. .	8.09	8.13
2 0	8.28	8.31	8.31
1 40	8.50	8.55	8.53
20	8.80	8.82	8.90
1 0	9.13	9.02	9.19
0 40	9.16	9.12	9.20
20	9.18	9.14	9.20
−0 0	9.19	9.12	9.19
+0 20	9.16	9.10	9.18
40	9.14	9.07	9.17
1 0	9.11	9.04	9.13
20	8.94	8.99	9.00
40	8.87	8.63	8.58
2 0	. .	8.35	. .
20	. .	8.16	. .
+2 40	. .	8.00	. .

Table III. Sperra.

Time from Minimum	1894–1897 Spring	1894–1897 Aut'n	Time from Minimum	1894–1897 Spring	1894–1897 Aut'n
h m	M	M	h m	M	M
−4 40	. .	7.14	0 0	9.10	9.06
20	. .	7.26	+0 20	9.08	9.03
4 0	. .	7.38	40	9.03	9.00
3 40	. .	7.50	1 0	8.90	8.95
20	. .	7.63	20	8.62	8.87
3 0	. .	7.78	40	8.35	8.76
2 40	7.91	7.96	2 0	8.10	8.61
20	8.24	8.18	20	7.89	8.47
2 0	8.55	8.50	40	7.71	8.33
1 40	8.83	8.89	3 0	7.54	8.14
20	8.99	9.08	20	7.36	7.94
1 0	9.07	9.10	40	7.19	7.74
0 40	9.10	9.10	4 0	. .	7.53
−0 20	9.11	9.08	+4 20	. .	7.30

Table III. — (Cont.)

Normal Light.

h m	M	h m	M
+12 8	6.92	+41 3	6.92
12 28	6.86	42 16	6.87
18 31	6.95	43 31	6.90
19 11	6.89	44 20	6.89
27 7	6.88	47 26	6.90
27 18	6.92	49 32	6.96
33 46	6.96	51 22	6.99
39 7	6.87	+54 12	7.12
+40 8	6.89	. .	. .

Table IV. Chandler.

Time from Minimum	1887–1888 Spring	1887–1888 Aut'n	Time from Minimum	1887–1888 Spring	1887–1888 Aut'n
h m	M	M	h m	M	M
−4 20	. .	7.54	−0 20	9.09	9.03
4 0	. .	7.57	0 0	9.07	9.02
3 40	7.64	7.61	+0 20	9.06	9.01
20	7.73	7.67	40	9.05	9.00
3 0	7.84	7.77	1 0	9.01	8.96
2 40	7.97	7.91	1 20	8.89	8.70
20	8.15	8.06	40	8.63	8.42
2 0	8.39	8.23	2 0	8.38	8.20
1 40	8.74	8.41	2 20	8.15	8.02
20	8.98	8.75	40	7.96	7.84
1 0	9.07	8 99	3 0	7.81	. .
−0 40	9.09	9.04	+3 20	7.69	. .

Table V. Plassmann.

Time from Minimum	1894 Spring	1901–1902 Spring	1901–1902 Aut'n
h m	M	M	M
−5 40	. .	. .	7.09
20	. .	. .	7.09
5 0	. .	. .	7.10
4 40	. .	. .	7.11
20	. .	. .	7.14
4 0	. .	7.17	7.18
3 40	. .	7.20	7.23
20	. .	7.24	7.30
3 0	7.37	7.29	7.44
2 40	7.66	7.40	7.72
20	7.95	7.66	8.03
2 0	8.26	8.16	8.31
1 40	8.59	8.61	8.56
20	8.94	8.87	8.80
1 0	9.03	9.02	8.91
0 40	9.03	9.00	8.91
−0 20	8.97	9.00	8.90
0 0	8.92	9.00	8.90
+0 20	8.93	9.00	8.90
40	8.98	9.00	8.90
1 0	8.96	8.97	8.86
1 20	8.80	8.73	8.53
40	8.50	8.34	. .
2 0	8.15	7.96	. .
+2 20	7.86	7.58	. .

Table VI. Schwab.

Time from Minimum	1900–1902 Spring	1900–1902 Aut'n	Time from Minimum	1900–1902 Spring	1900–1902 Aut'n	Time from Minimum	Normal Light	Time from Minimum	Normal Light
h m	M	M	h m	M	M	h m	M	h m	M
−5 0	. .	7.02	−0 40	9.12	9.20	+ 7 10	7.11	+37 38	6.91
4 40	. .	7.04	−0 20	9.15	9.20	8 31	7.01	40 48	6.86
20	. .	7.07	0 0	9.17	9.20	9 5	6.95	43 43	6.96
4 0	. .	7.12	+0 20	9.18	9.20	11 44	6.92	46 7	6.99
3 40	. .	7.20	40	9.16	. .	13 24	6.91	48 33	6.88
20	. .	7.33	1 0	9.11	. .	19 26	6.93	49 26	7.01
3 0	. .	7.50	1 20	8.90	. .	22 30	6.94	+51 10	6.92
2 40	. .	7.80	40	8.50	. .	22 57	6.98	. .	. .
20	. .	8.13	2 0	8.10	. .	23 11	6.94	. .	. .
2 0	. .	8.50	2 20	7.78	. .	24 20	6.96	. .	. .
1 40	. .	8.72	40	7.52	. .	25 12	6.93	. .	. .
20	8.92	9.03	3 0	7.34	. .	34 49	6.94	. .	. .
−1 0	9.06	9.18	+3 20	7.23	. .	+35 16	6.92	. .	. .

Table VII. Readings from Mean Curves.

		1888–1890		1891–1894				1895–1898		1899–1902			
		Autumn		Spring		Autumn		Autumn		Spring		Autumn	
	$T-t$	Before	After	Before	After	Before	After	Before	After	Before	After	Before	After
	h m	M	M	M	M	M	M	M	M	M	M	M	M
208	5 0	. .	. .	. .	. .	. .	. .	. .	. .	. .	7.17	7.16	7.18
194	4 40	. .	. .	. .	. .	. .	. .	7.15	7.12	. .	7.24	7.22	7.26
181	4 20	. .	. .	. .	. .	. .	. .	7.23	7.18	7.76	7.31	7.27	7.34
167	4 0	. .	. .	. .	. .	7.02	. .	7.30	7.24	7.86	7.37	7.34	7.42
153	3 40	. .	. .	. .	. .	7.10	. .	7.37	7.32	7.97	7.48	7.43	7.49
139	3 20	. .	. .	. .	. .	7.21	. .	7.44	7.40	8.09	7.60	7.53	7.58
125	3 0	. .	. .	. .	. .	7.34	. .	7.52	7.58	8.21	7.73	7.64	7.67
111	2 40	7.71	. .	. .	. .	7.60	7.66	7.67	7.68	8.35	7.88	7.77	7.75
097	2 20	7.84	. .	8.06	8.21	7.93	7.72	7.98	7.89	8.48	8.05	7.95	7.86
083	2 0	7.98	8.19	8.47	8.45	8.26	7.93	8.35	7.97	8.64	8.30	8.18	8.03
69	1 40	8.16	8.62	8.72	8.56	8.57	8.57	8.66	8.50	8.78	8.53	8.48	8.29
056	1 20	8.52	9.00	8.91	8.73	8.88	9.08	8.91	8.89	8.92	8.77	8.86	8.68
042	1 0	8.89	9.11	9.02	8.86	9.26	9.26	9.08	9.07	9.00	8.94	9.05	9.02
028	0 40	9.10	9.17	9.06	8.96	9.30	9.29	9.15	9.13	9.05	9.04	9.10	9.06
014	0 20	9.24	9.21	9.07	9.02	9.30	9.29	9.17	9.16	9.07	9.08	9.10	9.08
[illegible]	0 0	9.24	. . .	9.05	. .	9.29	. .	9.17	. .	9.08	. .	9.09	. .

Table VIII. Normals for Mean Light-Curve, 1888–1902.

East (Autumn)				East (Autumn)				East (Autumn)				West (Spring)			
$T-t$	Mag.	Obs.	v	$T-t$	Mag.	Obs.	v	$T-t$	Mag.	Obs.	v	$T-t$	Mag.	Obs.	v
h m				h m				h m				h m			
−4 55.8	7.14	10	+0.01	+ 0 8.1	9.16	20	−0.01	+21 35.6	7.09	9	0.00	−0 57.0	9.03	10	+0.02
4 31.4	7.38	10	+0.20	0 15.9	9.18	20	+0.01	25 11.7	7.12	10	+0.03	0 49.4	8.99	10	−0.04
4 13.0	7.12	8	−0.11	0 34.6	9.18	20	+0.03	27 44.5	7.14	6	+0.05	0 43.7	9.06	10	+0.02
3 44.8	7.37	15	+0.05	0 48.4	9.10	20	−0.02	29 26.1	7.10	6	+0.01	0 26.4	9.07	10	+0.02
3 34.7	7.41	14	+0.04	0 56.6	9.15	20	+0.03	32 34.8	7.06	4	−0.03	0 9.8	9.09	10	+0.03
3 17.3	7.40	15	−0.05	1 6.0	9.05	20	+0.02	42 18.3	7.07	6	−0.02	−0 1.6	9.05	10	−0.01
3 4.1	7.46	20	−0.08	1 13.8	8.96	20	+0.02	46 6.1	7.10	4	+0.01	+0 16.1	9.03	10	−0.01
2 52.0	7.67	20	+0.04	1 20.9	8.75	20	−0.10	49 17.8	7.08	11	−0.01	0 36.1	9.04	10	+0.02
2 40.1	7.82	20	+0.09	1 26.3	8.83	20	+0.10	52 27	7.03	3	−0.06	1 11.8	8.81	10	−0.10
2 22.8	7.86	20	−0.06	1 33.9	8.56	20	−0.02	+54 11.3	7.05	3	−0.04	1 28.0	8.65	10	−0.05
2 10.5	8.12	20	+0.06	1 39.7	8.39	20	−0.01	West (Spring)				1 49.7	8.48	10	+0.05
1 59.8	8.11	20	−0.08	1 47.9	8.33	20	+0.09	− 4 52.1	7.62	10	0.00	2 8.4	8.29	10	+0.07
1 51.6	8.43	20	+0.12	1 56.6	8.08	20	−0.02	3 45.3	7.77	10	+0.01	2 27.7	7.96	10	−0.05
1 44.2	8.41	20	0.00	2 4.6	7.97	20	−0.02	3 18.5	8.24	10	+0.38	2 49.8	7.82	7	+0.01
1 37.5	8.55	20	+0.02	2 21.2	7.81	20	−0.03	2 36.0	8.09	10	−0.03	+4 34.3	7.28	2	0.00
1 30.7	8.56	20	−0.08	2 48.8	7.69	20	0.00	2 30.6	8.10	10	−0.10				
1 21.8	8.84	20	+0.02	3 31.8	7.48	19	0.00	2 20.2	8.35	10	+0.04	+ 8 3	7.06	2	−0.03
1 16.2	8.87	20	−0.07	4 56.3	7.13	4	−0.03	1 59.7	8.56	10	0.00	19 13	7.11	2	+0.02
1 8.3	9.06	20	+0.04	5 44.5	7.11	6	+0.02	1 47.4	8.64	10	+0.08	20 31	7.11	2	+0.02
1 1.2	9.08	20	−0.01	7 9	7.09	3	0.00	1 42.1	8.73	10	−0.03	31 13	7.07	3	−0.02
0 52.9	9.14	20	+0.01	9 18.5	7.10	4	+0.01	1 31.7	8.81	10	−0.05	33 15	7.08	3	−0.01
0 41.0	9.16	20	0.00	13 58.1	7.09	6	0.00	1 23.9	8.84	10	−0.04	44 23.5	7.06	2	−0.03
0 28.1	9.16	20	−0.01	16 26.8	7.05	5	−0.04	1 14.9	8.96	10	+0.01	46 27	7.11	2	+0.02
−0 16.9	9.23	20	+0.06	+20 16.2	7.09	5	0.00	− 1 5.3	8.95	10	−0.04	+47 26	6.96	1	−0.13

TABLE IX.

Time from Minimum	East (Autumn) Before	East (Autumn) After	West (Spring) Before	West (Spring) After	Time from Minimum	East (Autumn) Before	East (Autumn) After	West (Spring) Before	West (Spring) After	Time from Minimum	East (Autumn) Before	East (Autumn) After	West (Spring) Before	West (Spring) After
h m	M	M	M	M	h m	M	M	M	M	h m	M	M	M	M
5 40	7.09	7.09	. .	. .	3 30	7.39	7.49	7.82	7.55	1 40	8.49	8.40	8.78	8.56
20	7.10	7.11	. .	. .	20	7.44	7.54	7.86	7.61	30	8.65	8.65	8.87	8.68
5 0	7.12	7.15	7.60	. .	10	7.50	7.58	7.91	7.67	20	8.84	8.85	8.93	8.80
4 50	7.14	7.17	7.62	. .	3 0	7.57	7.64	7.97	7.74	10	9.00	9.00	8.97	8.92
40	7.16	7.21	7.64	. .	2 50	7.64	7.68	8.03	7.81	1 0	9.10	9.07	9.00	8.97
30	7.18	7.25	7.65	7.26	40	7.73	7.73	8.10	7.90	50	9.14	9.11	9.03	9.00
20	7.21	7.29	7.67	7.30	30	7.84	7.78	8.19	7.99	40	9.17	9.14	9.04	9.02
10	7.24	7.33	7.69	7.34	20	7.95	7.85	8.31	8.10	30	9.17	9.16	9.05	9.03
4 0	7.27	7.37	7.72	7.39	10	8.06	7.94	8.43	8.20	20	9.17	9.17	9.06	9.04
3 50	7.30	7.41	7.75	7.44	2 0	8.19	8.05	8.56	8.32	10	9.17	9.18	9.06	9.05
3 40	7.34	7.45	7.78	7.49	1 50	8.34	8.20	8.68	8.43	0 0	9.18	. .	9.06	. .

TABLE X.

E	Date	Gr. M.T.	⊙	O—C	W.	Obs.
		h m				
63	1880 Nov. 27	8 56.9	+3.7	+ 3.2	5	B
65	Dec. 2	8 45.3	+3.7	+12.1	4	K
112	1881 Mar. 29	12 47.0	−1.9	+10.2	2	K
114	Apr. 3	12 13.3	−2.1	− 3.2	5	K
118	13	11 40.8	−2.6	+ 4.8	4	K
124	28	10 31.5	−3.1	− 3.5	2	K
126	May 3	10 19.1	−3.3	+ 4.4	5	K
128	8	10 1.5	−3.4	+ 7.2	3	K
187	Oct. 2	11 49.8	+1.7	+ 5.7	4	K
193	17	10 40.3	+2.5	− 1.5	4	K
219	Dec. 21	6 27.5	+3.4	+13.2	1	K
234	1882 Mar. 18	12 22.0	−1.2	+ 2.2	3	K
262	Apr. 7	11 2.6	−2.3	+ 3.6	5	K
262	7	11 7.0	−2.3	+ 8.0	5	B
268	22	10 8.0	−3.0	+ 9.9	1	K
270	27	9 42.0	−3.0	+ 4.1	3	K
345	Oct. 31	8 40.6	+3.1	− 1.8	3	K
353	Nov. 30	6 41.0	+3.7	+ 2.2	1	K
394	1883 Mar. 2	12 30.2	−0.3	+ 6.9	4	K
398	12	11 47.6	−0.8	+ 4.8	4	K
402	22	11 5.3	−1.4	+ 2.9	4	K
406	Apr. 1	10 25.2	−2.9	+ 2.3	4	K
487	Oct. 20	8 30.5	+2.7	+ 3.9	4	K
540	1884 Feb. 29	11 31.6	0.0	+ 5.8	4	K
548	Mar. 20	10 15.3	−1.4	+10.1	3	K
690	1885 Mar. 9	9 49.2	−0.6	+ 1.0	3	B
690	9	9 50.6	−0.6	+ 2.4	3	b
692	14	9 33.9	−1.1	+ 5.2	3	K
694	19	9 9.8	−1.4	+ 0.9	5	K
753	Aug. 3	11 54.2	−1.4	+ 9.3	2	B
753	3	11 50.7	−1.4	+ 5.8	3	b
832	1886 Feb. 26	9 34.3	+0.2	+ 3.2	3	B
832	26	9 37.3	+0.2	+ 6.2	5	b
966	1887 Jan. 26	10 31.0	+2.1	− 3.1	3	K
970	Feb. 5	9 51.5	+1.5	− 2.2	4	B
970	5	9 53.6	+1.5	− 0.1	5	b
978	25	8 32.0	+0.3	+ 9.4	3	K
1078	Nov. 1	15 32.1	+3.0	+ 4.6	3	C
1080	6	15 17.0	+3.3	+12.3	5	C
1084	16	14 28.6	+3.5	+ 3.2	2	C
1086	21	14 4.0	+3.6	+ 1.1	5	C
1090	Dec. 1	13 35.9	+3.7	+14.2	5	C
1092	6	13 8.7	+3.7	+ 7.5	4	C
1096	16	12 30.2	+3.6	+ 9.9	4	C
1133	1888 Mar. 17	18 6.6	−1.1	−19.3	1	C
1141	Apr. 6	16 35.0	−2.2	− 9.8	5	C
1145	16	16 7.7	−2.7	+ 3.4	5	C
1216	Oct. 10	16 2.8	+2.2	+10.5	3	Y
1299	1889 May 5	13 57.6	−3.3	+12.0	1	Y
1378	Nov. 18	12 32.2	+3.5	+23.5	4	Y
1382	28	11 52.4	+3.7	+24.9	3	Y
1512	1890 Oct. 18	13 20.3	+2.6	+ 4.9	5	Y
1516	28	12 55.4	+3.0	+21.4	2	Y
1910	1893 July 6	17 24.9	−2.9	+11.2	4	Y
1914	16	16 37.1	−2.4	− 0.8	5	Y
1916	21	16 24.6	−2.0	+ 7.6	5	Y
1922	Aug. 5	15 36.9	−1.2	+21.9	3	Y
1924	10	14 55.7	−1.0	+ 2.0	5	Y
1926	15	14 36.6	−0.7	+ 3.4	5	Y
1930	25	13 43.6	−0.3	+ 8.2	4	Y
2111	1894 Nov. 19	18 48.5	+3.6	− 4.2	4	Y
2111	19	18 41.6	+3.6	−10.1	5	S
2117	Dec. 4	17 39.1	+3.7	−11.0	2	Y
2117	4	17 36.6	+3.7	−13.5	5	S
2123	19	16 39.0	+3.4	− 9.8	5	S
2137	1895 Jan. 23	14 20.3	+2.2	− 6.1	3	Y
2141	Feb. 2	13 34.3	+1.7	−11.6	3	S
2184	May 20	18 15.8	−3.6	−14.5	2	S
2194	June 14	16 25.7	−3.3	−21.8	5	Y
2279	1896 Jan. 12	14 4.3	+2.6	−15.6	2	S
2328	May 13	17 43.6	−3.5	−10.0	5	Y
2541	1897 Oct. 26	17 23.0	+2.9	+ 0.2	3	S
2813	1899 Sept. 4	18 52.1	+0.1	− 4.1	4	Y
2815	9	18 36.9	+0.4	− 1.5	4	Y
2817	14	18 16.1	+0.7	− 1.5	1	Y
2821	24	17 36.6	+1.3	+ 3.7	4	Y
2825	Oct. 4	16 41.0	+1.9	−10.3	4	Y
2829	14	16 12.4	+2.4	+ 2.6	4	Y
2983	1900 Nov. 2	13 53.7	+3.1	+ 2.8	5	Y
3001	Dec. 17	10 34.5	+3.5	−10.3	2	Sch.
3011	1901 Jan. 11	9 0.2	+2.9	− 2.7	5	Sch.
3011	11	8 46.9	+2.9	−16.9	1	P
3013	16	8 40.3	+2.5	− 2.7	2	P
3013	16	8 35.8	+2.5	− 6.9	5	Sch.
3015	21	8 16.8	+2.4	− 5.5	5	Sch.
3062	May 18	12 16.0	−3.6	−10.3	2	P
3064	23	11 54.3	−3.7	−11.6	3	P
3119	Oct. 7	14 47.1	+2.5	+10.9	4	Y
3135	Nov. 16	11 42.6	+3.5	− 8.1	5	Sch.
3141	Dec. 1	10 38.9	+3.7	−10.2	5	Sch.
3239	1902 Aug. 2	18 0.4	−1.5	+ 9.0	5	Y
3255	Sept. 11	15 16.7	+0.8	− 6.0	5	Y
3269	Oct. 16	12 38.9	+2.5	−18.3	3	Y
3275	31	11 53.6	+3.1	− 1.6	5	Sch.
3277	Nov. 5	11 28.6	+3.3	− 5.9	5	Sch.
3283	20	10 29.8	+3.6	− 2.8	5	Sch.

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NOMENCLATURE OF NEWLY DISCOVERED VARIABLE STARS,*

Provis. Notation A.N.	Name	Position of 1900.0 R.A.	Decl.	Prec. 1900 R.A.	Decl.	Chart-Place R.A.	Decl.	Magnitude Max.	Min.	
		h m s	° ′	s	′	h m s	° ′			
11.1903	*RU Andromedae*	1 32 47	+38 9.5	+3.49	+0.31	1 30 11	+37 55.6	9	13	*ph*
15.1903	*Z Cephei*	2 12 48	+81 13	+7.81	+0.28	2 7 6	+81 0	9.10	<13	*ph*
56.1903	*RR Cephei*	2 29 23	+80 42.3	+8.03	+0.27	2 24 15	+80 30.2	9	<13	*ph*
14.1902	*Z Persei*	2 33 40	+41 46.1	+3.81	+0.26	2 30 50	+41 34.3	9	12	*v*
22.1903	*X Camelopardalis*	4 32 36	+74 56	+7.68	+0.12	4 26 48	+74 50	9	13	*ph*
5.1903	*RS Tauri*	5 46 3	+15 51.3	+3.45	+0.02	5 43 28	+15 50.3	8.9	1011	*v*
1.1903	*Z Aurigae*	5 53 39	+53 18.0	+4.86	+0.01	5 50 3	+53 16.9	9	11	*v*
20.1903	*W Camelopardalis*	6 12 0	+75 32	+8.25	−0.02	6 5 48	+75 32	10.11	12	*ph*
14.1903	*RS Geminorum*	6 55 14	+30 39.8	+3.84	−0.08	6 52 21	+30 43.3	9.10	11.12	*ph*
9.1903	*Z Geminorum*	7 1 36	+22 41.0	+3.61	−0.09	6 58 53	+22 44.9	9.10	<12	*v*
16.1903	*RR Monocerotis*	7 12 27	+ 1 16.6	+3.10	−0.10	7 10 7	+ 1 21.2	9	<13	*ph*
13.1903	*RR Geminorum*	7 15 11	+31 4.2	+3.83	−0.10	7 12 18	+31 9.0	10	11.12	*ph*
21.1903	*Y Camelopardalis*	7 27 39	+76 16.9	+8.15	−0.12	7 21 30	+76 22.3	9.10	<11.12	*ph*
4.1902	*Y Geminorum*	7 35 16	+20 39.6	+3.53	−0.13	7 32 37	+20 45.3	8.9	–	*ph*
2.1903	*Y Draconis*	9 31 5	+78 18.2	+6.98	−0.27	9 25 47	+78 30.1	9	13	*ph*
3.1903	*W Ursae Maj.*	9 36 44	+56 24.6	+4.25	−0.27	9 33 32	+56 36.7	8	9	*v*
4.1903	*Z Draconis*	11 39 49	+72 49.0	+3.45	−0.33	11 37 12	+73 4.0	9.10	12.13	*ph*
57.1903	*T Ursae min.*	13 32 38	+73 56.4	+1.25	−0.31	13 31 42	+74 10.2	9	<13	*ph*
29.1903	*ST Herculis*	15 47 47	+48 47.1	+1.79	−0.18	15 46 27	+48 55.4	7.8	8.9	*v*
18.1902	*W Coronae*	16 11 50	+38 2.7	+2.14	−0.15	16 10 14	+38 9.6	7.8	13	*v*
31.1903	*SU Herculis*	17 44 42	+22 34	+2.52	−0.02	17 42 48	+22 35	10	<12	*ph*
76.1901	*RT Ophiuchi*	17 51 51	+11 10.9	+2.81	−0.01	17 49 45	+11 11.5	9	<10	*v*
19.1903	*RZ Lyrae*	18 39 54	+32 41.7	+2.23	+0.06	18 38 14	+32 39.1	10	11.12	*ph*
17.1903	*RY Lyrae*	18 41 15	+34 34.0	+2.17	+0.06	18 39 38	+34 31.4	10	12	*ph*
17.1902	*RW Lyrae*	18 42 7	+43 31.9	+1.82	+0.06	18 40 45	+43 29.2	9	<12	*ph*
10.1903	*RX Lyrae*	18 50 27	+32 42.3	+2.23	+0.07	18 48 46	+32 39.0	11	<15	*ph*
55.1903	*VW Cygni*	20 11 21	+34 11.8	+2.31	+0.18	20 9 37	+34 3.7	9.10	11.12	*ph*
21.1902	*V Sagittae*	20 15 46	+20 47.3	+2.65	+0.19	20 13 47	+20 39.0	9.10	13	*ph*
16.1902	*Z Delphini*	20 28 3	+17 6.2	+2.74	+0.20	20 26 0	+16 57.2	9	<11	*ph*
15.1902	*Y Delphini*	20 36 52	+11 30.9	+2.86	+0.21	20 34 43	+11 21.5	9.10	<13	*v*
58.1903	*VX Cygni*	20 53 34	+39 47.5	+2.26	+0.23	20 51 52	+39 37.2	9	9.10	*ph*
20.1902	*VV Cygni*	21 2 20	+45 22.6	+2.12	+0.24	21 0 45	+45 11.9	11	<12	*ph*
19.1902	*RT Pegasi*	21 59 49	+34 38.2	+2.61	+0.29	21 57 51	+34 25.3	9.10	13.14	*v*
. .	*RS Andromedae*	23 50 19	+48 4.9	+3.01	+0.33	23 48 4	+47 49.5	7.8	8.9	*ph*
12.1903	*Nova Geminorum*	6 37 49	+30 2.6	+3.83	−0.05	6 34 56	+30 5.0	5	–	–

* *From Supplement to Nos. 549–550.*

The Committee for the A.G. Catalogue of Variable Stars:
DUNÉR, HARTWIG, MÜLLER, OUDEMANS.

ERROR IN THE PLACE OF (15) *EUNOMIA* IN THE *JAHRBUCH* FOR 1905.

BY J. C. HAMMOND AND W. W. DINWIDDIE.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent U.S. Naval Observatory.]

On August 20, a plate was exposed on the 6-inch camera by Mr. DINWIDDIE for the purpose of finding the minor planet (41) *Daphne*. A trail was found on the plate which appeared much too bright for that asteroid. The direction of the trail also showed that the motion was entirely different from that of *Daphne*.

After observing it on the 12-inch equatorial on August 21 and 23, Mr. HAMMOND computed a circular orbit from his observations and found that the elements agreed very closely with those of (15) *Eunomia*, which had been searched for photographically by Mr. PETERS in the place given in the *Jahrbuch* for 1905, but without success.

The position of *Eunomia* was then computed by Mr. HAMMOND from the elements and tables of SCHUBERT and was found to agree closely with the observed position. The differences O − C are -48^s in α and $-23'.5$ in δ. The position for August 23 is, $\alpha = 21^h\,52^m\,54^s$; $\delta = +0°\,18'.9$. The daily motion is 1^m in α and $+0.3$ in δ. The time of opposition is August 20 instead of July 29 as given in the *Jahrbuch*.

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THE INSTRUMENTAL CONSTANTS IN EQUATORIAL WORK,

By C. W. FREDERICK.

[Communicated by Rear-Admiral C. M. Chester, Superintendent U.S. Naval Observatory.]

In order to eliminate the effect of the constants in equatorial work it is necessary to determine the parallel of the micrometer at the place of each observation. But with a knowledge of the instrumental constants, and a sufficient theory as to their influence in varying the orientation of the micrometer, we could adopt a fixed setting for the parallel from careful determinations made in favorable parts of the sky. Then, at a given place of observation, corrections due to the error of this parallel would be computed for the quantities measured. In the following is given a method for obtaining the constants, with formulas for the deviation of the parallel produced by them, and the construction of tables to facilitate the computations.

The constants may be determined from observations of circumpolar and equatorial stars.

Since λ *Ursae Minoris* and *Polaris* are about six hours apart in right-ascension, one is near culmination when the other is near elongation. The azimuth of the instrumental pole may be obtained from the star at culmination, and the altitude from the star at elongation.

Before making the observations we should determine the reading of the micrometer scale when the movable wire is in the optical axis of the telescope, or better, place the fixed wire in the optical axis, so that coincidence is the required reading. This adjustment may be accomplished by starting the driving clock and reversing the micrometer a few times on a suitable star. To observe for azimuth we clamp the telescope at one side of the pier so the declination axis will lie in the plane of the six-hour circle. Also clamp the micrometer in a horizontal position. Then point the telescope at the star near culmination. As the star moves slowly across the field set the movable thread of the micrometer immediately ahead of it, and note the time of transit, also note the reading of the scale. Two or more such transits should be taken to serve as a check on each other. Then reverse the telescope to the opposite side of the pier, and repeat the same operation. This completes the observation for azimuth. For the altitude the star near elongation is observed in a similar manner, with the telescope above the pier, and clamped so the declination axis will lie in the plane of the meridian. The micrometer should remain clamped in position angle, and not be disturbed during the observations.

The formulas for reducing these observations are easily obtained, and it is not necessary to give their derivation. The quantities observed may be designated as follows:

R_0 = the reading of the micrometer scale when the movable wire is in coincidence with the optical axis of the telescope.

R = the mean of the settings of the movable wire in any given position of the telescope.

θ = the sidereal time corresponding to R; mean of the transits.

The position of the telescope may be shown by indices E, W, or U (east, west, up); and the position of the micrometer by subscripts E, W, U, or D. These may refer to the positive direction of the micrometer scale, or to the micrometer head if it is opposite the zero of the scale. Thus $R^{\mathrm{E}}_{\mathrm{W}}$ is the mean of the scale readings taken when the telescope is east of the pier, and the micrometer clamped so the scale increases westward, or head west. Put

α_c, ρ_c = the apparent right-ascension and polar distance of the star at culmination;

α_e, ρ_e = the same for the star at elongation;

r = the refraction of the atmosphere at the altitude of the pole;

φ = the latitude of the observatory;

$\tau = \theta - \alpha$.

Let the constants be as follows:

η = the distance of the instrumental pole westward from the true pole measured along the six-hour circle; termed azimuth above.

ξ = the distance of the instrumental pole above the true pole measured along the meridian; termed altitude above.

For the inclination of the axes we construct a right triangle upon sp as an hypothenuse, and side i_1 adjacent the pole, measured eastward when the telescope is east of the pier. The deviation is the angle at s,

$$\lambda = +i_1 \sec\delta$$

This term becomes negative when the telescope is west of the pier.

For the collimation we construct a right triangle upon sp as an hypothenuse, and side c adjacent s, measured eastward when the telescope is east of the pier. The deviation is the complement of the angle at s,

$$\lambda = -c \tan\delta$$

This term becomes positive when the telescope is west of the pier.

The flexure of the declination axis need not be considered, as its component in right-ascension does not affect the parallel, and its component in declination is included in i_1.

The torsion of the tube we assume proportional to the component of gravity taken perpendicular to the plane of the tube and declination axis. When the telescope is east of the pier we have

$$\lambda = +\mu(\sin\varphi\cos\delta - \cos\varphi\sin\delta\cos\tau)$$

Substituting the observed quantity f for $\mu\sin\varphi$,

$$\lambda = +f\cos\delta - f\cot\varphi\sin\delta\cos\tau$$

The signs are changed in this expression when the telescope is west of the pier. If f' should be appreciable we should have, $\lambda = \pm f'\sin\tau$.

Now assume a fundamental parallel, or setting of the position-circle, p_m, the error of which we are to compute for any position of the telescope. It is most natural to take the parallel determined by the trail of an equatorial star along the fixed short wire of the micrometer when the telescope is east of the pier, and in the meridian. Then λ for this position should be zero. But taking the sum of the above equations when δ and τ are zero,

$$\lambda = i_1 - \eta + f$$

Therefore subtracting this expression from the sum of the above equations we have the deviation for the setting p_m.

Taking the sum of the terms which are independent of the position of the telescope,

(7a)

$$\lambda_1 = \eta - i_1 - f - \eta\sec\delta\cos\tau + (\xi\sec\delta - e\cos\varphi\tan\delta)\sin\tau$$

Taking the sum of the terms which change sign when the telescope is reversed in position,

(7b) $\lambda_2 = i_1\sec\delta - c\tan\delta + f\cos\delta - f\cot\varphi\sin\delta\cos\tau$

Then we have,

$$(7)\quad \begin{aligned} \lambda^E &= \lambda_1 + \lambda_2 \quad , \text{ for Telescope East of Pier} \\ \lambda^W &= \lambda_1 - \lambda_2 \quad , \text{ for Telescope West of Pier} \end{aligned}$$

These designations for the position of the telescope are not sufficiently general. But if we call the direction of increasing right-ascensions, east, for any given part of the sky at which the telescope is pointed, the designation becomes general.

In order to avoid laborious computing in the reduction of observations it is necessary to tabulate the values of λ^E and λ^W. It will be sufficient to compute them for every five degrees in declination, and every thirty minutes in hour-angle.

The value of p_m may now be easily deduced from observations made in any part of the sky, and either position of the telescope by adding λ from the table, and a correction for refraction, to the observed parallel. By taking many observations in different regions of the sky, in different positions of the instrument, and by different methods we should arrive at an accurate mean value of p_m, and incidentally test the theory of the constants. The stability of p_m would also appear with continued observing.

Adopting a value of p_m we are ready for the reduction of observations. For position angles we have

$$p_0 = p + \lambda - p_m \tag{8}$$

in which p_0 is the true, and p the observed angle. In satellite work where many observations are made in one neighborhood, p_m should be derived principally from parallel determinations made in the same neighborhood. In this case the parallel will be determined by the trail of a star on the long wire of the micrometer, and p_m will be reading of the position-circle less ninety degrees.

When observing by rectangular coordinates the position-circle is set at p_m, and $90° + p_m$. Then we have

$$\begin{aligned} \Delta\alpha'_0 &= \Delta\alpha' + \Delta\delta \ \sin\lambda \\ \Delta\delta_0 &= \Delta\delta - \Delta\alpha' \sin\lambda \end{aligned} \tag{9}$$

in which $\Delta\alpha'_0$, $\Delta\alpha'$, are the true and observed micrometer measurements in right-ascension, not reduced to the equator; $\Delta\delta_0$, $\Delta\delta$, are the true and observed measurements in declination. For this reduction the values of the natural sine of λ in units of the fifth decimal place should be tabulated for every five degrees in declination, and thirty minutes in hour-angle.

When $\Delta\alpha$ is determined by transits the formulas become

$$\begin{aligned} \Delta\alpha_0 &= \Delta\alpha + \tfrac{1}{15}\Delta\delta\sin\lambda\sec\delta \\ \Delta\delta_0 &= \Delta\delta \end{aligned} \tag{10}$$

The values of $\frac{1}{15}\sin\lambda\sec\delta$ should be tabulated for this reduction.

These instrumental corrections will be found of about the same magnitude as the corrections for differential refraction. But there is some degree of uncertainty about them, as there are mechanical imperfections of the instrument, and temperature disturbances, the effect of which cannot be computed. However, the action of the known constants is positive, and observations are improved with as much certainty by applying the above corrections as they are in the case of refraction.

The stability of the constants for the 26-inch equatorial may be seen from the following results:

Date	Temp. Fah.	η	i_1-c	ξ	c from stars	c from coll's	$\epsilon \cos\varphi$	$e \cos\varphi$ from stars	e from coll's	Adopted Constants
	°	″	″	″	″	″	″	″	″	″
1902 July 13	69	+116	−54	−71						$\eta = +115$
27	76	+113	−55	−73						$\xi = -72$
Aug. 3	80	+115	−51	−73						$i_1 - c = -54$
11	70	+115	−55	−75						$c = +113$
28	67	+121	−55	−74						$i_1 = +59$
Sept. 23	70	+115	−52	−73	. .	+113	. .	. .	. .	$e \cos\varphi = +5$
Oct. 22	51	+116	−52	−73	+114	. .	+94	. .	. .	
27	64	. .	. .	. .	. .	+113	. .	. .	+10	
Nov. 5	56	. .	. .	. .	. .	. .	. .	. .	+4	
28	35	+116	−52	−68	+107	+108	. .	. .	. .	
1903 Jan. 9	27	. .	. .	. .	+113	+119	+96	+2	−2	
9	20	+114	−57	−67	. .	. .	. .	. .	. .	
Feb. 17	14	+111	−56	−63	+115	. .	+95	+4	. .	
Mar. 13	55	+114	−50	−70	. .	. .	. .	. .	. .	
Apr. 21	48	+113	−55	−65	+113	. .	+98	+6	. .	
June 21	70	+113	−53	−74	+114	. .	+96	+5	. .	

The collimators used in the above determinations of c and e were attached to the dome in a vertical position, and sighted into each other by means of mirrors. They were very unstable, the images of the wires moving as much as ten or fifteen seconds vertically, and half as much horizontally during an observation.

Torsion of the Tube from Levels Placed on the Micrometer Box.

Date	Temp.	f	f'	ξ
	°	°	°	″
1903 May 5	. .	+0.021	+0.001	−97
26	64	+0.023	−0.001	−73
June 25	70	+0.017	−0.004	−81
26	68	+0.016	0.000	−80
1903 July 13	74	+0.022	+0.004	−94
13	75	+0.013	+0.002	−69
24	72	+0.014	0.000	−78
24	72	+0.017	+0.002	−79

Adopt $f = +0°.018$. For these determinations the levels were fastened to the micrometer box with lead wire. The position-circle was read by careful estimation to half-hundredths of a degree, the verniers reading only to fiftieths. The quality of the observations may be judged, aside from the agreement of the results in f, by the knowledge that f' should be zero, and the value of ξ about $-72''$.

ON THE SPIRAL CHARACTER OF THE NEBULOSITIES SURROUNDING γ *CASSIOPEAE*,

By J. M. SCHAEBERLE.

Half-hour exposure photographs of the region surrounding γ *Cassiopeae*, taken with the 13-inch reflector, show that in various position-angles and at various distances there are rows of stars connected by nebulous streams all of which seem to originate in γ *Cassiopeae*, and many of these streams plainly appear to return to the star, so that in their course other streams are crossed at various angles, producing appearances strongly suggestive of similar structures to be found in the Great Cluster in *Hercules*. Especially is this the case near the beginning of the 3d quadrant where, up to distances of more than 15′ from the central star, these intersections are so numerous that much confusion of detail exists in this region. There are isolated patches of nebulosity in various position-angles. The two known objects* lie on a heavy stream which seems to leave the central star near the middle of the 3d quadrant. With γ *Cassiopeae* on the optical axis all these patches are so far from the center of the plate that considerable uncertainty still exists for the regions which have not yet been photographed near the optical axis. With this axis half-way between the two known condensations, a broad nebulous spiral-like band can be traced from near the beginning of the second quadrant (where it is something more than a quarter of a degree from *Gamma*) through the brighter parts of the two known objects, on towards and nearly up to the naked-eye star in the 4th quadrant. Within 5′ of the optical axis this band appears to be made up largely of star-like condensations, each surrounded by a nebulosity which has a tendency to form outlines similar to the two neighboring nebulas; the width of this band varies from 1′ to 5′ or more. A second fainter band from 5′ to 7′ inside of and nearly parallel with the one just described can be traced from the middle of the 1st quadrant well into the 4th quadrant; its outline is somewhat

* First photographed by Barnard and Wolff.

irregular, the average width being about 1′, and the distance from *Gamma* 15′±.

A composite drawing, made with the aid of many negatives having different pointings, covering the whole region within half a degree of γ *Cassiopeae* will be required before definite conclusions can be drawn from photographs having such a limited field of view. During the past month but one night was suitable for work with this instrument, and owing to local conditions observations can only be made near the meridian.

Ann Arbor, *December* 16, 1903.

ON THE PHYSICAL STRUCTURE OF THE GREAT CLUSTER IN *HERCULES*,

BY J. M. SCHAEBERLE.

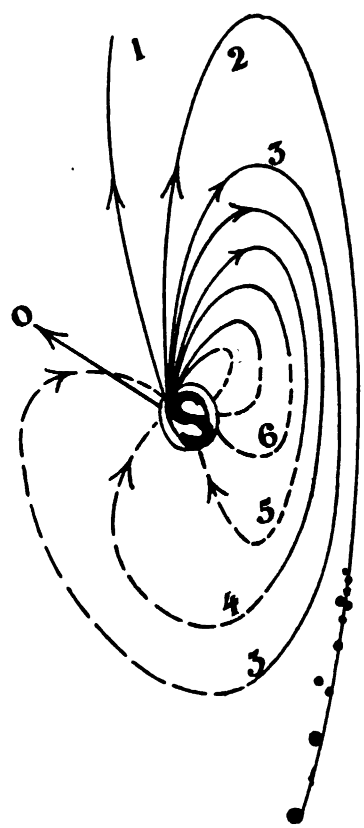
Several months ago a few photographs of the cluster in *Hercules* were taken with the 13-inch reflector; these negatives were found to give unmistakable evidence of a spiral structure in this object. Nebulous streams joining certain stars in curved lines could be traced up to the very center of the cluster. The puzzling feature, however, was that there seemed to be two spirals, the more pronounced one being clock-wise, the other counter clock-wise. As certain other observations were demanding attention at that time, the matter was laid aside with the intention of making this cluster a special study when it again got into good position for observation. The discovery, however, that a precisely similar structure on a much larger scale exists in the stars and nebulosity surrounding γ *Cassiopeae*,* led me again to examine the above mentioned negatives with the result that the physical structure of this cluster seems to be very simple.

The clock-wise spiral is formed by the inner streams of *outgoing* matter; the seeming counter clock-wise spiral is formed by that part of each stream which contains *returning* matter. The plane of the spiral is not normal to the line of sight.

My interpretations of these negatives can, perhaps, best be followed with the aid of the accompanying figure roughly deduced from the photographs. To avoid confusion of detail, streams from only one branch are represented; the broken lines refer to the seeming counter clock-wise spiral. For purposes of approximate orientation a few conspicuous star-images are inserted.

Streams with the greatest velocity (and consequently least density) have the initial direction 0. All outgoing streams between 0 and 2 have velocities still too great to form closed curves. Individual masses in any of the streams 3 to 8 describe orbits in which the minor axes are nearly zero. The periodic time for masses in stream 3, for instance, is about equal to three-fourths of the time required for the central mass to make one complete rotation on its axis.

The slower moving masses in streams with large initial inclinations to the normal will describe orbits of less eccentricity; a number of these are seen to encircle the two branches (in the shape of the letter S) near the origin; and as the lower branch is less dense than the upper, these encircling streams give a horse-shoe-like outline to this rather conspicuous central nebulosity; they also form the outer boundary of the *lanes* which Lord ROSSE first observed, the inner boundary being formed by the upper part of the central figure S. The third, nearly radial lane, is included between streams 1 and 2, returning streams also play an important part in the formations at this place. A similar, less conspicuous structure exists on the other branch.



The star-like masses in this cluster are probably of various shapes and have various velocities of axial rotation; this seems to be indicated by the variations in brightness which some of these objects are known to undergo.

If two *dot* tracings of the figure are made (the distances between the dots (star images) increasing with the distance from the origin, measured on the curve) and one of these placed upside down against the other — thus giving the

* In the opinion of the writer a majority of the stars—both bright and faint—within half a degree of γ*Cassiopeae* belong to a single physical system.

combination for both branches — a resemblance between the transparency and the original cluster will be recognized.

It is hoped that a negative suitable for enlargement, showing the essential features, will be obtained when this cluster is again in a favorable position for observation. It is quite probable that a long exposure taken with a suitable instrument will show that the whole cluster is but the central condensation, made up of the heavier masses, of a great spiral nebula.

Ann Arbor, November 20, 1903.

OBSERVATIONS OF COMETS AND MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By THEO I. KING.

[Communicated by Rear-Admiral C. M. Chester, U.S.N., Superintendent.]

Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					Comet *b* 1902 III (Perrine).					
1902 d h m s			m s	′ ″	h m s	° ′ ″			s	″
Sept. 4 14 12 13.7	1–2	12 , 6	−0 38.20	+ 0 2.0	3 14 2.04	+36 13 18.2	*n*9.4773	0.0000	+4.07	+ 1.2
5 13 47 32.9	3	12 , 6	−0 30.74	+ 9 43.8	3 12 45.71	+36 40 56.2	*n*9.5341	0.0544	+4.11	+ 1.4
6 11 38 11.4	4	8 , 4	−0 47.44	− 5 4.6	3 11 27.73	+37 6 53.8	*n*9.7255	0.4523	+4.17	+ 1.4
7 12 43 37.7	5	13 , 7	+0 1.86	+ 2 6.4	3 9 47.47	+37 38 39.8	*n*9.6468	0.2233	+4.24	+ 1.7
					Comet *a* 1903 I (Giacobini).					
1903 Feb. 26 7 7 36.6	6	19 , 4	+4 21.33	+ 2 11.3	23 55 28.25	+14 20 15.0	9.6711	0.7110	−0.20	+ 0.6
Mar. 3 7 18 57.8	7	28 , 6	+2 32.75	+ 3 35.4	0 5 41.54	+16 5 32.8	9.6773	0.7198	−0.17	0.0
4 7 19 44.2	8	33 , 9	−0 25.82	+ 0 36.9	0 7 43.82	+16 23 42.7	9.6780	0.7210	−0.16	− 0.1
12 7 4 31.0	9	21 , 7	+0 1.66	− 6 59.6	0 21 58.84	+17 15 1.0	9.6800	0.7205	−0.14	− 0.8
13* 6 45 38.6	10	5 , 1	−1 25.59	− 3 0.2	0 23 14.56	+17 1 26.8	9.6785	0.7191	−0.14	− 1.0
					(42) *Isis*.					
1902 July 2 13 2 50.5	11	6 , 8	+1 36.01	− 1 10.9	20 16 9.10	−27 30 11.7	*n*8.8552	0.9037	+3.75	+19.8
					(6) *Hebe*.					
1903 Apr. 17 12 14 31.8	12	28 , 6	+0 5.84	+13 41.2	13 40 25.58	+11 5 7.7	8.4756	0.6107	+2.29	− 9.9
21 11 14 23.3	13	30 , 6	−0 49.23	−13 31.7	13 37 0.21	+11 28 21.2	*n*8.7292	0.6057	+2.30	− 9.6
27 12 1 29.4	14	30 , 6	−1 7.04	− 0 12.3	13 31 54.99	+11 57 2.9	9.0012	0.6012	+2.31	− 8.9
28 10 1 5.8	14	39 , 6	−1 51.83	+ 3 23.9	13 31 10.20	+12 0 39.2	*n*9.1259	0.6003	+2.31	− 8.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	3 14 36.17	+36 13 15.0	B.D. +36°680 †	8	0 8 9.80	+16 23 5.9	Berlin, A.G., A, 30
2	3 14 7.52	+36 20 4.9	Lund, A.G. 1719	9	0 21 57.32	+17 22 1.4	Berlin, A.G., A, 100
3	3 13 12 34	+36 31 11.0	Lund, A.G. 1712	10	0 24 40.29	+17 4 28.0	Berlin, A.G., A, 119
4	3 12 11.00	+37 11 57.0	Lund, A.G. 1704	11	20 14 29.34	−27 29 20.6	Gould Gen. Catal. 27849
5	3 9 41.37	+37 36 31.7	Lund, A.G. 1680	12	13 40 17.45	+10 51 36.4	Leipzig, A.G., I, 4923
6	23 51 7.12	+14 18 3.1	Leipzig, A.G., I, 9490	13	13 37 47.14	+11 42 2.5	Leipzig, A.G., I, 4914
7	0 3 8.96	+16 1 57.4	Berlin, A.G., A, 11	14	13 32 59.72	+11 57 24.1	Leipzig, A.G., I, 4893

* This observation was made by Mr. J. C. Hammond, and could not be completed on account of clouds.

† Micrometrical comparison with *2.

PERIOD OF 320 *U CEPHEI*,

By S. C. CHANDLER.

The appearance of Mr. Yendell's paper on the light-curve of *U Cephei*, in the last number of this journal, makes timely the following remarks on its period. A new investigation of this has been made within the past year. For this calculation I reduced anew all the series of observations whose details were available in such form as to enable me to determine the times of minima on a homogeneous plan, independent as possible of the peculiarities in the form of the light-curve. This object seemed to be best secured by adopting as the point of reference, or in-

stant of normal minimum, the mean of the times, before and after minimum, when the star is near the middle of its variation, and fluctuating most rapidly. The points selected for this use were the magnitudes 8.3 and 8.6. The series so treated embraced the observations of KNOTT, WILSING, YENDELL, SPERRA, and my own. The results were grouped to form normals, as follows:

No. Min.	Mean E	Wt.	O—C	No. Min.	Mean E	Wt.	O—C
			m				m
3	55	2	+0.6	3	1139	5	−0.8
5	122	6	+1.3	1	1216	2	+8.9
4	199	6	+2.8	2	1514	1	+7.0
6	258	9	−1.2	7	1920	7	+4.4
4	337	4½	−2.4	5	2117	9	−2.6
5	400	8	−3.5	3	2154	4	−3.5
5	471	9½	−5.0	1	2328	1	−11.3
4	546	5	−6.1	5	2821	5	+3.0
2	692	3	−0.8	1	2983	1	+7.3
2	972	4	−7.4	4	3246	3½	−4.8
7	1086	10½	+9.8				

The column O−C shows the representation of the normal observed times by the elements,*

1880 June 23 $7^h\ 46^m.0$ (Gr.) + $2^d\ 11^h\ 49^m\ 44^s.55$ E

It may be concluded from this comparison that during the twenty-four years' interval since 1880 there has been no sensible deviation from a uniform period. On the contrary, these elements give, for the minimum indicated by SCHWERD's observations in 1828 (Epoch −7636), a deviation, O−C = $+947^m$. Unless, therefore, we are prepared to reject this indication, the period cannot be constant. I have discussed the circumstances relating to SCHWERD's data in *A.J.* IX, p. 50. For the present we must give over the attempt to ascertain the nature of the inequality of the period, and rest on the value of the above elements.

* These values differ very slightly from those used by YENDELL in *A.J.* 551 (p. 214), (1880 June 23 $7^h\ 43^m.5$ + $2^d\ 11^h\ 49^m\ 44^s.7$ E), which were furnished him before I had incorporated some of the recent observations.

EPHEMERIS OF WINNECKE'S COMET FOR THE APPEARANCE OF 1903–1904.

[BY C. HILLEBRAND, FROM *A.N.* 3916.]

For Berlin Midnight.

1904	App. α h m s	App. δ ° ′ ″	log r	log Δ	1904	App. α h m s	App. δ ° ′ ″	log r	log Δ
Jan. 0	17 30 51.40	−17 46 46.7	9.988836	0.272241	Jan. 22	19 23 28.07	−20 53 18.6		
1	35 47.19	18 0 34.6			23	28 39.41	20 55 18.0		
2	40 44.73	18 13 56.4			24	33 50.27	20 56 41.8	9.966004	0.271006
3	45 43.71	18 26 51.5			25	39 0.52	20 57 29.8		
4	50 44.16	18 39 19.1	9.981012	0.270012	26	44 10.08	20 57 42.6		
5	17 55 46.06	18 51 17.6			27	49 18.85	20 57 20.6		
6	18 0 49.31	19 2 49.2			28	54 26.72	20 56 24.0	9.968430	0.273468
7	5 53.80	19 13 50.2			29	19 59 33.56	20 54 52.7		
8	10 59.47	19 24 20.8	9.974601	0.268606	30	20 4 39.30	20 52 47.4		
9	16 6.23	19 34 19.9			31	9 43.88	20 50 8.8		
10	21 14.02	19 43 47.6			Feb. 1	14 47.17	20 46 57.1	9.972633	0.276601
11	26 22.67	19 52 43.1			2	19 49.09	20 43 12.5		
12	31 32.15	20 1 5.7	9.969782	0.268027	3	24 49.57	20 38 55.9		
13	36 42.33	20 8 54.7			4	29 48.53	20 34 8.0		
14	41 53.11	20 16 10.0			5	34 45.89	20 28 49.3	9.978474	0.280344
15	47 4.38	20 22 51.3			6	39 41.55	20 23 0.2		
16	52 16.03	20 28 58.1	9.966685	0.268248	7	44 35.45	20 16 41.5		
17	18 56 27.91	20 34 29.7			8	49 27.55	20 9 54.2		
18	19 2 40.09	20 39 26.2			9	54 17.77	20 2 38.9	9.985794	0.284645
19	7 52.27	20 43 47.7			10	20 59 6.02	19 54 56.3		
20	13 4.39	20 47 33.7	9.965417	0.269255	11	21 3 52.26	19 46 47.2		
21	19 18 16.37	−20 50 43.9			12	21 8 36.47	−19 38 12.6		

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